Image formation
Matlab tutorial
Tuesday, Sept 2

Physical parameters of image formation

- Geometric
  - Type of projection
  - Camera pose
- Optical
  - Sensor’s lens type
  - focal length, field of view, aperture
- Photometric
  - Type, direction, intensity of light reaching sensor
  - Surfaces’ reflectance properties

Pinhole camera

- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the aperture
  - How does this transform the image?

Pinhole camera

- Pinhole camera is a simple model to approximate imaging process, perspective projection.

  If we treat pinhole as a point, only one ray from any given point can enter the camera.

Image formation

- How are objects in the world captured in an image?

Slide by Steve Seitz

Pinhole camera
"Reinier Gemma-Frisius", observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book De Radio Astronomica et Geometrica, 1545. It is thought to be the first published illustration of a camera obscura.

Hammond, John H., The Camera Obscura, A Chronicle

Jetty at Margate England, 1898.

An attraction in the late 19th century

Camera obscura at home

Perspective effects

- Far away objects appear smaller
### Perspective effects
- Parallel lines in the scene intersect in the image
- Converge in image on horizon line

![Perspective effects diagram](image)

### Projection properties
- Many-to-one: any points along same ray map to same point in image
- Points $\rightarrow$ points
- Lines $\rightarrow$ lines (collinearity preserved)
- Distances and angles are **not** preserved
- Degenerate cases:
  - Line through focal point projects to a point.
  - Plane through focal point projects to line
  - Plane perpendicular to image plane projects to part of the image.

### Perspective and art
- Use of correct perspective projection indicated in 1st century B.C. frescoes
- Skill resurfaces in Renaissance: artists develop systematic methods to determine perspective projection (around 1480-1515)

#### Perspective projection equations
3d world mapped to 2d projection in image plane

![Perspective projection equations diagram](image)

### Homogeneous coordinates
**Is this a linear transformation?**
- *no*—division by $z$ is nonlinear

**Trick:** add one more coordinate:

\[
\begin{pmatrix}
  x \\
  y \\
  1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix}
\]

Converting **from** homogeneous coordinates

\[
\begin{pmatrix}
  x \\
  y \\
  w
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
  x/w, y/w \\
  z/w
\end{pmatrix}
\]

### Perspective Projection Matrix
- Projection is a matrix multiplication using homogeneous coordinates:

\[
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1/f' & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
  x/f', y/f', z/f'
\end{bmatrix}
\]

**Complete mapping from world points to image pixel positions?**

Perspective projection & calibration

- Perspective equations so far in terms of camera’s reference frame.
- Camera’s intrinsic and extrinsic parameters needed to calibrate geometry.

Weak perspective

- Approximation: treat magnification as constant
- Assumes scene depth << average distance to camera

Orthographic projection

- Given camera at constant distance from scene
- World points projected along rays parallel to optical access

Pinhole size / aperture

How does the size of the aperture affect the image we’d get?

Adding a lens

- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the focal length $f$
**Pinhole vs. lens**

- A lens focuses parallel rays onto a single focal point.
- Gather more light, while keeping focus; make pinhole perspective projection practical.

**Cameras with lenses**

- Optical center (Center Of Projection)
- Focal point
- Aperture

**Human eye**

- Rough analogy with human visual system:
  - Pupil/iris – control amount of light passing through lens
  - Retina - contains sensor cells, where image is formed
  - Fovea – highest concentration of cones

**Thin lens**

- Rays entering parallel on one side go through focus on other, and vice versa.
- In ideal case – all rays from P imaged at P'.

**Thin lens equation**

\[
\frac{1}{u} + \frac{1}{v} = \frac{1}{f}
\]

- Any object point satisfying this equation is in focus.

**Focus and depth of field**

- Image credit: cambridgeincolour.com
Focus and depth of field

• Depth of field: distance between image planes where blur is tolerable

Thin lens: scene points at distinct depths come in focus at different image planes. (Real camera lens systems have greater depth of field.)

Focus and depth of field

• How does the aperture affect the depth of field?

A smaller aperture increases the range in which the object is approximately in focus

Depth from focus

Images from same point of view, different camera parameters

3d shape / depth estimates

[Fig from H. Jin and P. Favaro, 2002]

Field of view

• Angular measure of portion of 3d space seen by the camera

Field of view depends on focal length

• As f gets smaller, image becomes more wide angle
  - more world points project onto the finite image plane

• As f gets larger, image becomes more telescopic
  - smaller part of the world projects onto the finite image plane

Field of view depends on focal length

\[ \varphi = \tan^{-1}\left(\frac{d}{2f}\right) \]

Smaller FOV = larger Focal Length
Resolution

- sensor: size of real world scene element a that images to a single pixel
- image: number of pixels
- Influences what analysis is feasible, affects best representation choice.

Digital cameras

- Film → sensor array
- Often an array of charge coupled devices
- Each CCD is light sensitive diode that converts photons (light energy) to electrons

Digital images

Think of images as matrices taken from CCD array.

Intensity: [0, 255]

Color sensing in digital cameras

Estimate missing components from neighboring values (demosaicing)

Color images, RGB color space
Historical context

- **Pinhole model**: Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses)**: Alhacen (965-1039 CE)
- **Camera obscura**: Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo**: Joseph Nicphore Niepce (1822)
- **Daguerreotypes** (1839)
- **Photographic film**: Eastman, 1889
- **Cinema**: Lumière Brothers, 1895
- **Color Photography**: Lumière Brothers, 1908
- **Television**: Baird, Farnsworth, Zworykin, 1920s
- **First consumer camera with CCD**: Sony Mavica (1981)
- **First fully digital camera**: Kodak DCS100 (1990)

Summary

- Image formation affected by geometry, photometry, and optics.
- Projection equations express how world points mapped to 2d image.
- Homogenous coordinates allow linear system for projection equations.
- Lenses make pinhole model practical.
- Parameters (focal length, aperture, lens diameter,…) affect image obtained.

Next

**Problem set 0** due Thursday

```
turnin --submit harshd pset0 <filename>
```

**Thursday**: Color

- Read F&P Chapter 6