Brief Announcement: Consistent Fixed Points and Negative Gain

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Abstract. We discuss the stabilization properties of networks that are composed of "displacement elements". Each displacement element is defined by an integer K, called the displacement of the element, an input variable x, and an output variable y, where the values of x and y are non-negative integers. An execution step of this element assigns to y the maximum of 0 and $K + x$. The objective of our discussion is to demonstrate that two principles play an important role in ensuring that a network N is stabilizing, i.e. starting from any global state, network N is guaranteed to reach a global fixed point. The first principle, named consistent fixed points, states that if a variable is written by two subnetworks of N, then the values of this variable, when these two subnetworks reach fixed points, are equal. The second principle, named negative gain, states that the sum of displacements along every directed loop in network N is negative.

1 Introduction

A network of communicating elements is stabilizing iff, starting from any global state, the network is guaranteed to reach a fixed point [1]. The theory of network stabilization, though highly studied, has not yet identified the general principles which can explain the stabilization of rich classes of networks. We identify two such general principles, and illustrate their utility by showing how these principles can be used to design interesting classes of stabilizing networks. In our study, we focus on a simple model, called displacement networks.

A displacement network consists of one or more displacement elements. Each element is defined by an integer K, called the displacement of the element, an input variable x, and an output variable y, where the values of x and y are non-negative integers. The execution step of this element assigns to y the maximum of 0 and $K + x$ (provided that this assignment changes the value of y). Later, we generalize the model to allow elements with multiple inputs.

We propose two general principles to adopt while designing a stabilizing displacement network N: consistent fixed points and negative gain. The principle of consistent fixed points states that, if a variable is written by two or more subnetworks of N, then the values of this variable written by these subnetworks, when

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these subnetworks reach fixed points, are equal. The principle of negative gain states that the sum of the displacements along every directed loop in network N is negative.

These two principles have counterparts in control theory. Specifically, the principle of consistent fixed points is analogous to the requirement that a control system be free from self-oscillations; the principle of negative gain is analogous to the requirement that the feedback loop of a control system be negative.

2 Results

1. An acyclic network N is stabilizing iff every two chains, that terminate at the same variable in N, are consistent.
2. Given a particular assignment of input values, a stabilizing acyclic network has exactly one fixed point.
3. A loop network is stabilizing iff one of the following two conditions holds:
   - the total gain of the loop is negative.
   - the total gain of the loop is zero and the loop has no more than two elements.
4. A stabilizing loop network has exactly one fixed point.
5. A composite network (composed of stabilizing acyclic and loop networks) is stabilizing iff it is consistent.
6. If the gain of each directed loop in an m-bow network N is negative, then N is stabilizing.
7. A composition of a stabilizing loop or m-bow network with a stabilizing acyclic network, is stabilizing if every variable shared by both components is an input variable of the acyclic network.

3 Concluding Remarks

Our two principles, of consistent fixed points and negative gain, are seen to be sufficient (and sometimes also necessary) to establish stabilization of many classes of networks. It would be interesting to study whether these principles remain sufficient to ensure self-stabilization if we allow a richer network model, in which an element can perform any linear operation on its inputs and output the result.

Reference

1. Dijkstra, E.W.: Self-stabilizing systems in spite of distributed control. Commun. ACM 17(11), 643–644 (1974)