Problem Set 4

Randomized Algorithms

Due Tuesday, October 31

1. [Karger] Suppose you are given a graph whose edge lengths are all integers in the range from 0 to B. Suppose also that you are given the all-pairs distance matrix for this graph (it can be constructed by a variant of Seidel's deterministic distance algorithm). Prove that you can identify the (successor matrix representation of the) shortest paths in $O(B^2 MM(n) \log^2 n)$ time, where $MM(n)$ is the time to multiply $n \times n$ matrices.

2. Let S be an unknown set of n items (with n known). Suppose that you receive a sample T of k items chosen from S uniformly at random *without replacement*. Show how to construct a sample T' of k items from S, whose distribution is identical to a uniform sample of k items from S drawn *with replacement*.

3. In class we presented an efficient randomized algorithm for bipartite matching on d-regular graphs.

   (a) What goes wrong if the graph is not d-regular?

   (b) Additionally, we showed that the algorithm achieves $O(n \log n)$ time in expectation. Show an algorithm that achieves $O(n \log n)$ time with high probability. **Hint:** it may help to recall how we showed that the coupon collector takes $O(n \log n)$ samples with high probability.

4. Consider the example given in class for how online bipartite matching using random edges achieves a competitive ratio of $R = 1/2$: each arriving vertex x_i has an edge to y_i as well as all of $y_{n/2}, \ldots, y_n$. Show

that the algorithm that the algorithm given in class, which randomly ranks the right vertices y_i, has $R \leq 3/4$ on this example.