CS395T: Numerical Optimization for Graphics and AI: Homework V

1 Guideline

- Please complete **3** problems out of **5** problems, and please complete at least one problem in the theory session.
- You are welcome to complete more problems.

2 Programming

Each problem in this section counts as two.

Problem 1 and Problem 2. Implement at least two solvers (e.g., simplex method, barrier method, interior point method and augmented Lagragian method) for solving linear systems in the standard form:

$$\begin{aligned} \text{minimize} \quad & \langle \boldsymbol{c}, \boldsymbol{x} \rangle \\ \text{subject to} \quad & A\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq 0 \end{aligned} \tag{1}$$

For testing data, you may choose $A = \text{rand}(100, 200)$, $\boldsymbol{b} = \text{rand}(100, 1)$ and $\boldsymbol{c} = \text{rand}(200, 1)$.

3 Theory

Problem 3.

- Given a symmetric matrix $A \in \mathbb{R}^{n \times n}$. Suppose we look for approximating A in the space of positive definite matrices, i.e.,
$$X^\star = \min_{X \succeq 0} \ \|X - A\|_{\mathcal{F}}^2.$$
Then $X^\star = U \max(\Sigma, 0) U^T$, where $A = U\Sigma U^T$ is the spectral decomposition of A.
- Given a matrix $A \in \mathbb{R}^{n \times m}$. Suppose we look for a rank r approximation of A, i.e.,
$$X^\star = \min_{rank(X)=r} \ \|X - A\|_{\mathcal{F}}^2.$$
Then $X^\star = U_r \Sigma_r V_r^T$, where U_r, Σ_r and V_r encode the first r singular vectors and singular values of $A = U\Sigma V^T$.

Problem 4. Construct a linear program, where the simplex method does not converge in polynomial time.

Problem 5. In the class we have covered augmented Lagrangian method. Please apply it to the following problem and analyzes its convergence property:

$$\begin{aligned} \text{minimize} \quad & \|A\|_\star + \lambda \|E\|_1 \\ \text{subject to} \quad & D = A + E \end{aligned} \tag{2}$$

where $D \in \mathbb{R}^{m \times n}$ is a given matrix, $\|A\|_\star = \sum_{i=1}^{\min(m,n)} \sigma_i(A)$ is the sum of the singular values of A and $\|E\|_1 = \sum_{i,j} |E_{i,j}|$ is the sum of element-wise absolute values.