CS341 Automata Theory

Summer 2007
Midterm
June 21, 2007

Name:
EID:

1. For each of the following languages, state whether the language is (I) regular, or (II) context-free but not regular. Prove your answer.

   (a) $L = \{wa^n a^n w^R | w \in \{a,b\}^* \text{ and } n > 0\}$

   (II)
   Not regular: Apply regular language pumping lemma with $w = b^p aab^p$.

   context-free:
   $S \rightarrow aSa|bSb|Y$
   $Y \rightarrow aYa|aa$

   (b) $L = \{xyx^R | x \in \{a,b\}^+, y \in \{a,b\}^*\}$

   (I)
   L contains all strings of a's and b's of length 2 or more that begin and end with the same symbol.
   Regular expression: $a(a \cup b)^* a \cup b(a \cup b)^* b$

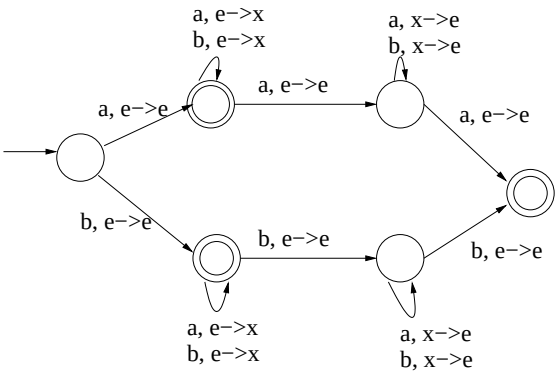
   (c) $L = \{x\#y | x \in \{0,1\}^*$ and when x and y are considered as binary numbers, $x = 2y\}$ (example: $10\#1 \in L$ since $2 = 2(1)$, and $100\#0010 \in L$ since 4=2(2)).
   This was actually a typo - this language is not context-free.

2. Is $L = \{0^m 1^n | m \neq 2n\}$ regular or non-regular? Prove your answer.

   Show that $L' = \{0^m 1^n | m = 2n\}$ is not regular using a pumping lemma proof (e.g., apply the pumping lemma with $w = 0^{2p}1^p$). Then use closure properties to show that L is also not regular.

3. Consider the language $L = \{w \in \{a, b\}^* || w| \geq 1$ and the first, middle and last symbols in w are the same$\}$

   (a) Give the state diagram of a pushdown automaton that recognizes L. Do **not** use the conversion algorithm to convert your CFG in b) to a PDA.

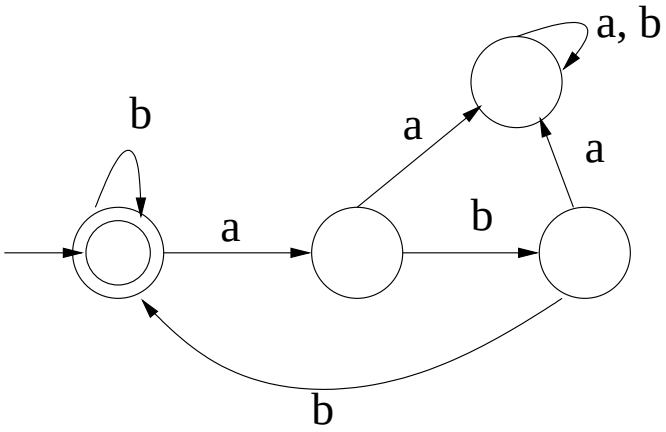


Note: e stands for ε

   (b) Define a context-free grammar that generates L.
   $S \rightarrow aAa|bBb$
   $A \rightarrow aAa|aAb|bAa|bAb|a$
   $B \rightarrow aBa|aBb|bBa|bBb|b$

4. Consider the language $L = \{w \in \{a, b\}^* |$ every a in w is followed immediately by the string $bb\}$

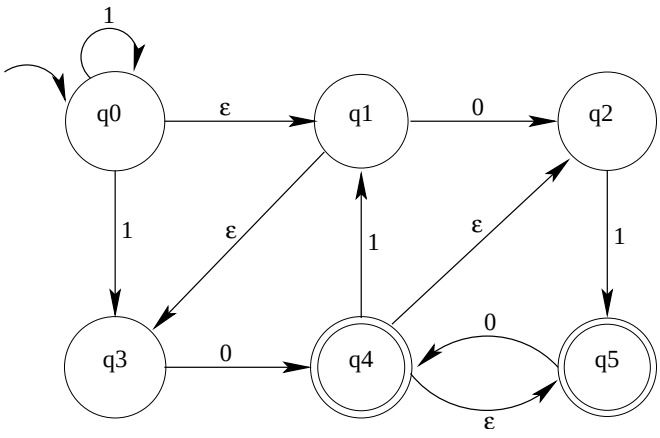
   (a) Draw the state diagram of a deterministic finite automaton that recognizes L.



   (b) Write a regular expression that describes L. Do not ues the conversion algorithm to convert your FA in a) to a regular expression. $(b \cup abb)^*$

5. For the following NFA N, use the construction we discussed in class to produce an equivalent DFA D. Draw D's state diagram. You must use the algorithm presented in

class and show your work to receive credit.



6. Let L be a regular language over $\{0, 1\}$. Is it possible for the language $PLUS(L) = \{xx | x \in L\}$ to be non-regular? Prove your answer.
   Yes. Let $L = (0 \cup 1)^*$. Then $PLUS(L) = \{xx | x \in \{0, 1\}^*\}$, which is not regular (apply the pumping lemma with $w = 0^p 1^p 0^p 1^p$).