

# Lecture 10: Stereo

Tuesday, Oct 2

## Grad student extension ideas for problem set 2

- Implement textons approach for texture recognition [Leung & Malik]
  - Possible data sources: Vistex, Curet databases
- Build a shape-based object detector using the generalized Hough transform
- Clustering approach to video shot boundary detection
- Build a deformable contour tracker

# Exam

- Next Tuesday, Oct 9, in class
- Bring one handwritten 8.5 x 11", one-sided sheet with any notes
- Closed book/laptop/calculator

# Review all material covered so far

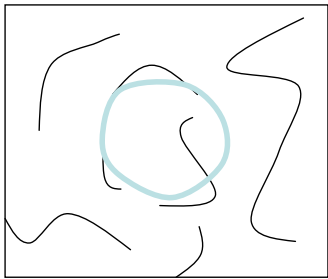
- Image formation
  - Perspective, orthographic projection properties, equations, effects
  - Pinhole cameras
  - Thin lens
  - Field of view, depth of field
- Color
  - BRDF
  - Spectral power distribution
  - Color mixing
  - Color matching
  - Color spaces
  - Human perception
- Binary image analysis
  - Histograms and thresholding
  - Connected components
  - Morphological operators
  - Region properties and invariance
  - Distance transform, Chamfer distance
- Filters
  - Application/effects of
  - Convolution properties
  - Noise models
  - Mean, median, Gaussian, derivative filters
  - Separability
- Edges, pyramids, sampling
  - Image gradients
  - Effects of noise
  - Derivative of Gaussian, Laplacian filters
  - Canny edge detection
  - Corner detection
  - Sampling and aliasing
  - Pyramids – construction and applications
- Texture
  - Analysis vs. synthesis
  - Representations
- Grouping
  - Gestalt principles
  - Clustering: agglomerative, k-means, mean shift, graph-based
  - Graphs and affinity matrices
- Fitting
  - Hough transform
  - Generalized Hough transform
  - Least squares
  - Incremental line fitting, k-means
  - Robust fitting: RANSAC, M-estimators
  - Deformable contours, energy functions
- Stereo vision

# Outline

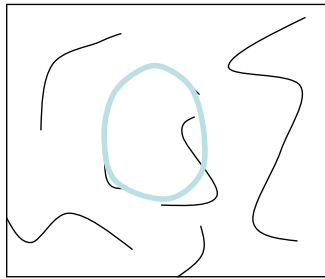
- Brief review of deformable contours
- Fundamentals of stereo vision
- Epipolar geometry

# Last time: deformable contours

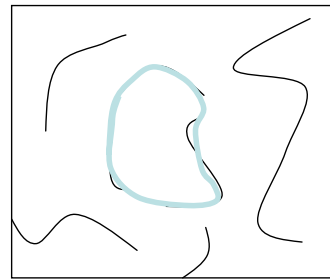
a.k.a. active contours, snakes



initial



intermediate



final

# Snake energy function

The total energy of the current snake defined as

$$E_{total} = E_{in} + E_{ex}$$

Internal energy encourages smoothness  
or any particular shape

External energy encourages curve onto  
image structures (e.g. image edges)

Internal energy incorporates **prior knowledge** about object boundary, which allows a boundary to be extracted even if some image data is missing

We will want to iteratively *minimize* this energy for a good fit between the deformable contour and the target shape in the image

# Discrete energy terms

- If the curve is represented by  $n$  points

$$v_i = (x_i, y_i) \quad i = 0 \dots n-1$$

$$\frac{dv}{ds} \approx \frac{v_{i+1} - v_i}{2} \quad \frac{d^2v}{ds^2} \approx (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - 2v_i + v_{i-1}$$

$$E_{in} = \sum_{i=0}^{n-1} \alpha |v_{i+1} - v_i|^2 + \beta |v_{i+1} - 2v_i + v_{i-1}|^2$$

Elasticity,  
Tension;  
Want to favor close points

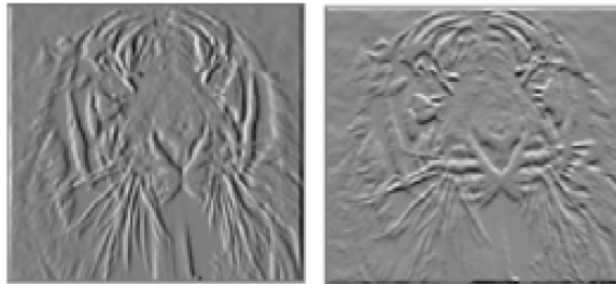
Stiffness  
Curvature;  
Want to favor smoothly shaped  
curve (not corners)



## Discrete energy terms

- An external energy term for a (discrete) snake based on image edge

$$E_{ex} = - \sum_{i=0}^{n-1} |G_x(x_i, y_i)|^2 + |G_y(x_i, y_i)|^2$$



# Energy minimization

- Many algorithms proposed to fit deformable contours
  - Greedy search
  - Gradient descent
  - Dynamic programming (for 2d snakes)

## Problems with snakes

- Depends on number and spacing of control points
- Snake may oversmooth the boundary
- Not trivial to prevent curve self intersecting

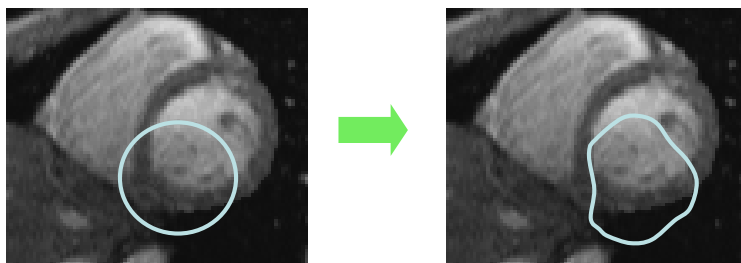


- Cannot follow topological changes of objects



## Problems with snakes

- May be sensitive to initialization, get stuck in local minimum



- Accuracy (and computation time) depends on the convergence criteria used in the energy minimization technique

# Problems with snakes

- External energy: snake does not really “see” object boundaries in the image unless it gets very close to it.

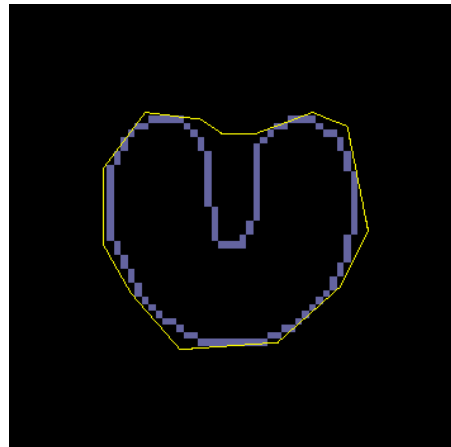
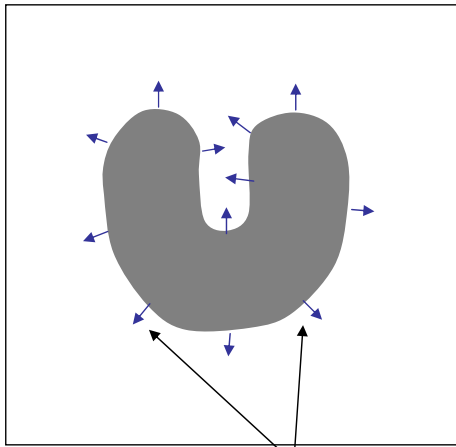
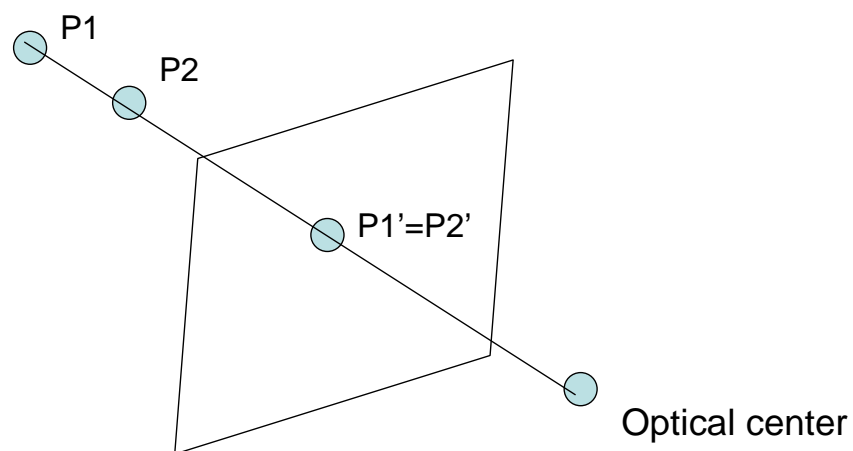


image gradients  $\nabla I$   
are large only directly on the boundary



## Depth unavailable in single views

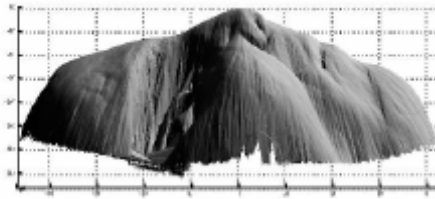


What cues can indicate 3d shape?

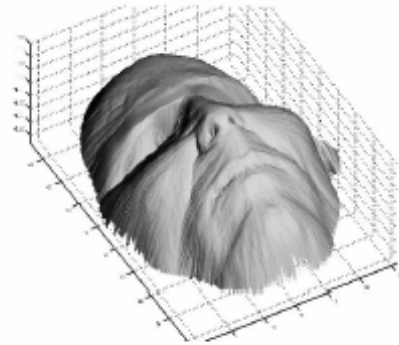
# Shading



a)



b)

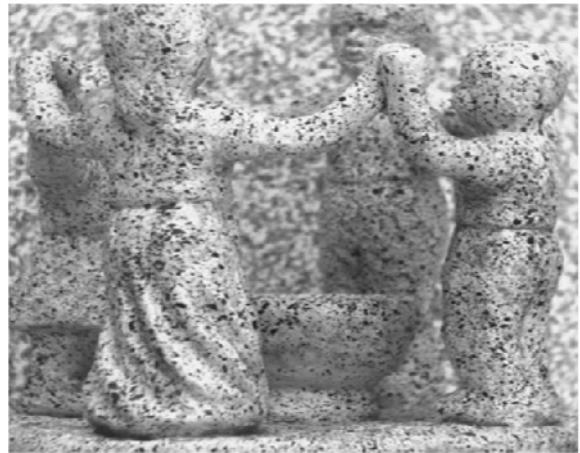


c)

[Figure from Prados & Faugeras 2006]

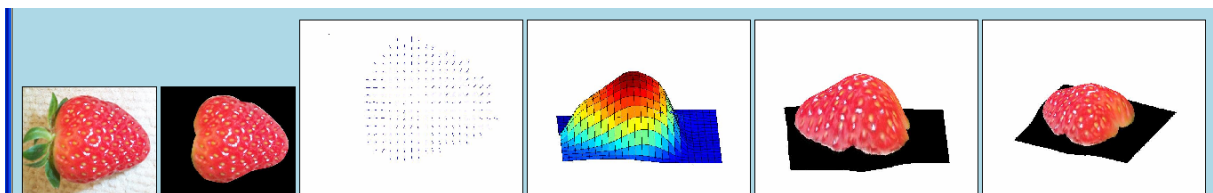
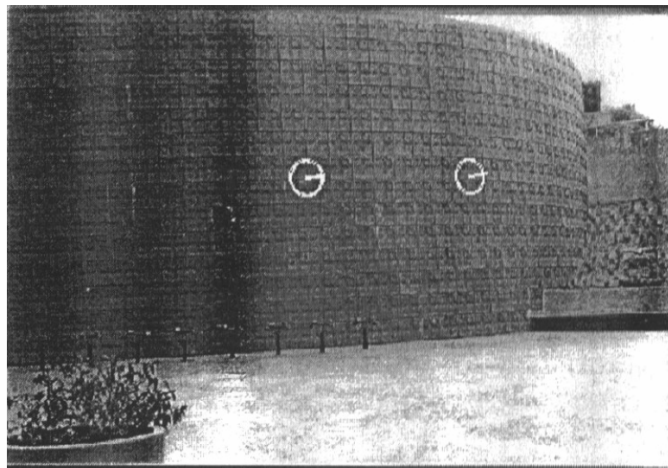


# Focus/Defocus



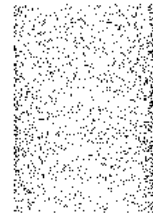
[Figure from H. Jin and P. Favaro, 2002]

# Texture



[From [A.M. Loh. The recovery of 3-D structure using visual texture patterns.](#) PhD thesis]

# Motion



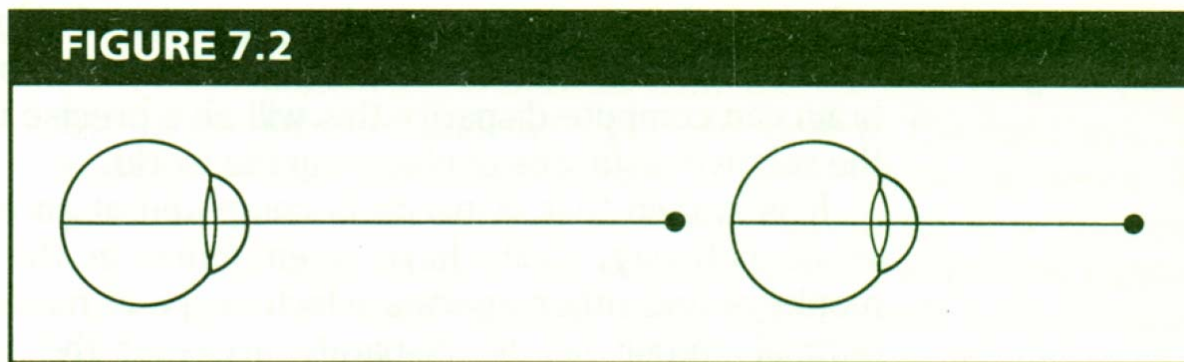
Figures from L. Zhang

<http://www.brainconnection.com/teasers/?main=illusion/motion-shape>

# Estimating scene shape

- Shape from X: Shading, Texture, Focus, Motion...
- Stereo:
  - shape from motion between two views
  - infer 3d shape of scene from two (multiple) images from different viewpoints

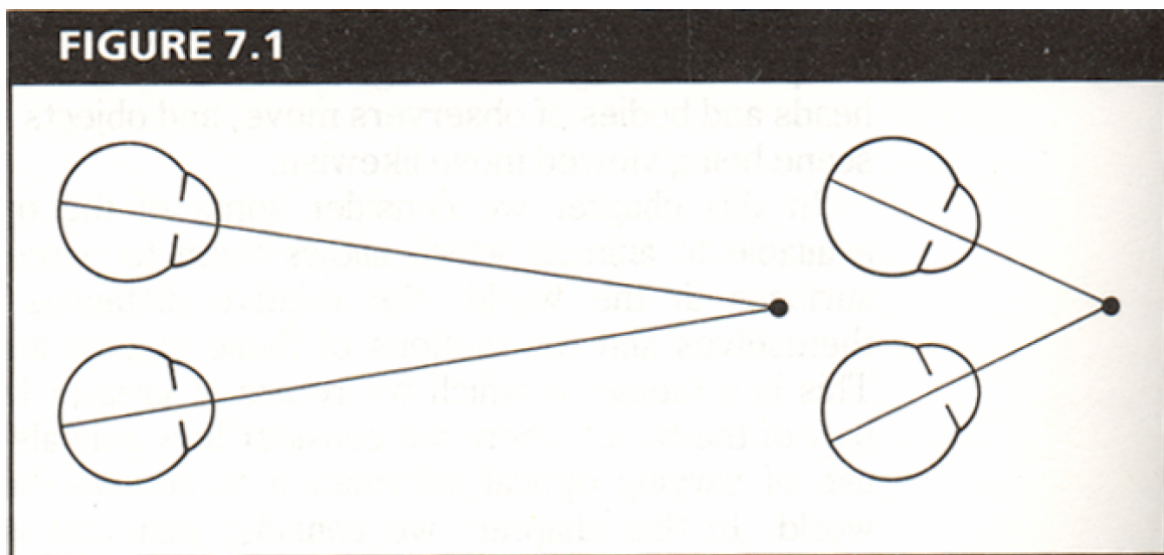
# Accommodation and focus



From Bruce and Green, Visual Perception,  
Physiology, Psychology and Ecology

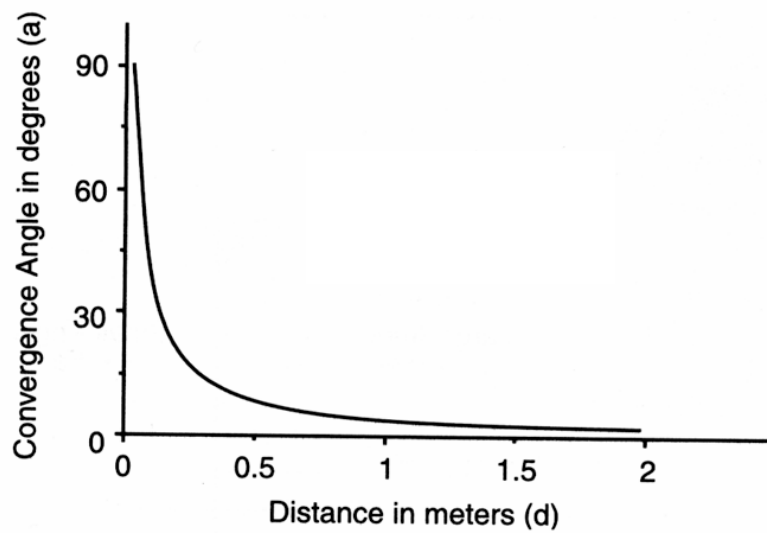
The lens modifies the image focus by adjusting  
its focal length.

# Fixation, convergence



From Bruce and Green, Visual Perception,  
Physiology, Psychology and Ecology

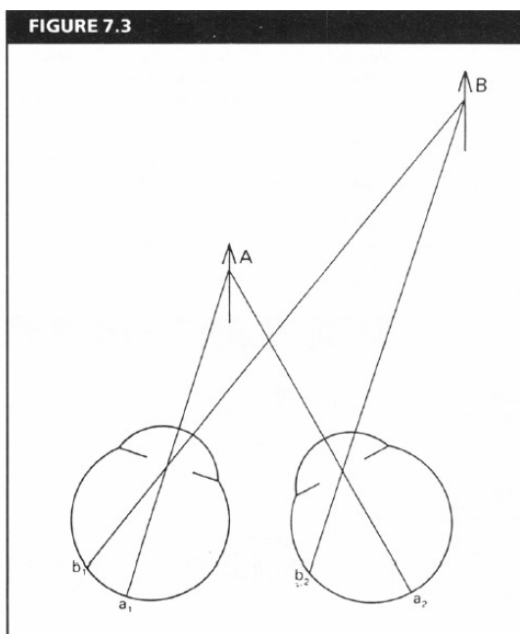
# Fixation, convergence



**Figure 5.2.3** Convergence as a function of distance. The angle of convergence changes rapidly with distances up to a meter or two but very little after that.

From Palmer, "Vision Science", MIT Press

# Human stereopsis: disparity



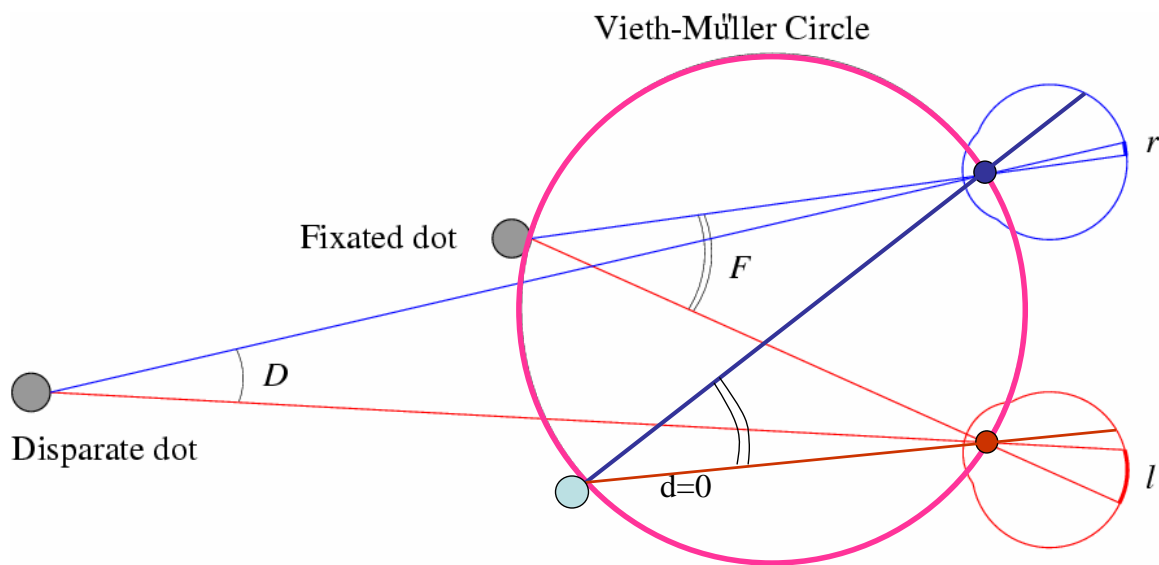
Disparity occurs when eyes verge on one object; others appear at different visual angles

From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Adapted from David Forsyth, UC Berkeley



# Human stereopsis: disparity

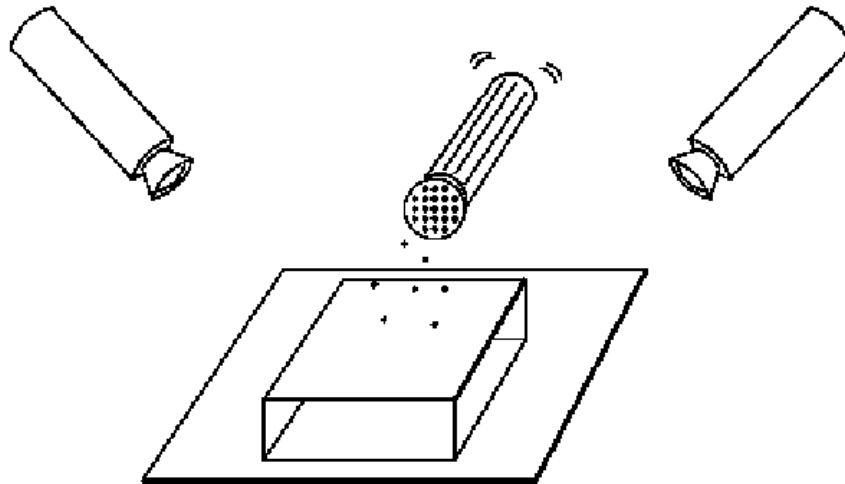


Disparity:  $d = r - l = D - F$ .

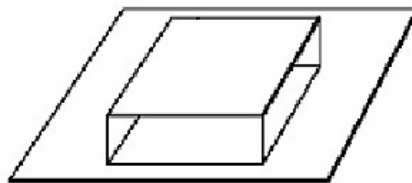
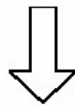
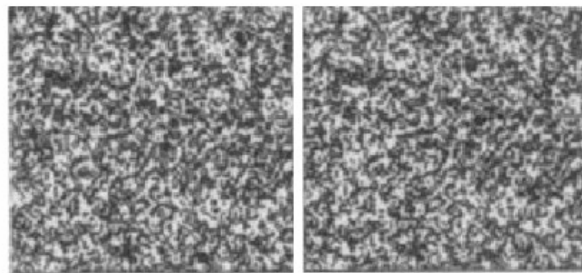
## Random dot stereograms

- Julesz 1960: Do we identify local brightness patterns before fusion (monocular process) or after (binocular)?
- To test: pair of synthetic images obtained by randomly spraying black dots on white objects

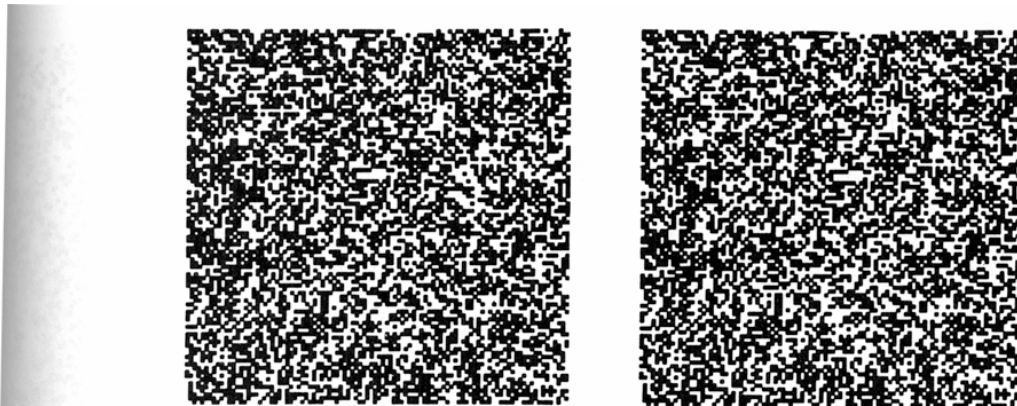
# Random dot stereograms



# Random dot stereograms



# Random dot stereograms



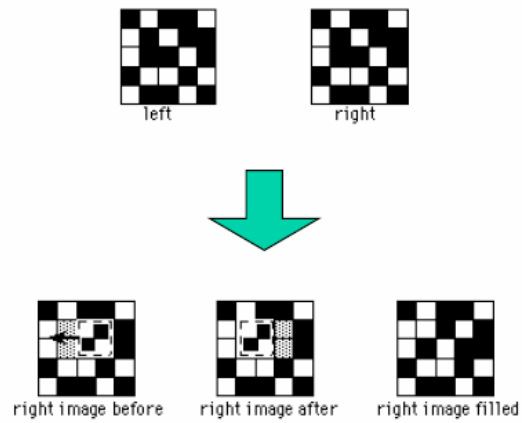
**Figure 5.3.8** A random dot stereogram. These two images are derived from a single array of randomly placed squares by laterally displacing a region of them as described in the text. When they are viewed with crossed disparity (by crossing the eyes) so

that the right eye's view of the left image is combined with the left eye's view of the right image, a square will be perceived to float above the page. (See pages 210–211 for instructions on fusing stereograms.)

## Random dot stereograms

- When viewed monocularly, they appear random; when viewed stereoscopically, see 3d structure.
- Conclusion: human binocular fusion not directly associated with the physical retinas; must involve the central nervous system
- Imaginary “*cyclopean retina*” that combines the left and right image stimuli as a single unit

# Generating a random dot stereogram



<http://www.wellesley.edu/CS/LiDPC/OnParallaxis/Braunl.paper20.html>

# Autostereograms



Exploit disparity as  
depth cue using single  
image

(Single image random  
dot stereogram, Single  
image stereogram)

Images from [magiceye.com](http://magiceye.com)

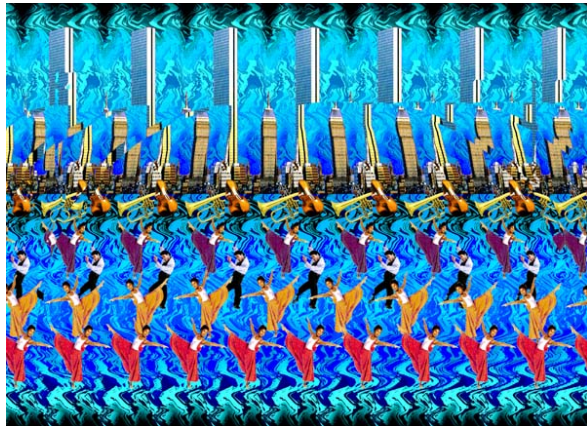


# Autostereograms



Images from [magiceye.com](http://magiceye.com)

# Autostereograms



Images from [magiceye.com](http://magiceye.com)

# Stereo photography and stereo viewers

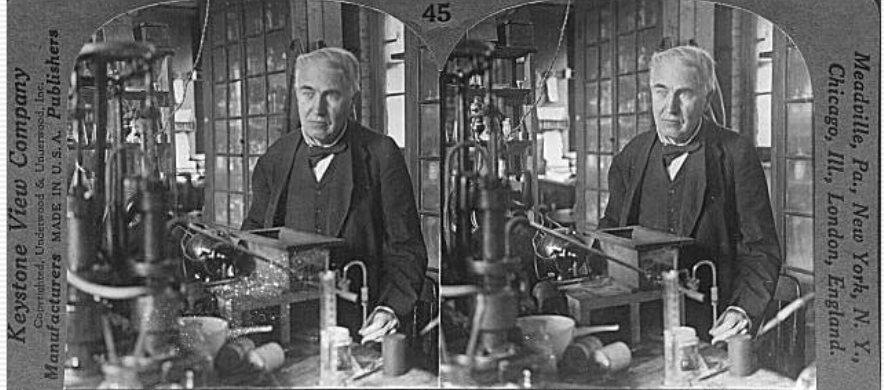
Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.



Invented by Sir Charles Wheatstone, 1838



Image courtesy of fisher-price.com



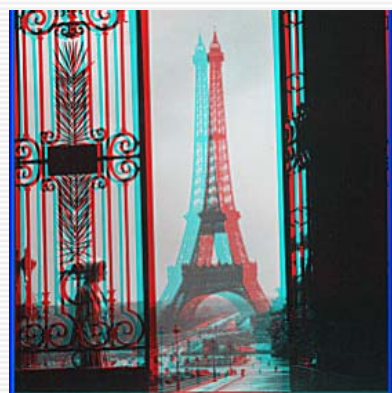
Keystone View Company  
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Manufacturers MADE IN U.S.A. Publishers

Meadville, Pa., New York, N. Y.,  
Chicago, Ill., London, England.

V28007 Most Famous Inventor of the Age, Thomas A. Edison in His Laboratory.

© Copyright 2001 Johnson-Shaw Stereoscopic Museum

<http://www.johnsonshawmuseum.org>



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<http://www.johnsonshawmuseum.org>



Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923

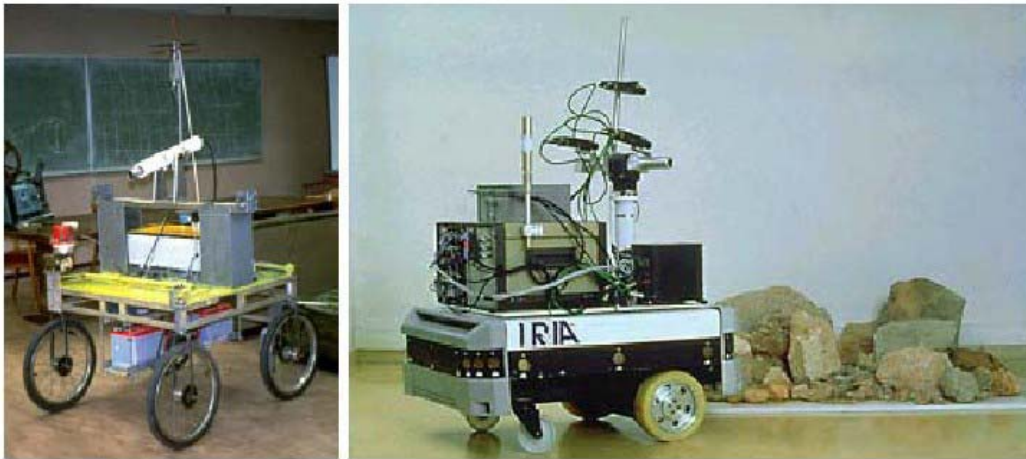






[http://www.well.com/~jimg/stereo/stereo\\_list.html](http://www.well.com/~jimg/stereo/stereo_list.html)

# Stereo in machine vision systems



Left : The Stanford cart sports a single camera moving in discrete increments along a straight line and providing multiple snapshots of outdoor scenes

Right : The INRIA mobile robot uses three cameras to map its environment



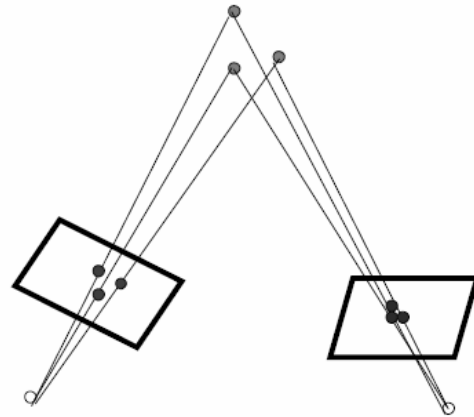
# Stereo

- Main issues
  - Geometry: what information is available, how do the camera views relate?
  - Correspondences: what feature in view 1 corresponds to feature in view 2?
  - Triangulation, reconstruction: inference in presence of noise

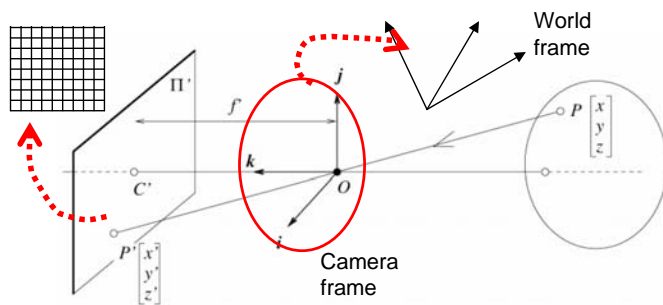
# Multi-view geometry

Relate

- 3-D points
- Camera centers
- Camera orientation
- Camera intrinsics



# Camera parameters



Extrinsic:  
Camera frame  $\leftrightarrow$  Reference frame

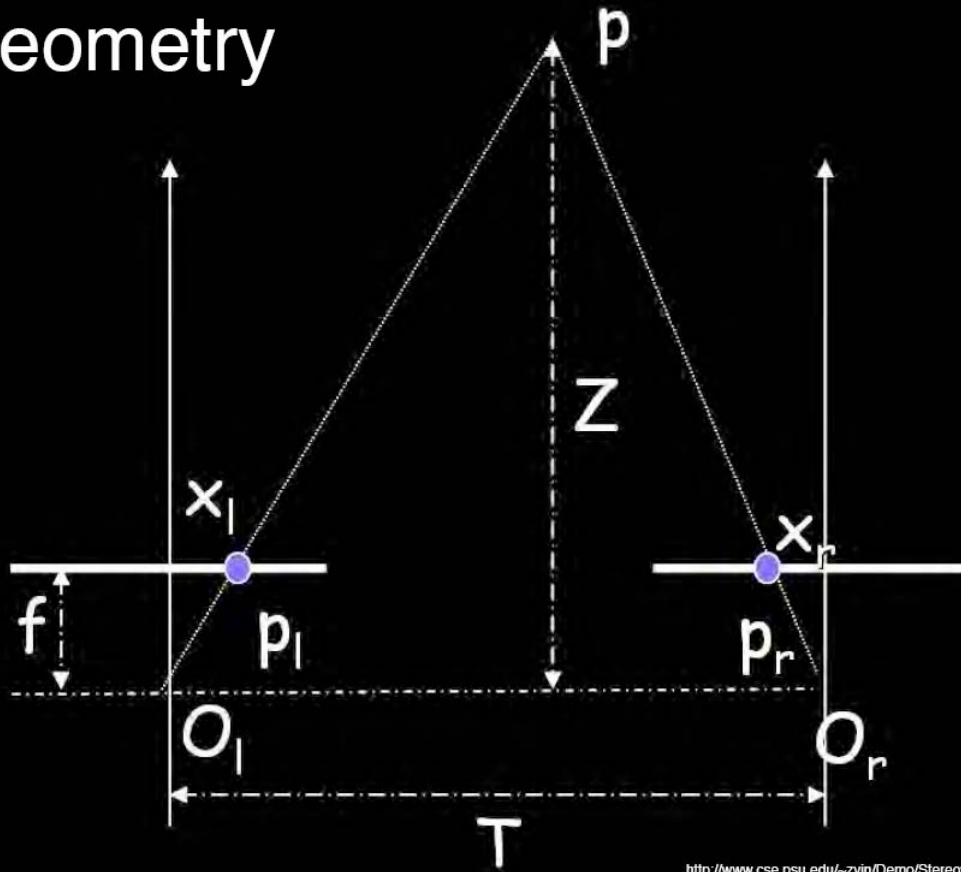
Intrinsic:  
Image coordinates relative to  
camera  $\leftrightarrow$  Pixel coordinates

- *Extrinsic* params: rotation matrix and translation vector
- *Intrinsic* params: focal length, pixel sizes (mm), image center point, radial distortion parameters

## Geometry for a simple stereo system

- First, assuming parallel optical axes, known camera parameters:

# Geometry

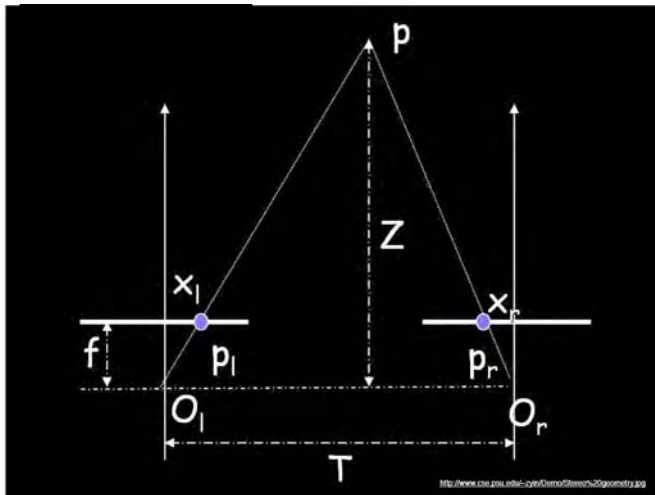


## Geometry for a simple stereo system

- Parameters in this case:
  - Camera centers ( $O_l$ ,  $O_r$ )
  - Focal length ( $f$ )
  - Baseline ( $T$ )

# Geometry for a simple stereo system

- First, assuming parallel optical axes, known camera parameters:



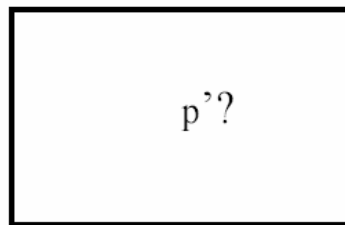
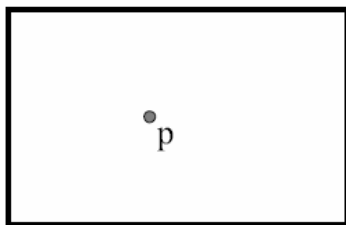
Similar triangles ( $p_l, P, p_r$ )  
and ( $O_l, P, O_r$ )

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f T/d$$

where  $d = x_r - x_l$

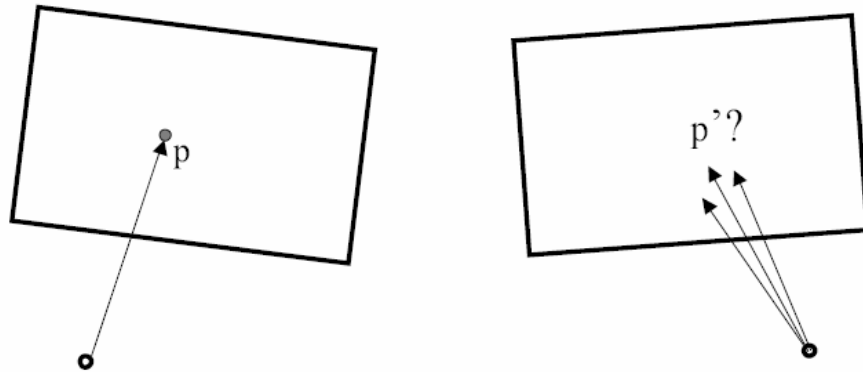
## Stereo constraints



- Given  $p$  in left image, where can corresponding point  $p'$  be?

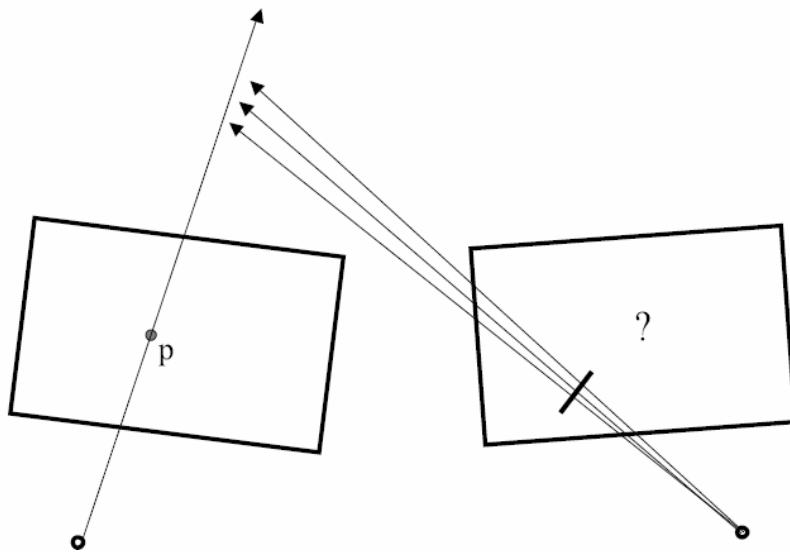


## Stereo constraints



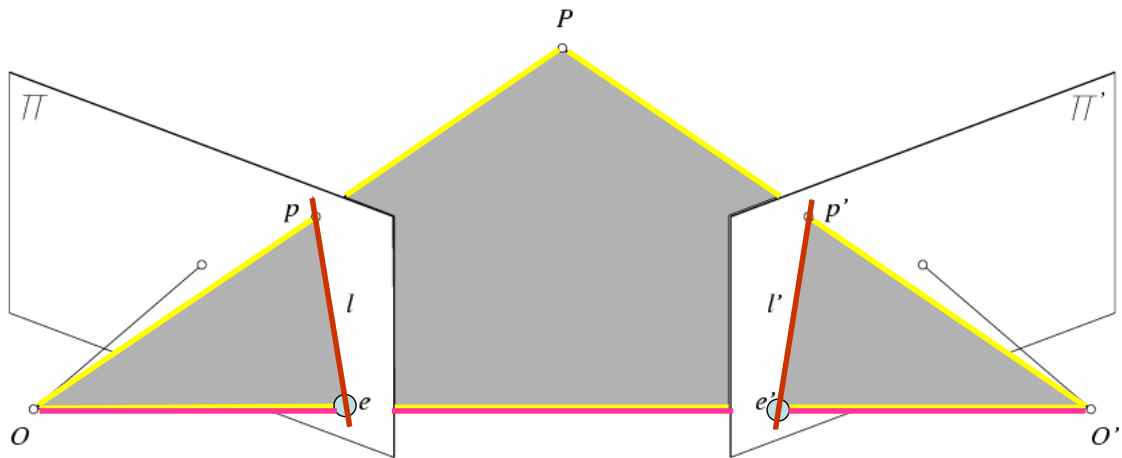
- Given  $p$  in left image, where can corresponding point  $p'$  be?

# Stereo constraints



# Epipolar geometry

Now the optical axes are not necessarily parallel.



• Epipolar Plane

• Baseline

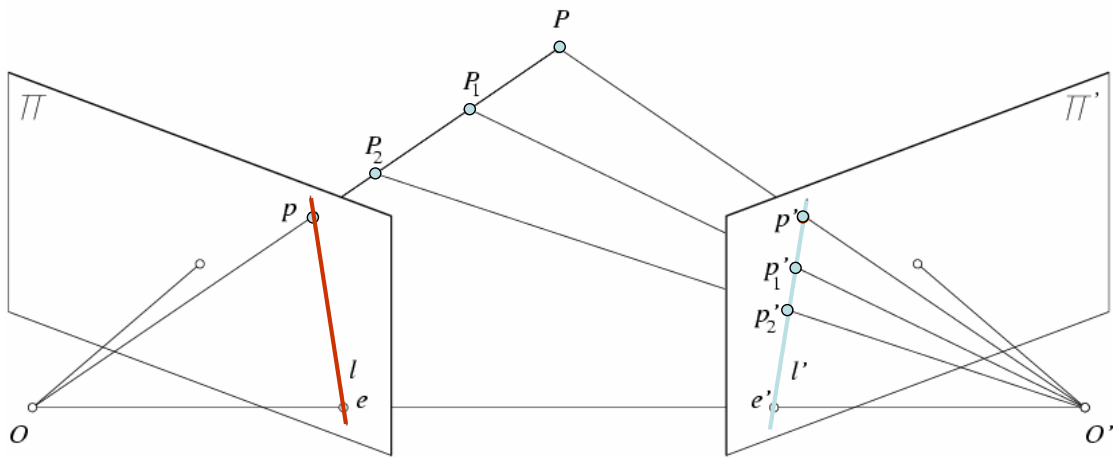
• Epipoles

• Epipolar Lines

# Epipolar geometry

- Baseline: line joining the camera centers
- Epipole: point of intersection of baseline with the image plane
- Epipolar plane: plane containing baseline
- Epipolar line: intersection of epipolar plane with the image plane
  
- All epipolar lines intersect at the epipole
- An epipolar plane intersects the left and right image planes in epipolar lines

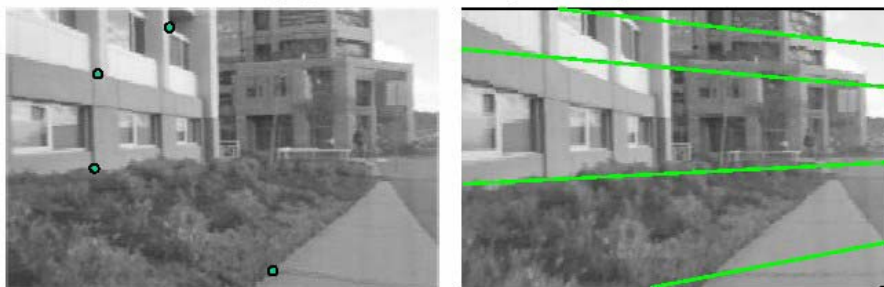
# Epipolar constraint



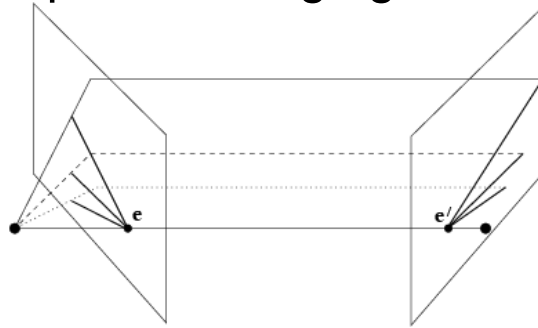
- Potential matches for  $p$  have to lie on the corresponding epipolar line  $l'$ .
- Potential matches for  $p'$  have to lie on the corresponding epipolar line  $l$ .

Slide credit: M. Pollefeys

# Epipolar constraint example



## Example: converging cameras



As position of 3d point varies, epipolar lines “rotate” about the baseline

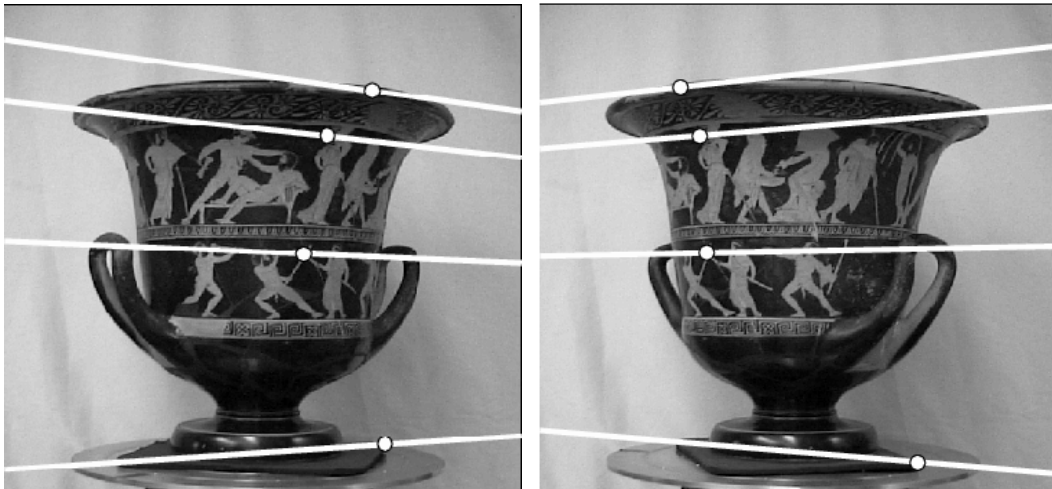


Figure from Hartley & Zisserman

## Example: motion parallel with image plane

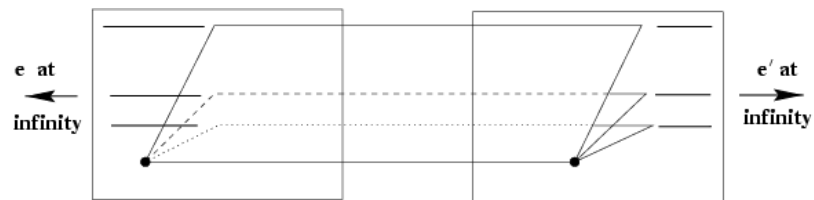
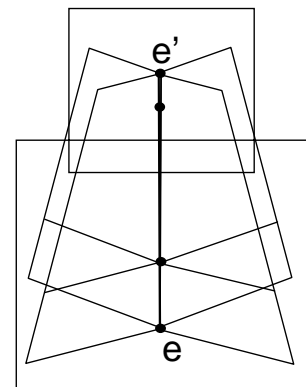
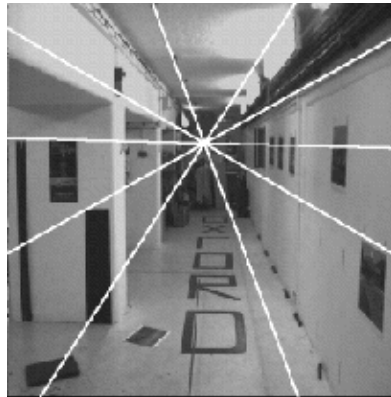
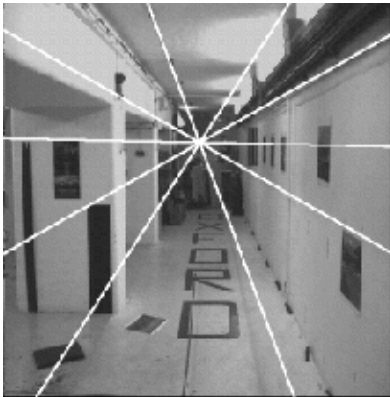


Figure from Hartley & Zisserman



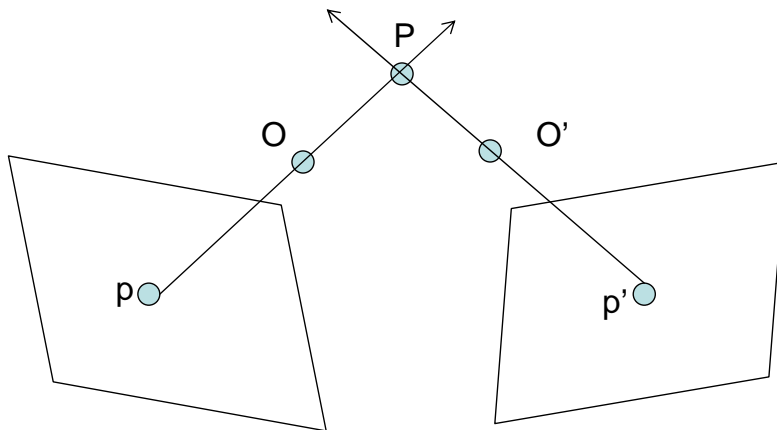
## Example: forward motion



Epipole has same coordinates in both images.  
Points move along lines radiating from  $e$ : "Focus of expansion"

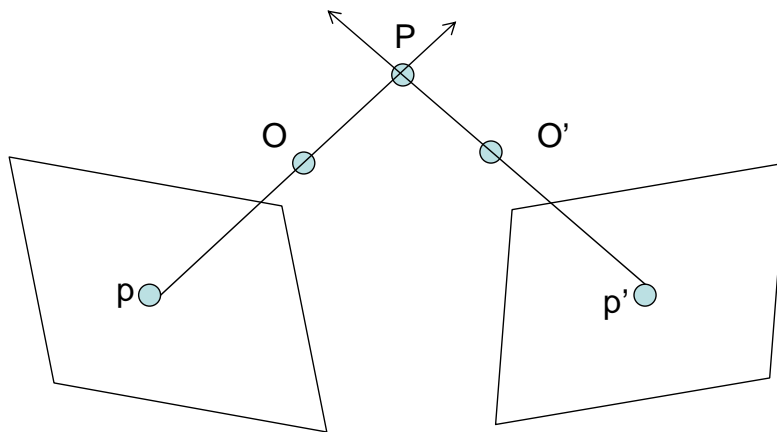
# Reconstruction by triangulation

- Assuming intrinsic and extrinsic parameters are known, compute 3d location of point  $P$  from projections  $p$  and  $p'$ :
- Intersect rays  $R = Op$  and  $R' = O'p'$ .



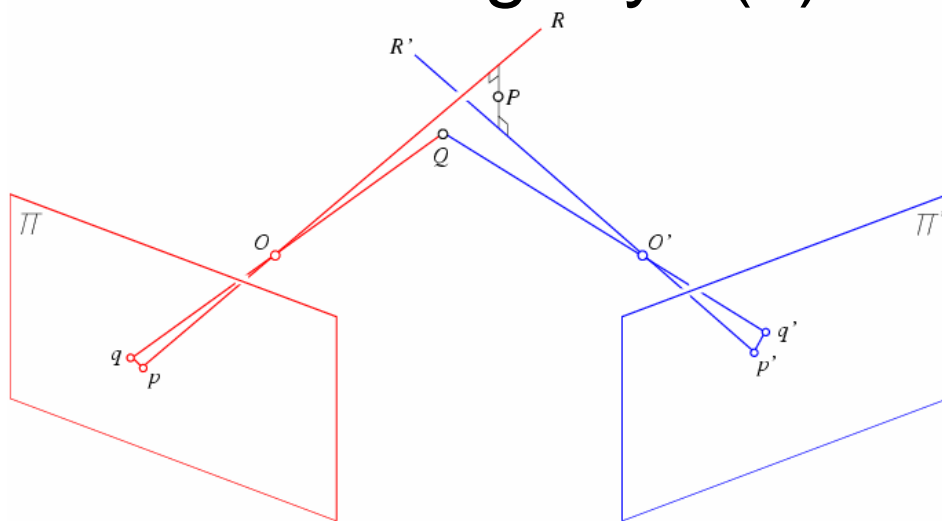
# Reconstruction by triangulation

- Assuming intrinsic and extrinsic parameters are known, compute 3d location of point  $P$  from projections  $p$  and  $p'$ :
- Intersect rays  $R = O_p$  and  $R' = O'_p$ .



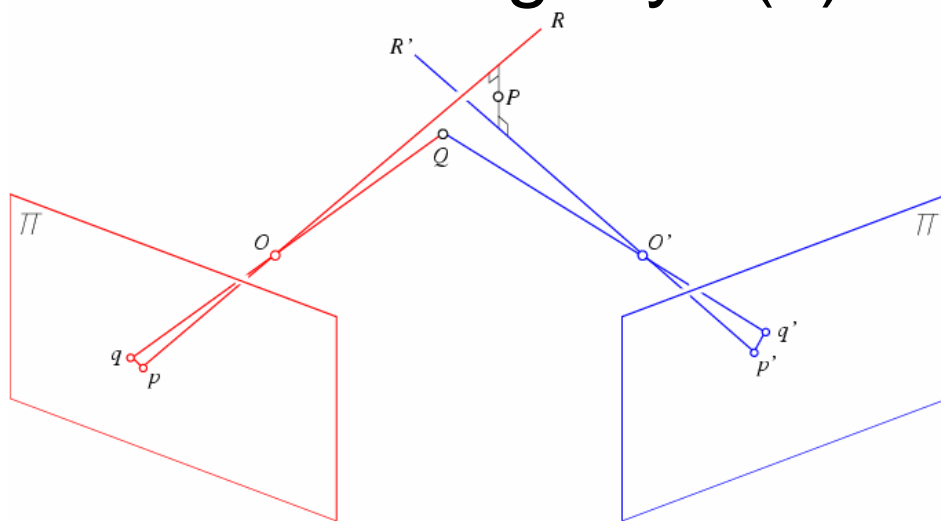
But, in practice, parameters and image locations only approximately known...

# Triangulation with non-intersecting rays (1)



Construct line segment perpendicular to  $R$  and  $R'$  that intersects both rays  
Midpoint of this segment is closest point to the two rays, use as  $P$  estimate of scene point

## Triangulation with non-intersecting rays (2)



Estimate scene point  $Q$  as the point that minimizes summed squared distance between  $p$  and  $q$ , and  $p'$  and  $q'$  (non-linear least squares, iterative, not closed form)

# Next

- 3d reconstruction
- Building stereo algorithms
  - correspondences