CS395T: Numerical Optimization for Graphics and AI: Homework II

1 Guideline

- Please complete **3** problems out of **6** problems, and please complete at least one problem in the theory session.
- You are welcome to complete more problems.

2 Programming

Each problem in this section counts as one.
Attention. The underlying cameras of the images involved in these three problems are different.
Problem 1. In this problem, we are interested in solving the camera calibration problem. This amounts to predict the intrinsic and extrinsic camera parameters. Mark feature points are provided. Please submit your results for

- The estimated projection matrix;
- The factored intrinsic and extrinsic camera parameters;
- (**Optional**) Apply bundle adjustment (i.e., local optimization) to improve the results.

Problem 2. In this problem, we are interested in two-view reconstruction. This amounts to solve both the intrinsic camera parameters, the relative transformation, and 3D locations of the mark feature points. Please submit your results for

1. Fundamental matrix;
2. Intrinsic camera parameters;
3. Extrinsic camera parameters;
4. 3D locations of the marked feature points;

Note that without assumptions on the intrinsic camera parameters, the reconstruction is not unique. Moreover, to improve the quality of the reconstruction, you may perform bundle adjustment to improve the solution.
Problem 3. In this problem, we are interested in solving the multi-view reconstruction problem. This amounts to detect features from the input images, compute consistent feature correspondences, estimate the intrinsic/extrinsic camera parameters, and compute 3D locations of the corresponding feature points. Please submit your results for

- Detected feature points;
- Consistent correspondences;
- Fundamental matrices;

- Intrinsic camera parameters;
- Extrinsic camera parameters;
- 3D locations of the detected feature points;

Please consider using the SIFT features and descriptors for the feature points. For this particular problem, you can think of using the first image as the base-image and estimates the correspondences to other images and initialize the intrinsic and extrinsic camera parameters. However, it is recommended to jointly optimize all the variables in the end.

Data can be downloaded from `www.cs.utexas.edu/~huangqx/CS395T_HomeworkII_data.zip`.

3 Theory

Problem 4. A rank condition for epipolar constraint. Show that

$$\boldsymbol{x}_2^T\big((T\times)R\big)\boldsymbol{x}_1 = 0$$

if and only if

$$\text{rank}[(\boldsymbol{x}_2\times)R\boldsymbol{x}_1, (\boldsymbol{x}_2\times T] \leq 1.$$

Problem 5.

- Given a symmetric matrix $A \in \mathbb{R}^{n\times n}$. Suppose we look for approximating A in the space of positive definite matrices, i.e.,
$$X^\star = \min_{X\succeq 0} \ \|X - A\|_{\mathcal{F}}^2.$$
Then $X^\star = U\max(\Sigma, 0)U^T$, where $A = U\Sigma U^T$ is the spectral decomposition of A.
- Given a matrix $A \in \mathbb{R}^{n\times m}$. Suppose we look for a rank r approximation of A, i.e.,
$$X^\star = \min_{rank(X)=r} \ \|X - A\|_{\mathcal{F}}^2.$$
Then $X^\star = U_r\Sigma_r V_r^T$, where U_r, Σ_r and V_r encode the first r singular vectors and singular values of $A = U\Sigma V^T$.

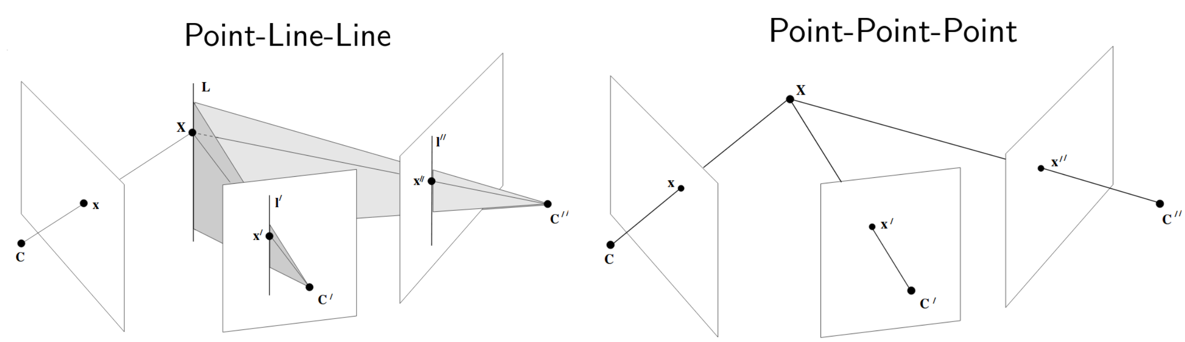

Figure 1: Illustration of three-view reconstruction under point-line-line constraints(Left) and point-point-point constraints(Right).

Problem 6. In this problem, we study trilinear relations. We have studied correspondences of type "line-line-line" in class. This problem asks you to show similar properties for two other types of correspondences:

- **point-line-line.** Show that the correspondence $(\boldsymbol{x}, \boldsymbol{l}', \boldsymbol{l}'')$ satisfies the following constraint:

$$\boldsymbol{l}'^T\big(\sum_i x^i T_i\big)\boldsymbol{l}'' = 0 \tag{1}$$

- **point-point-point.** Show that the correspondence $(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{x}'')$ satisfies the following constraint:

$$\boldsymbol{x}' \times \big(\sum_i x^i T_i\big)(\boldsymbol{x}'' \times) = 0 \tag{2}$$

Across these three cases, the tensor (T_1, T_2, T_3) is only dependent on the projection matrices associated with the input images.