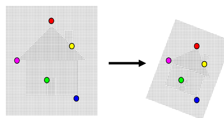
Fitting a transformation: feature-based alignment

Wed, Feb 23

Prof. Kristen Grauman

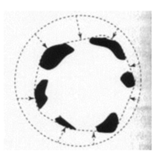
UT-Austin



Announcements

- Reminder: Pset 2 due Wed March 2
- Midterm exam is Wed March 9
  (2 weeks from now)

Last time: Deformable contours



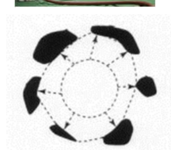


Image from http://www.healthline.com/blogs/exercise_fitness/uploaded_images/HandBand2-795868.JPG

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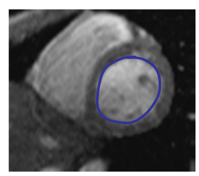
Last time: Deformable contours

a.k.a. active contours, snakes

Given: initial contour (model) near desired object

Goal: evolve the contour to fit exact object boundary



Main idea: elastic band is iteratively adjusted so as to
- be near image positions with high gradients, **and**
- satisfy shape "preferences" or contour priors

[Snakes: Active contour models, Kass, Witkin, & Terzopoulos, ICCV1987]

Figure credit: Yuri Boykov

Last time: Deformable contours

<u>Pros:</u>
- Useful to track and fit non-rigid shapes
- Contour remains connected
- Possible to fill in "subjective" contours
- Flexibility in how energy function is defined, weighted.

<u>Cons</u>:
- Must have decent initialization near true boundary, may get stuck in local minimum
- Parameters of energy function must be set well based on prior information

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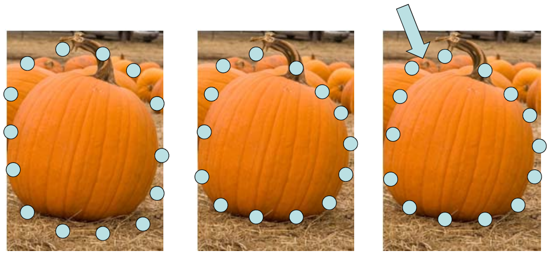
Today

- Interactive segmentation
- Feature-based alignment
  - 2D transformations
  - Affine fit
  - RANSAC

Interactive forces



How can we implement such an *interactive* force with deformable contours?

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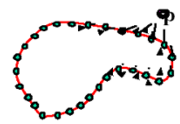
Interactive forces

- An energy function can be altered online based on user input – use the cursor to push or pull the initial snake away from a point.
- Modify external energy term to include:



$$E_{push} = \sum_{i=0}^{n-1} \frac{r^2}{|v_i - p|^2}$$

Nearby points get pushed hardest

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Intelligent scissors

Another form of interactive segmentation:

Compute optimal paths **from every point to the seed** based on edge-related costs.

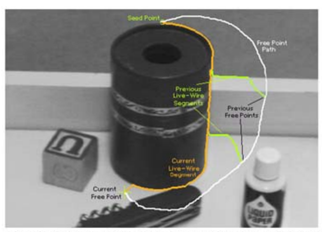
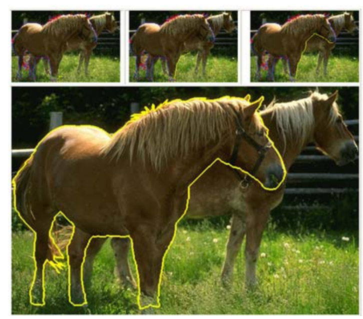


Figure 2: *Image demonstrating how the live-wire segment adapts and snaps to an object boundary as the free point moves (via cursor movement). The path of the free point is shown in white. Live-wire segments from previous free point positions (t_0, t_1, and t_2) are shown in green.*

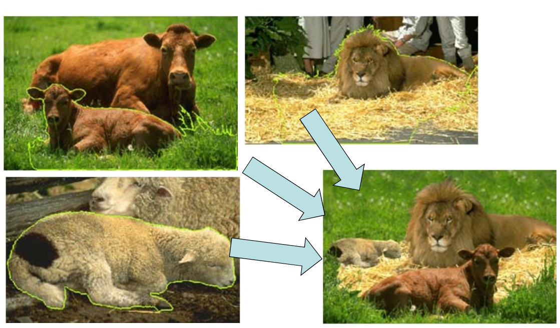
[Mortensen & Barrett, SIGGRAPH 1995, CVPR 1999]

Intelligent scissors



- http://rivit.cs.byu.edu/Eric/Eric.html

Intelligent scissors



- http://rivit.cs.byu.edu/Eric/Eric.html

Beyond boundary snapping…

- Another form of interactive guidance: specify regions
- Usually taken to suggest foreground/background color distributions

User Input

Result

Boykov and Jolly (2001)

How to use this information?

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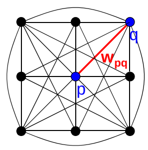
Recall: Images as graphs

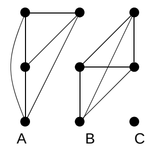


Fully-connected graph

- node for every pixel
- link between *every* pair of pixels, **p**,**q**
- similarity w_{pq} for each link
  - » similarity is *inversely proportional* to difference in color and position

Steve Seitz

Recall: Segmentation by Graph Cuts

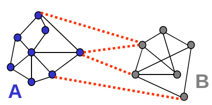


Break graph into segments

- Delete links that cross between segments
- Easiest to break links that have low similarity
  - similar pixels should be in the same segments
  - dissimilar pixels should be in different segments

Steve Seitz

Recall: Segmentation by Graph Cuts



Link Cut

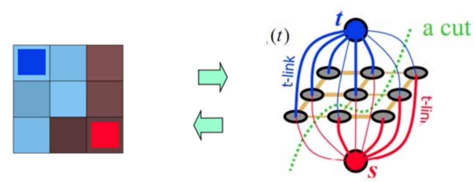
- set of links whose removal makes a graph disconnected
- cost of a cut: $$cut(A,B) = \sum_{p\in A, q\in B} w_{p,q}$$

Find minimum cut

- gives you a segmentation
- fast algorithms exist for doing this

Source: Steve Seitz

Graph cuts for interactive segmentation



Adding hard constraints:

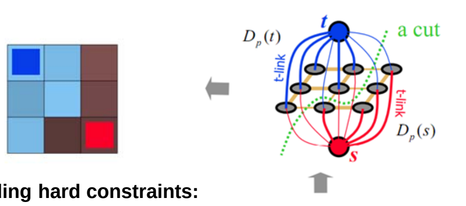
Add two additional nodes, object and background "terminals"

Link each pixel

- To both terminals
- To its neighboring pixels

Yuri Boykov

Graph cuts for interactive segmentation



Adding hard constraints:

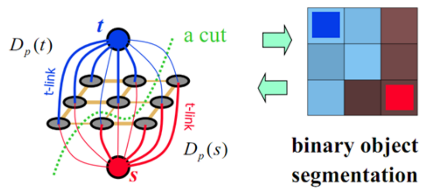
Let the edge weight to object or background terminal reflect similarity to the respective seed pixels.

$$D_p(s) = const - |I_p - I^s|$$
$$D_p(t) = const - |I_p - I^t|$$

Yuri Boykov

Graph cuts for interactive segmentation

$$E(L) = \underbrace{\sum_p -D_p(L_p)}_{\text{t-links}}^{\text{Regional term}} + \underbrace{\sum_{pq\in N} w_{pq}\cdot\delta(L_p \neq L_q)}_{\text{n-links}}^{\text{Boundary term}} \qquad L_p \in \{s,t\}$$

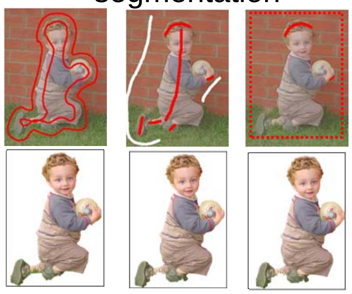


binary object segmentation

Boykov and Jolly (2001)

Yuri Boykov

Graph cuts for interactive segmentation



Another interaction modality: specify bounding box

Intelligent Scissors Mortensen and Barrett (1995)

Graph Cuts Boykov and Jolly (2001)

GrabCut Rother et al. (2004)

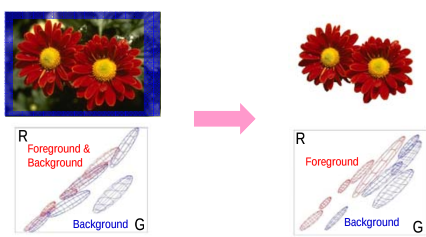
"Grab Cut"

- Loosely specify foreground region
- Iterated graph cut

Rother et al (2004)

"Grab Cut"

- Loosely specify foreground region
- Iterated graph cut



Gaussian Mixture Model (typically 5-8 components)

Rother et al (2004)

"Grab Cut"

Rother et al (2004)

Today

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  - Affine fit
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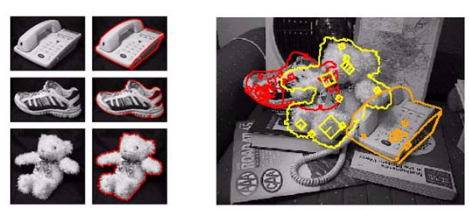
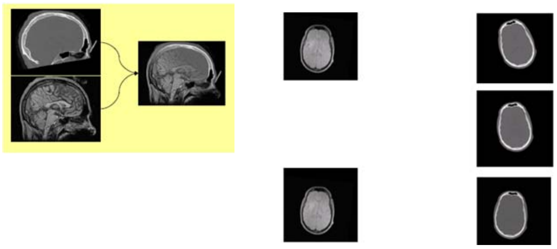
Motivation: Recognition



Figures from David Lowe

Motivation: medical image registration



Motivation: mosaics

(In detail next week)

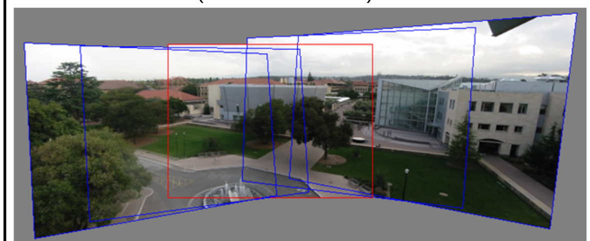
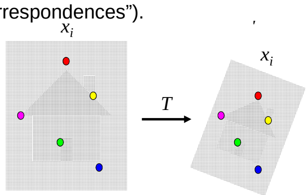


Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fa

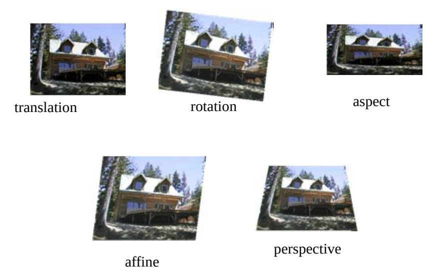
Alignment problem

- We have previously considered how to **fit a model to image evidence**
  - e.g., a line to edge points, or a snake to a deforming contour
- In alignment, we will **fit the parameters of some transformation** according to a set of matching feature pairs ("correspondences").



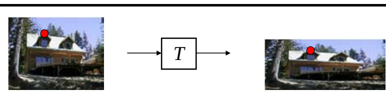
Parametric (global) warping

Examples of parametric warps:



Source: Alyosha Efros

Parametric (global) warping



p = (x,y) **p'** = (x',y')

Transformation T is a coordinate-changing machine:

p' = *T*(p)

What does it mean that *T* is **global**?

- Is the same for any point p
- can be described by just a few numbers (parameters)

Let's represent *T* as a matrix:

p' = **M**p

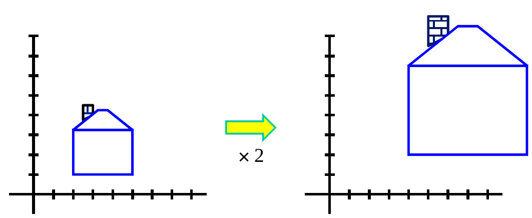
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Source: Alyosha Efros

Scaling

Scaling a coordinate means multiplying each of its components by a scalar

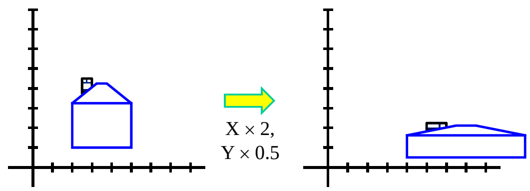
Uniform scaling means this scalar is the same for all components:



Source: Alyosha Efros

Scaling

Non-uniform scaling: different scalars per component:



Source: Alyosha Efros

Scaling

Scaling operation:

$$x' = ax$$
$$y' = by$$

Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix S}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Source: Alyosha Efros

What transformations can be represented with a 2x2 matrix?

2D Scaling?

$$x' = s_x * x$$
$$y' = s_y * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Rotate around (0,0)?

$$x' = \cos\Theta * x - \sin\Theta * y$$
$$y' = \sin\Theta * x + \cos\Theta * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$x' = x + sh_x * y$$
$$y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Source: Alyosha Efros

What transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Translation?

$$x' = x + t_x$$
$$y' = y + t_y$$

NO!

Source: Alyosha Efros

2D Linear Transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Only linear 2D transformations can be represented with a 2x2 matrix.

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

Source: Alyosha Efros

Homogeneous coordinates

To convert to homogeneous coordinates:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Homogeneous Coordinates

Q: How can we represent 2d translation as a 3x3 matrix using homogeneous coordinates?

$$x' = x + t_x$$
$$y' = y + t_y$$

A: Using the rightmost column:

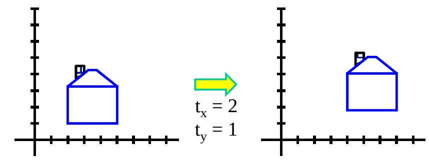
$$\textbf{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Source: Alyosha Efros

Translation

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



Source: Alyosha Efros

Basic 2D Transformations

Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} & 0 \\ \boldsymbol{sh_y} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

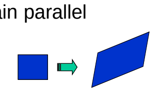
Source: Alyosha Efros

2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …
- Linear transformations, and
- Translations

Parallel lines remain parallel



Today

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- Feature-based alignment
  - 2D transformations
  - Affine fit
  - RANSAC

Alignment problem

- We have previously considered how to **fit a model to image evidence**
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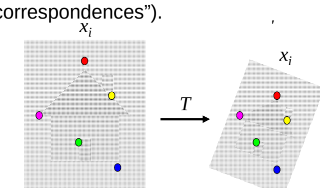

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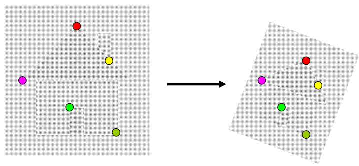
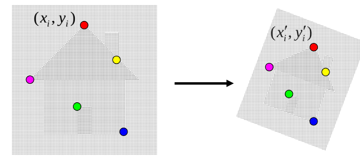
Image alignment



- Two broad approaches:
  - Direct (pixel-based) alignment
    - Search for alignment where most pixels agree
  - Feature-based alignment
    - Search for alignment where *extracted features* agree
    - Can be verified using pixel-based alignment

Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

An aside: Least Squares Example

Say we have a set of data points (X1,X1'), (X2,X2'), (X3,X3'), etc. (e.g. person's height vs. weight)

We want a nice compact formula (a line) to predict X's from Xs: Xa + b = X'

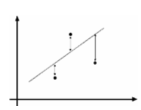
We want to find a and b

How many (X,X') pairs do we need?

$$X_1 a + b = X_1' \qquad X_2 a + b = X_2' \qquad \begin{bmatrix} X_1 & 1 \\ X_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} X_1' \\ X_2' \end{bmatrix} \qquad \text{Ax=B}$$

What if the data is noisy?

$$\begin{bmatrix} X_1 & 1 \\ X_2 & 1 \\ X_3 & 1 \\ \dots & \dots \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ \dots \end{bmatrix} \qquad \min \|Ax - B\|^2$$

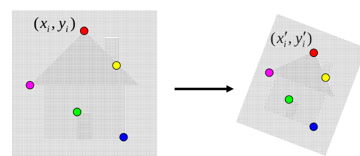


overconstrained

Source: Alyosha Efros

Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \qquad \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- How many matches (correspondence pairs) do we need to solve for the transformation parameters?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for (x_{new}, y_{new}) ?
- Where do the matches come from?

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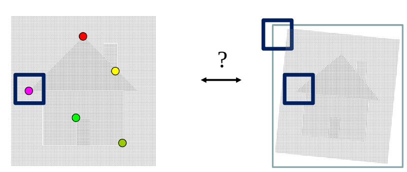
What **are** the correspondences?



- Compare content in **local** patches, find best matches.
  e.g., simplest approach: scan with template, and compute SSD or correlation between list of pixel intensities in the patch
- Later in the course: how to select regions according to the geometric changes, and more robust descriptors.

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Fitting an affine transformation

Affine model approximates perspective projection of planar objects.

Figures from David Lowe, ICCV 1999

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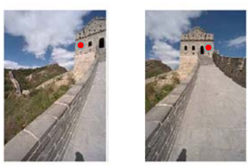
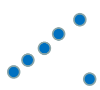
Outliers

- **Outliers** can hurt the quality of our parameter estimates, e.g.,
  - an erroneous pair of matching points from two images
  - an edge point that is noise, or doesn't belong to the line we are fitting.





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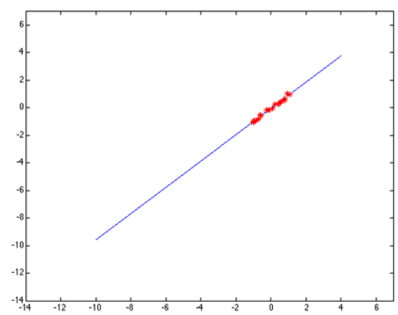
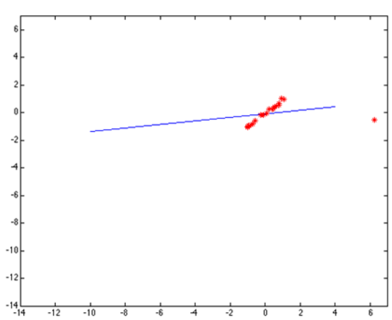
Outliers affect least squares fit



Outliers affect least squares fit



RANSAC

- RANdom Sample Consensus
- **Approach**: we want to avoid the impact of outliers, so let's look for "inliers", and use those only.
- **Intuition**: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.

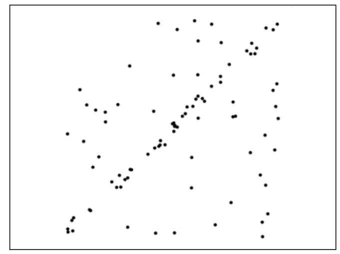
RANSAC: General form

- RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute estimate of transformation on all of the inliers

- Keep the transformation with the largest number of inliers

RANSAC for line fitting example



Source: R. Raguram

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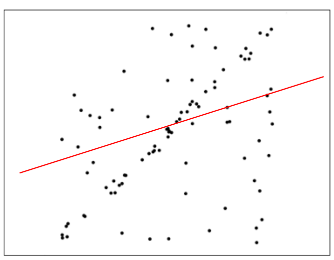
RANSAC for line fitting example



Least-squares fit

Source: R. Raguram

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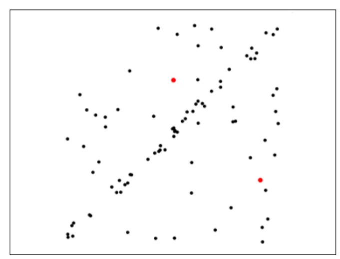
RANSAC for line fitting example



1. Randomly select minimal subset of points

Source: R. Raguram

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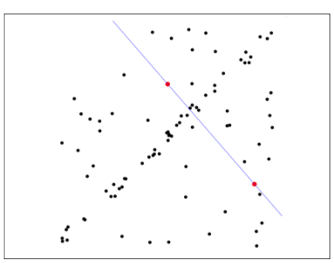
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model

Source: R. Raguram

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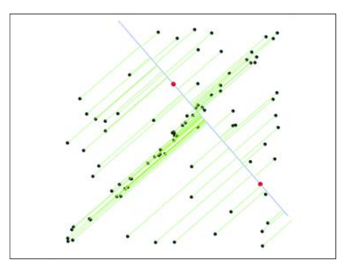
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

Source: R. Raguram

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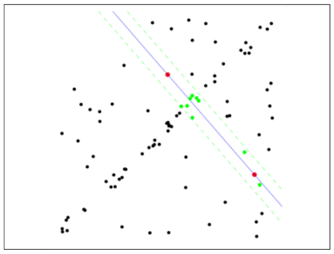
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

Source: R. Raguram

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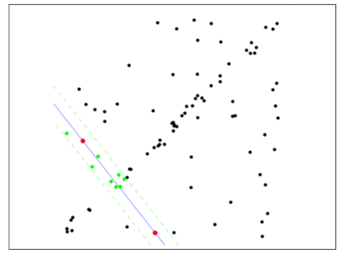
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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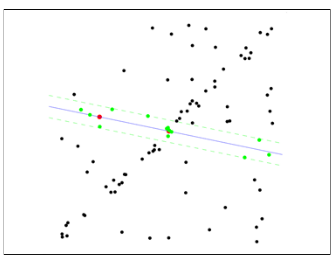
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

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Source: R. Raguram

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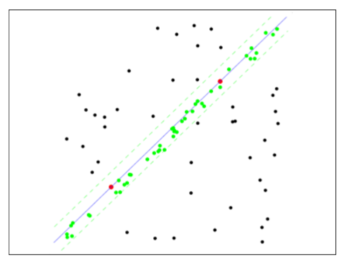
RANSAC for line fitting example

Uncontaminated sample



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

64

Source: R. Raguram

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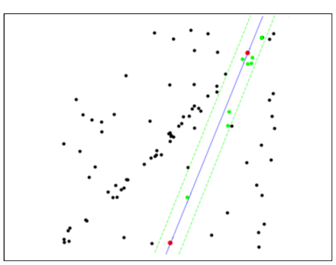
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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RANSAC for line fitting

Repeat ***N*** times:

- Draw ***s*** points uniformly at random
- Fit line to these ***s*** points
- Find inliers to this line among the remaining points (i.e., points whose distance from the line is less than ***t***)
- If there are ***d*** or more inliers, accept the line and refit using all inliers

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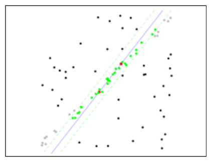
RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Lots of parameters to tune
  - Doesn't work well for low inlier ratios (too many iterations, or can fail completely)
  - Can't always get a good initialization of the model based on the minimum number of samples



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Today

- Interactive segmentation
- Feature-based alignment
  - 2D transformations
  - Affine fit
  - RANSAC

Coming up: alignment and image stitching