Image formation
Matlab tutorial

Tuesday, Sept 2

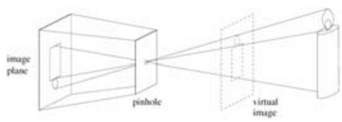


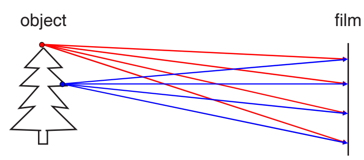
Image formation

- How are objects in the world captured in an image?

Physical parameters of image formation

- Geometric
  - Type of projection
  - Camera pose
- Optical
  - Sensor's lens type
  - focal length, field of view, aperture
- Photometric
  - Type, direction, intensity of light reaching sensor
  - Surfaces' reflectance properties

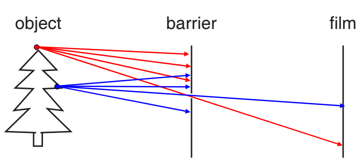
Image formation



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Slide by Steve Seitz

Pinhole camera

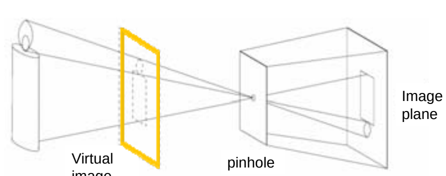


- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**
  - How does this transform the image?

Slide by Steve Seitz

Pinhole camera

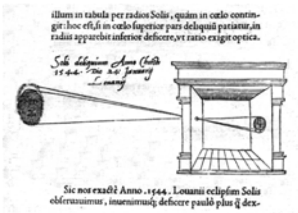
- Pinhole camera is a simple model to approximate imaging process, perspective **projection**.



If we treat pinhole as a point, only one ray from any given point can enter the camera.

Fig from Forsyth and Ponce

Camera obscura



In Latin, means 'dark room'

"**Reinerus Gemma-Frisius**, observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book De Radio Astronomica et Geometrica, 1545. It is thought to be the first published illustration of a camera obscura..."
Hammond, John H., The Camera Obscura, A Chronicle

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html

Camera obscura

Jetty at Margate England, 1898.

Around 1870s

An attraction in the late 19th century

http://brightbytes.com/cosite/collection2.html
Adapted from R. Duraiswami

Camera obscura at home

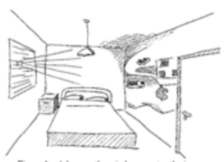


Figure 1 - A lens on the window creates the image of the external world on the opposite wall and you can see it every morning, when you wake up.

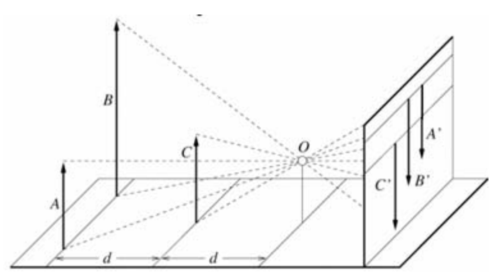
Sketch from http://www.funsci.com/fun3_en/sky/sky.htm

http://blog.makezine.com/archive/2006/02/how_to_room_sized_camera_obscu.html

Perspective effects

Perspective effects

- Far away objects appear smaller

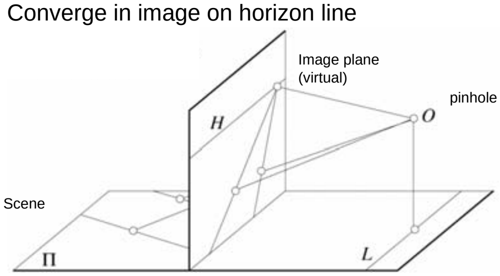


Forsyth and Ponce

Perspective effects

Perspective effects

- Parallel lines in the scene intersect in the image
- Converge in image on horizon line



Projection properties

- Many-to-one: any points along same ray map to same point in image
- Points → points
- Lines → lines (collinearity preserved)
- Distances and angles are **not** preserved
- Degenerate cases:
  - Line through focal point projects to a point.
  - Plane through focal point projects to line
  - Plane perpendicular to image plane projects to part of the image.

Perspective and art

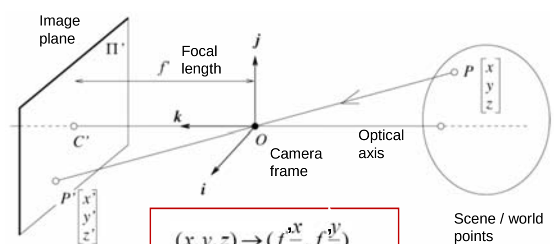
- Use of correct perspective projection indicated in 1st century B.C. frescoes
- Skill resurfaces in Renaissance: artists develop systematic methods to determine perspective projection (around 1480-1515)


Raphael


Durer, 1525

Perspective projection equations

- 3d world mapped to 2d projection in image plane



$$(x, y, z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

Scene point → Image coordinates

Forsyth and Ponce

Homogeneous coordinates

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Slide by Steve Seitz

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f' & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f' \end{bmatrix} \Rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

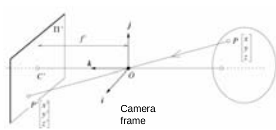
divide by the third coordinate to convert back to non-homogeneous coordinates

Complete mapping from world points to image pixel positions?

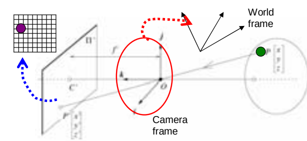
Slide by Steve Seitz

Perspective projection & calibration

- Perspective equations so far in terms of *camera's* reference frame….
- Camera's *intrinsic* and *extrinsic* parameters needed to calibrate geometry.

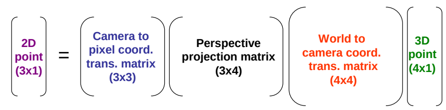


Perspective projection & calibration



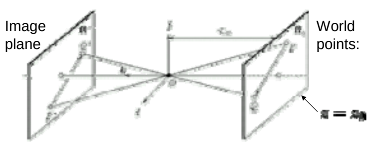
Extrinsic:
Camera frame ←→World frame

Intrinsic:
Image coordinates relative to camera ←→ Pixel coordinates



Weak perspective

- Approximation: treat magnification as constant
- Assumes scene depth << average distance to camera



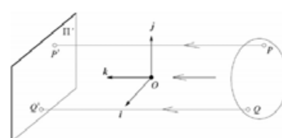
$$x' = f\frac{x}{z} \approx \frac{f}{z_0}x$$

$$y' = f\frac{y}{z} \approx \frac{f}{z_0}y$$

$z = z_0$

Orthographic projection

- Given camera at **constant** distance from scene
- World points projected along rays parallel to optical access



$$x' = x$$

$$y' = y$$

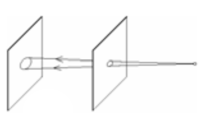
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Pinhole size / aperture

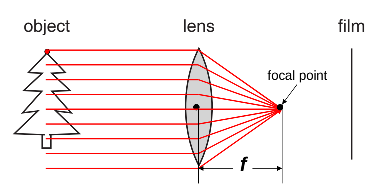
How does the size of the aperture affect the image we'd get?

Larger

Smaller



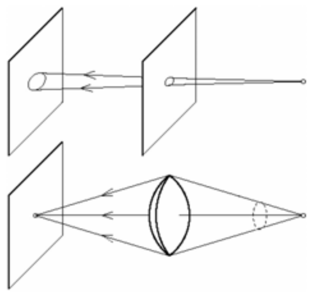
Adding a lens



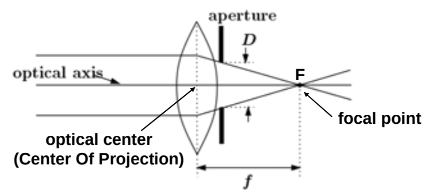
- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length* *f*

Slide by Steve Seitz

Pinhole vs. lens



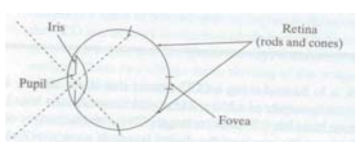
Cameras with lenses



- A lens focuses parallel rays onto a single focal point
- Gather more light, while keeping focus; make pinhole perspective projection practical

Human eye

Rough analogy with human visual system:



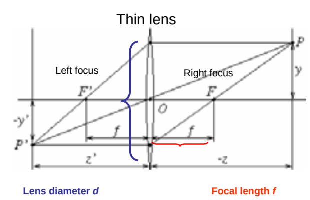
Pupil/Iris – control amount of light passing through lens

Retina - contains sensor cells, where image is formed

Fovea – highest concentration of cones

Fig from Shapiro and Stockman

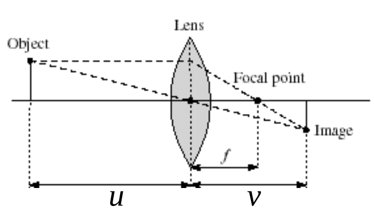
Thin lens



Rays entering parallel on one side go through focus on other, and vice versa.

In ideal case – all rays from P imaged at P'.

Thin lens equation



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- Any object point satisfying this equation is in focus

Focus and depth of field

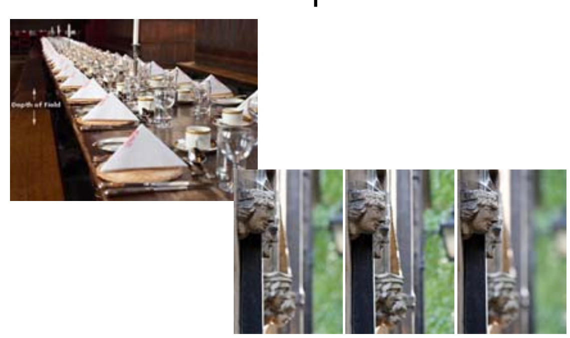
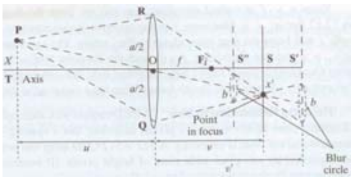


Image credit: cambridgeincolour.com

Focus and depth of field

- Depth of field: distance between image planes where blur is tolerable



←— "circles of confusion" —→

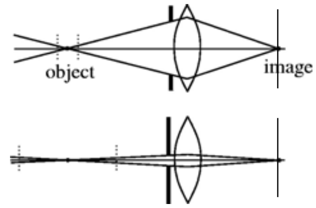
Thin lens: scene points at distinct depths come in focus at different image planes.

(Real camera lens systems have greater depth of field.)

Shapiro and Stockman

Focus and depth of field

- How does the aperture affect the depth of field?



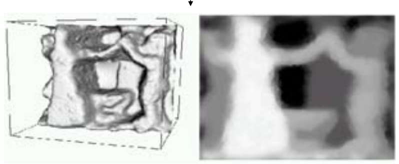
- A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

Slide from S. Seitz

Depth from focus

Images from same point of view, different camera parameters



3d shape / depth estimates

[figs from H. Jin and P. Favaro, 2002]

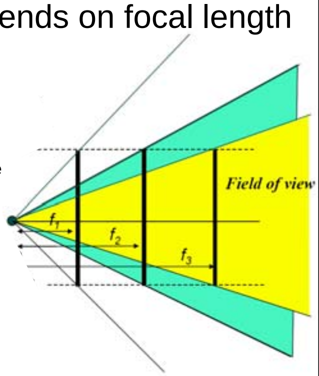
Field of view

- Angular measure of portion of 3d space seen by the camera

Images from http://en.wikipedia.org/wiki/Angle_of_view

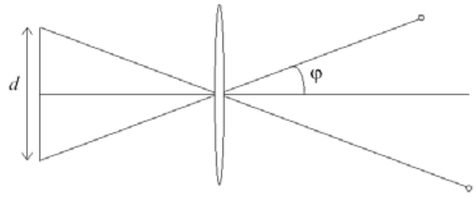
Field of view depends on focal length

- As ***f*** gets smaller, image becomes more *wide angle*
  - more world points project onto the finite image plane
- As ***f*** gets larger, image becomes more *telescopic*
  - smaller part of the world projects onto the finite image plane



from R. Duraiswami

Field of view depends on focal length



Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller FOV = larger Focal Length

Slide by A. Efros

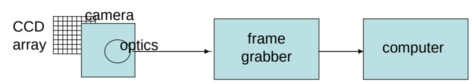
Resolution

- sensor: size of real world scene element a that images to a single pixel
- image: number of pixels
- Influences what analysis is feasible, affects best representation choice.

[fig from Mori et al]

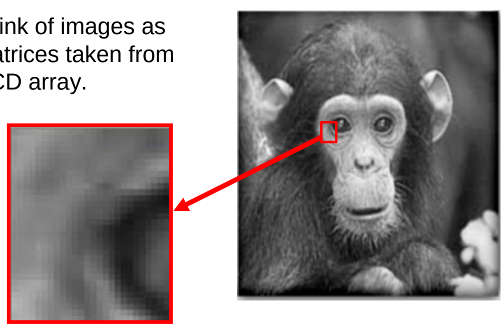
Digital cameras

- Film → sensor array
- Often an array of charge coupled devices
- Each CCD is light sensitive diode that converts photons (light energy) to electrons



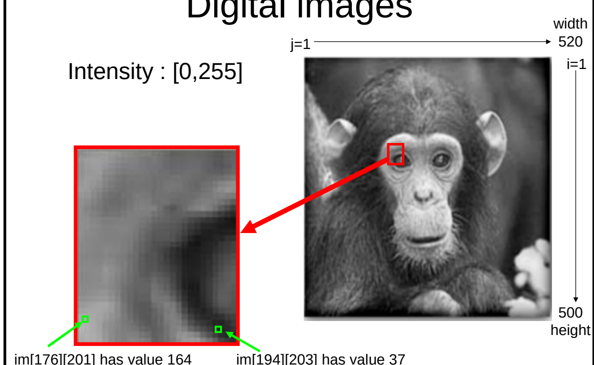
Digital images

Think of images as matrices taken from CCD array.



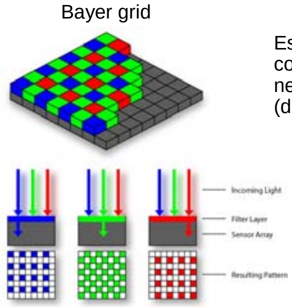
Digital images

Intensity : [0,255]



Color sensing in digital cameras

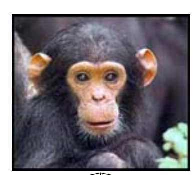
Bayer grid

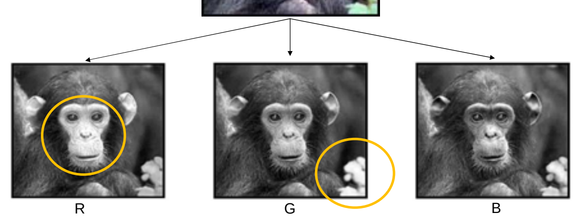


Estimate missing components from neighboring values (demosaicing)

Source: Steve Seitz

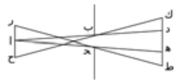
Color images, RGB color space





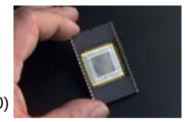
Historical context

- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD**: Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes

Niepce, "La Table Servie," 1822



CCD chip

Slide credit: L. Lazebnik

Summary

- Image formation affected by geometry, photometry, and optics.
- Projection equations express how world points mapped to 2d image.
- Homogenous coordinates allow linear system for projection equations.
- Lenses make pinhole model practical.
- Parameters (focal length, aperture, lens diameter,…) affect image obtained.

Next

Problem set 0 due Thursday

```
turnin --submit harshd pset0 <filename>
```

Thursday: Color

- Read F&P Chapter 6