CS341 Automata Theory

Final Exam Review Sheet - Solutions

1. For each of the following languages, determine if the language is (I) regular, (II) context-free but not regular, (III) not context-free. Prove your answer.

   (a) $L = \{a^n b^{n+2} | n \geq 0\}$
   (II) CF, not regular
   CF: $S \rightarrow aSb|bb$
   not regular: Apply pumping lemma to $w = a^p b^{p+2}$ and pump either way.

   (b) $L = \{xay | x, y \in \{a, b\}^+\}$
   (I) This language contains all strings over $\{a, b\}$ that have length of at least 3 and contain at least one a other than the first and last symbols. $(a \cup b)^+ a (a \cup b)^+$

   (c) $L = \{0^n 1^{m+n} 2^{m+n} | m, n \geq 0\}$
   (III) not CF. Apply pumping lemma to $w = 0^p 1^{2p} 2^{2p}$

2. Give both a CFG and a PDA for the following languages.

   (a) $L = \{a^k c^3 b^r | r > k\}$
   CFG: $S \rightarrow aSb|B$
   $B \rightarrow bB|bC$
   $C \rightarrow ccc$

   (b) $L = \{w \in \{0, 1\}^* | w = w^R\}$

   (c) $L = \{w \in \{a, b\}^* | |w|$ is odd and the middle symbol of w is a$\}$

3. Give a regular expression and a FA for the following languages.

   (a) $L = \{w \in \{0, 1\}^* | w$begins with a 1 and ends with a 0$\}$

   (b) $L = \{w \in \{0, 1\}^* |$w contains substrings 010 and 101$\}$

4. Give the state diagram of a recognizer for the following.

   (a) $L = \{a^i b^i c^j | i, j \geq 0, j > i\}$

   (b) $L = \{0^n 1^{2n+1} | n \geq 0\}$

5. Define a TM that computes the function f where $f(x) = 2x + 1$.

6. Prove that the decidable languages are closed under complement.

7. Prove that $L = \{< M > |$ M is a TM and M accepts all strings of length less than 5 in 100 steps or less$\}$ is decidable.

8. Assuming only that A_{TM} is not decidable, prove that $HALT_{TM}$ is not decidable.

9. Prove that $\{< M > | M$ is a TM and if M accepts any string w, then M also accepts $w^r\}$ is not decidable.

10. Prove the following lanuages are SD but not decidable:

    (a) $L = \{< M > | M$ is a TM and M accepts at least 2 strings $\}$

    (b) $L = \{< M_1, M_2 > | M_1, M_2$ are TMs and $aa \in L(M_1) \cup L(M_2)\}$