

CS341 Automata Theory

Midterm 2 Review

Covered topics include:

All material on CFLs: CFGs, PDAs, Chomsky Normal Form, ambiguity of a grammar, closure properties of a CFL, decision procedures for CFLs, pumping lemma for CFLs, conversion of a CFG to a PDA.

TMs: definition of semi-decidable/decidable languages, configurations of a TM, defining recognizers and deciders, defining TM that computes a function on integers.

1. For each of the following languages, state whether the language is (I) regular, (II) context-free but not regular, or (III) not context-free. Prove your answer.

(a) $\{0^n 0^{2n} 0^{3n} \mid n \geq 0\}$

(b) $\{xwx^R \mid x, w \in \{0, 1\}^+\}$

(c) $\{xyz \mid x, y, z \in \{0, 1\}^*, |x| = |z| > 0 \text{ and } \#_0(x) \geq \#_0(z)\}$

(d) $\{1^{n^2} \mid n \geq 0\}$

(e) $L(11(01 \cup 0)^*(01 \cup 0) \cup 10)$

(f) $\{w\#w^R \mid w \in \{a, b\}^*\}$

(g) $\{w \mid w \in \{a, b\}^*, w \neq w^R\}$

(h) $\{a^i b^j c^k \mid i < j \rightarrow k < j\}$

2. Prove that the context-free languages are closed under union and concatenation. (Don't copy your class notes - this should be good practice for proving a closure property on the exam).
3. Suppose L_1, L_2 are context-free languages. Is it necessarily true that $L_1 - L_2$ is context-free? Prove your answer.

4. Convert the following grammar to Chomsky Normal Form. $S \rightarrow A|B$

$$A \rightarrow aA|aC$$

$$B \rightarrow bB|bC$$

$$C \rightarrow pqrW$$

$$W \rightarrow TV$$

$$T \rightarrow t|epsilon$$

5. Show that the following CFG is ambiguous:

$$E \rightarrow E + E \mid E - E \mid ExE \mid (E) \mid a \mid b.$$

6. For CFG G and string w , give an algorithm to decide if $w \in L(G)$.

7. Define a PDA that recognizes $\{0^n w 1^m \mid w \in \{0, 1\}^*, \#_0(w) = \#_1(w) \text{ and } m \geq n\}$.

8. Define a CFG for $L = \{a^n b^m \mid n \neq m\}$.
9. Define a recognizer for:
- (a) $L = \{a^n b^m c^m\}$
 - (b) $L = \{w \in \{0, 1\}^* \mid \#_0(w) = 2\#_1(w)\}$
 - (c) $L = \{tat \mid t \in \{b, c\}^*\}$
10. For the languages in the previous problem, are they: (I) regular, (II) context-free but not regular, (III) semi-decidable but not context-free? Prove your answer.