Announcements

- Reminder: check eGradebook to see all your scores
- Thursday: course recap and exam review
- **Pset 4 hardcopy turnin: two options**
  - Bring to class this Thursday (last class day), or
  - Anytime after Thursday’s class, drop in drop box on Taylor first floor in front of undergrad advising office
  - write “CS378 Computer Vision” on top of your hardcopy

Outline

- Last time:
  - Using optical flow (dense motion estimates) to recognize activities
  - Tracking as inference
    - Linear models of dynamics
    - Kalman filters
- **Today**:
  - Kalman filter recap, updates for n-d
  - Limitations of Kalman filtering
  - Other issues in tracking

Last time: Linear dynamic model

- Describe the *a priori* knowledge about
  - System dynamics model: represents evolution of state over time, with noise.
    \[
    x_i \sim N(Dx_{i-1}; \Sigma_d)
    \]
  - Measurement model: at every time step we get a noisy measurement of the state.
    \[
    y_i \sim N(Mx_i; \Sigma_m)
    \]

Last time: Kalman filter

- Know corrected state from previous time step, and all measurements up to the current one → Predict distribution over next state.
- Know prediction of state, and next measurement → Update distribution over current state.

<table>
<thead>
<tr>
<th>Time update (“Predict”)</th>
<th>Measurement update (“Correct”)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x_i</td>
<td>y_{i-1}, y_{i-2}, \ldots) )</td>
</tr>
<tr>
<td>( \mu_i^+, \sigma_i^+ )</td>
<td>( \mu_{i+1}, \sigma_{i+1} )</td>
</tr>
<tr>
<td>Mean and std. dev. of predicted state:</td>
<td>Mean and std. dev. of corrected state:</td>
</tr>
</tbody>
</table>

1D Kalman filter: prediction vs. correction

\[
\mu_i^+ = \frac{\mu_i^* \sigma_i^2 + m y_i (\sigma_i^2)}{\sigma_i^2 + m^2 (\sigma_i^2)}
\]
\[
(\sigma_i^+)^2 = \frac{\sigma_i^2 (\sigma_i^2)}{\sigma_i^2 + m^2 (\sigma_i^2)}
\]

- What if there is no prediction uncertainty \((\sigma_i^* = 0)\)?
  \[
  \mu_i^+ = \mu_i^*
  \]
  \[
  (\sigma_i^+)^2 = 0
  \]
  The measurement is ignored!

- What if there is no measurement uncertainty \((\sigma_m = 0)\)?
  \[
  \mu_i^+ = \frac{y_i}{m}
  \]
  \[
  (\sigma_i^+)^2 = 0
  \]
  The prediction is ignored!
Kalman filter: General case (> 1dim)

What if state vectors have more than one dimension?

**PREDICT**

\[
x_t^- = D_x x_{t-1}
\]

\[
\Sigma_t^- = D_x \Sigma_{t-1} D_x^T + \Sigma_x
\]

**CORRECT**

\[
K_t = \Sigma_t^- M_t^T \left( M_t \Sigma_t^+ M_t^T + \Sigma_z \right)^{-1}
\]

\[
x_t^+ = x_t^- + K_t (y_t - M_t x_t^-)
\]

\[
\Sigma_t^+ = (I - K_t M_t) \Sigma_t^-
\]

Less weight on residual as a priori estimate error covariance approaches 0.

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Kalman filter: pros and cons

- Gaussian densities, linear dynamic model:
  - Simple updates, compact and efficient
  - But, restricted class of motions defined by linear model
  - Unimodal distribution = only single hypothesis

\[x \sim N(\mu, \Sigma)\]

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When is a single hypothesis too limiting?

![Diagram](image1)

Consider this example: say we are tracking the face on the right using a skin color blob to get our measurement.

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Alternative: particle-filtering and non-Gaussian densities

- Can represent distribution with set of weighted samples ("particles")
- Allows us to maintain multiple hypotheses.

Alternative: particle-filtering and non-Gaussian densities

Monitor is a distractor; multiple hypotheses necessary. Kalman filter fails once it starts tracking the monitor.

http://www.robots.ox.ac.uk/~vgd/dynamics.html

Tracking: issues

- Initialization
- Data association
- Multiple tracked objects
- Deformable and articulated objects
- Constructing accurate models of dynamics
- Drift

Tracking: issues

- Initialization
  - Often done manually
  - Background subtraction, detection can also be used
- Data association, multiple tracked objects
  - Occlusions

Data association

- We’ve assumed entire measurement (y) was cue of interest for the state
- But, there are typically uninformative measurements too—clutter.
- Data association: task of determining which measurements go with which tracks.

Data association

- Simple strategy: only pay attention to the measurement that is “closest” to the prediction

Data association

- Simple strategy: only pay attention to the measurement that is “closest” to the prediction

Doesn’t always work…
Alternative: keep track of multiple hypotheses at once.
Tracking: issues

- Initialization
  - Often done manually
  - Background subtraction, detection can also be used
- Data association, multiple tracked objects
  - Occlusions
- Deformable and articulated objects
- Constructing accurate models of dynamics
  - e.g., parameters for a linear dynamics model
- Drift
  - Accumulation of errors over time

Drift


Tracking people by learning their appearance

- Person model = appearance + structure (+ dynamics)
- Structure and dynamics are generic, appearance is person-specific
- Trying to acquire an appearance model "on the fly" can lead to drift
- Instead, can use the whole sequence to initialize the appearance model and then keep it fixed while tracking
- Given strong structure and appearance models, tracking can essentially be done by repeated detection (with some smoothing)


Tracking people by learning their appearance

Use a part-based model to encode part appearance + relative geometry.

Bottom-up initialization: Clustering

Top-down initialization: Exploit “easy” poses


Example results


Example results

Tracking: summary

- Tracking as inference
  - Goal: estimate posterior of object position given measurement
  - Know where to look, can survive even with poor measurements
- Linear models of dynamics
  - Represent state evolution and measurement models
- Kalman filters
  - Recursive prediction/correction updates to refine measurement
  - Single hypothesis can be limiting → alternative models use non-Gaussian distributions
- Drift: as error accumulates we may gradually start tracking something else.
  - Tracking via detection one way to mitigate drift (though lose out on prediction help)