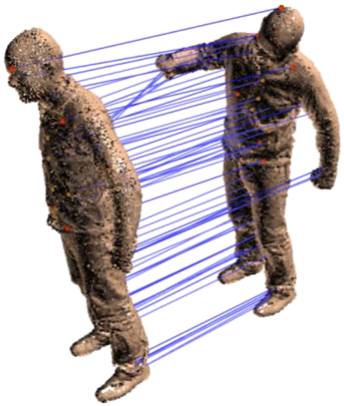
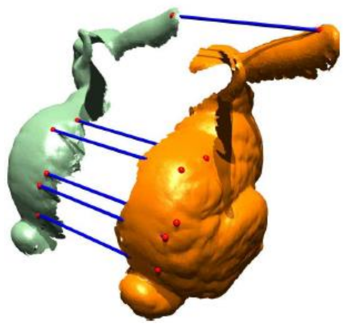
Data-Driven Geometry Processing
Map Synchronization I

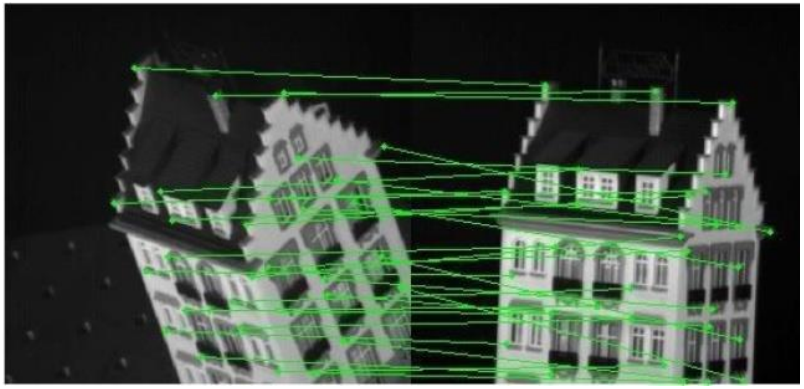
Qixing Huang

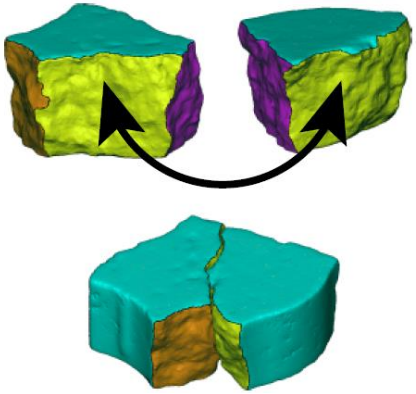
Nov. 28th 2018

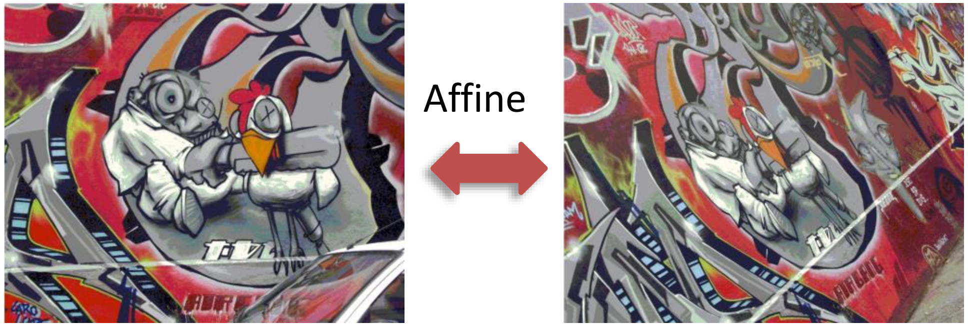
Shape matching



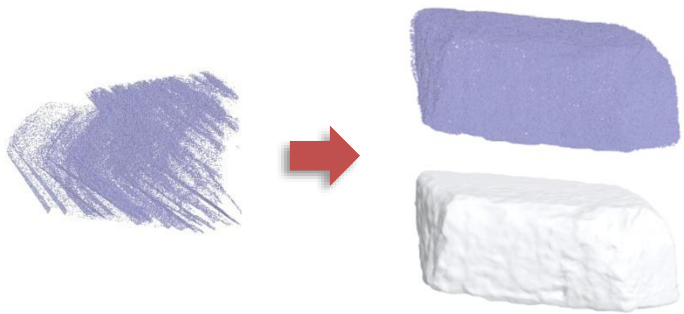




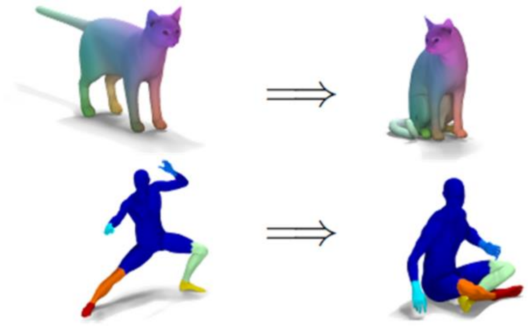




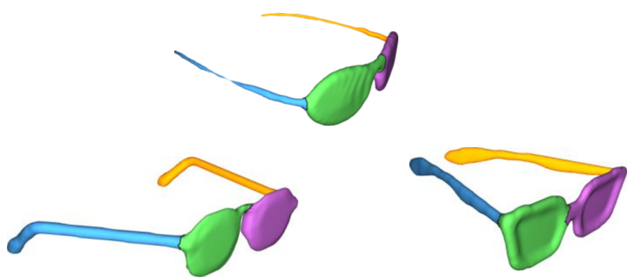
Applications



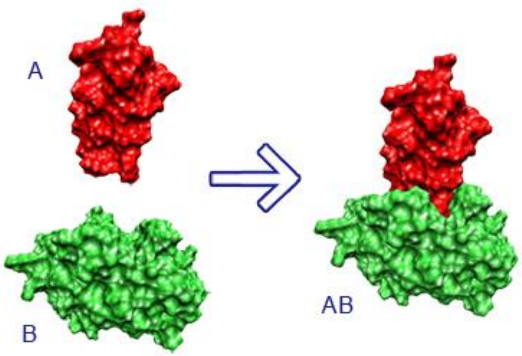
Shape reconstruction



Transfer information



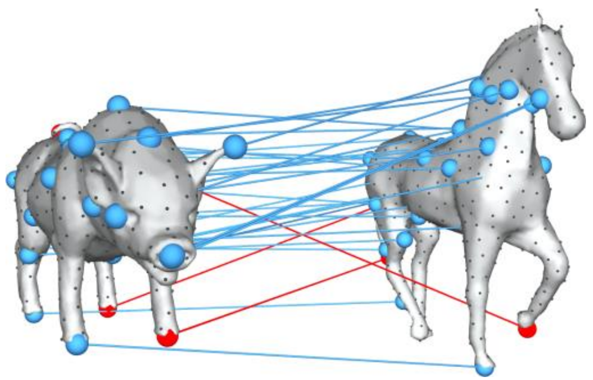
Aggregate information



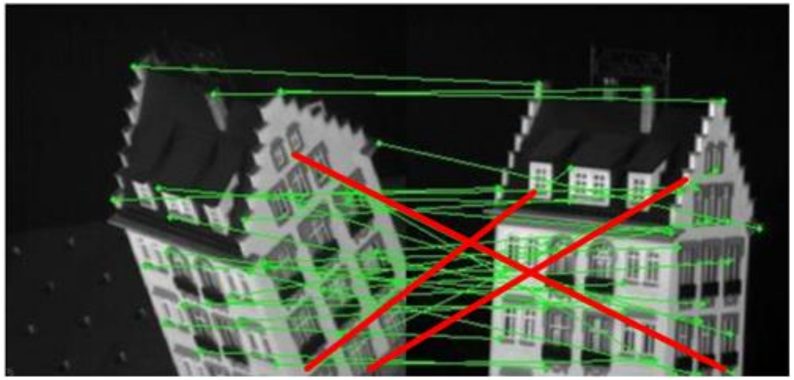
Protein docking

Pair-wise matching is unreliable

State-of-the-art techniques:

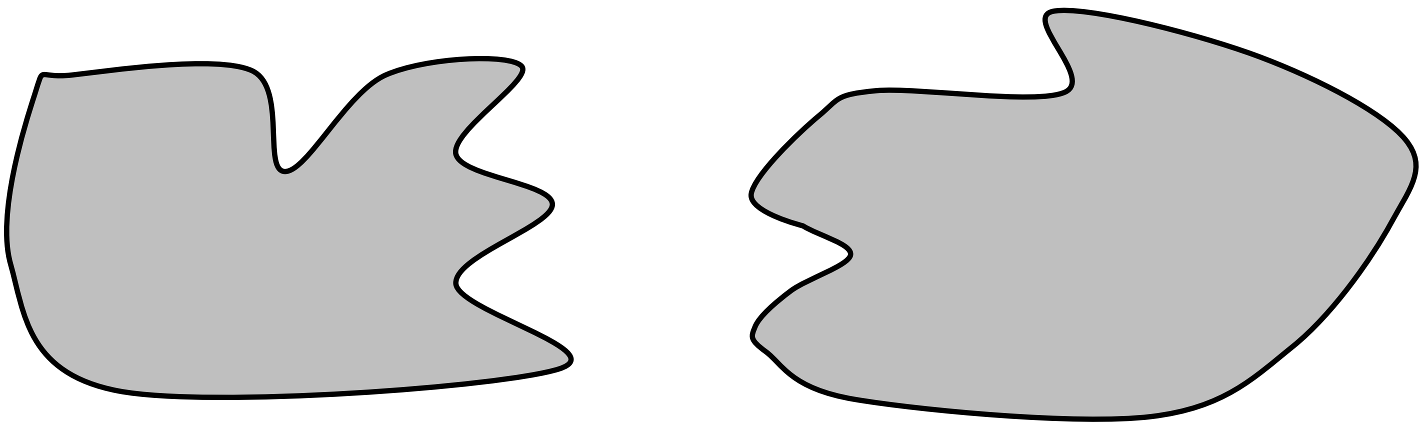


Blended intrinsic maps
[Kim et al. 11]

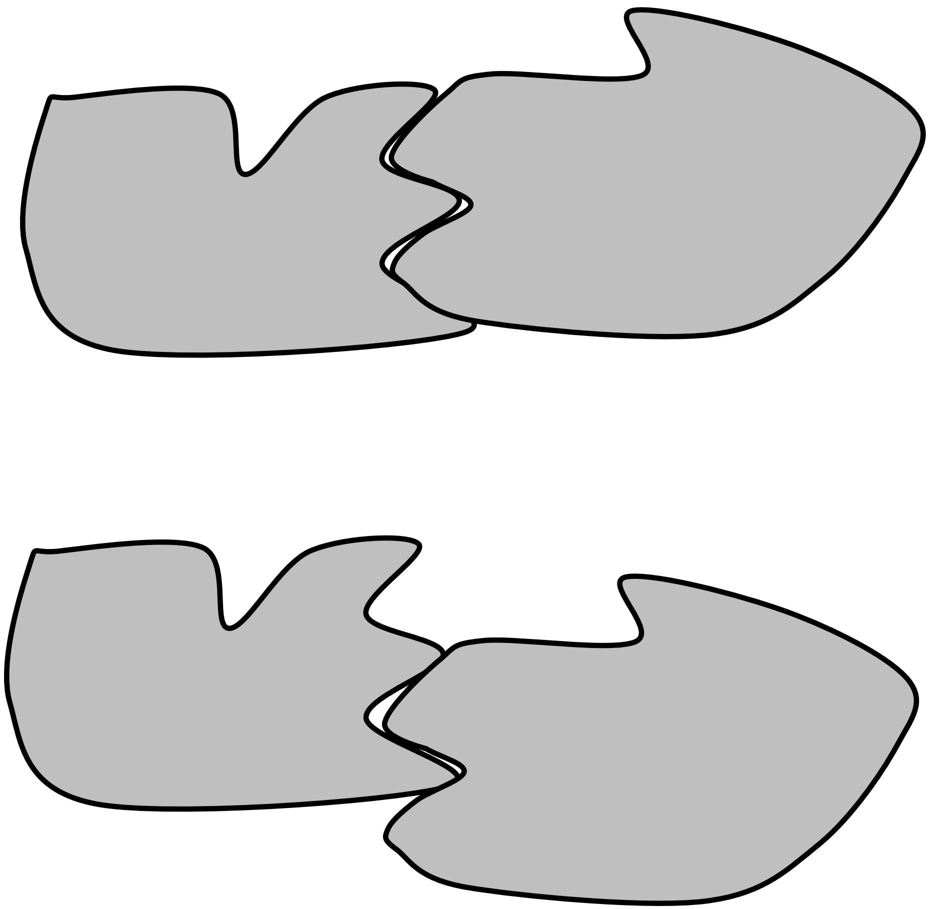


Learning-based graph matching
[Leordeanu et al. 12]

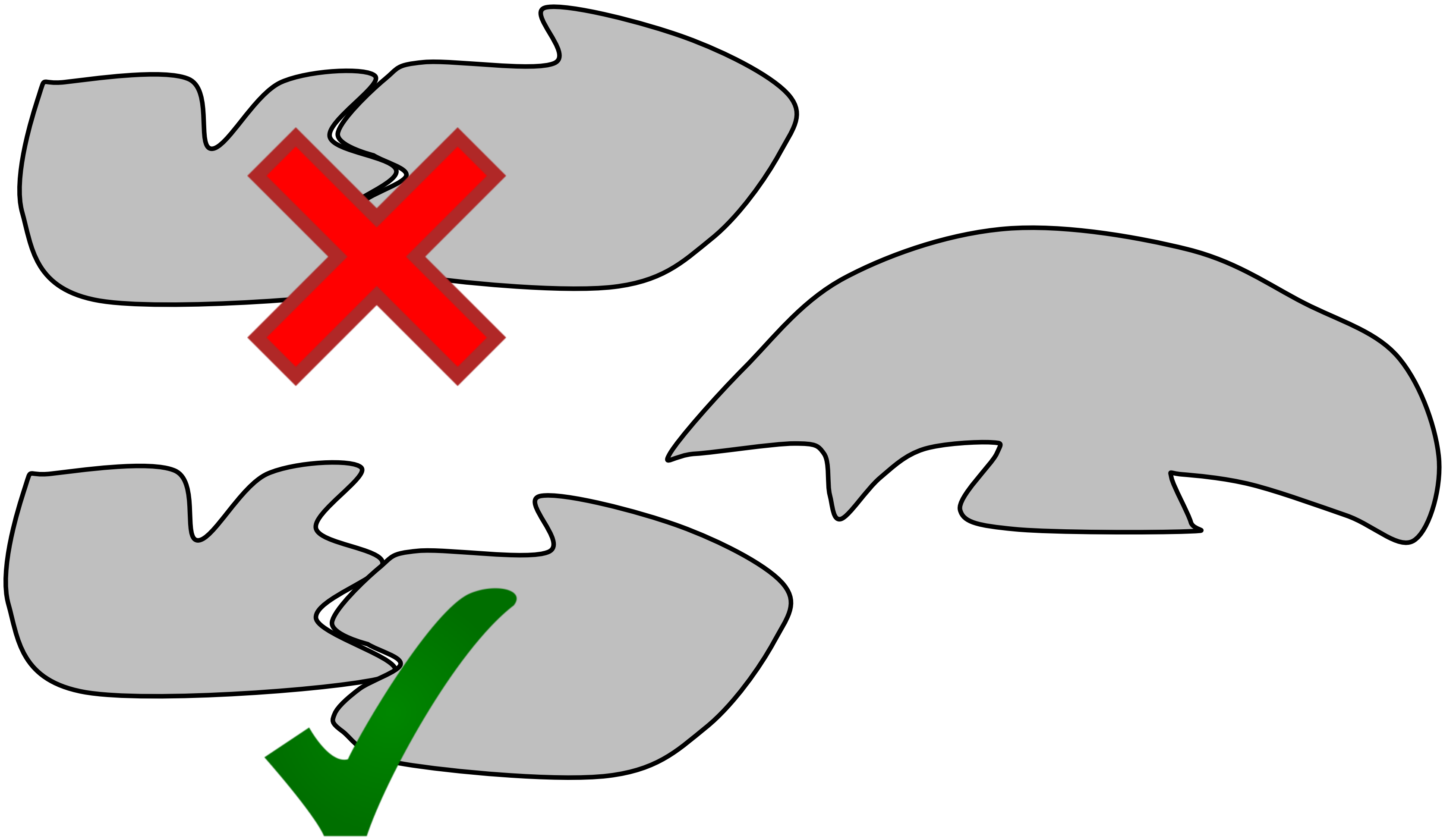
Piece assembly



Ambiguous matches

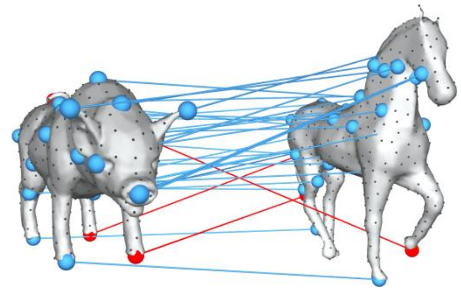


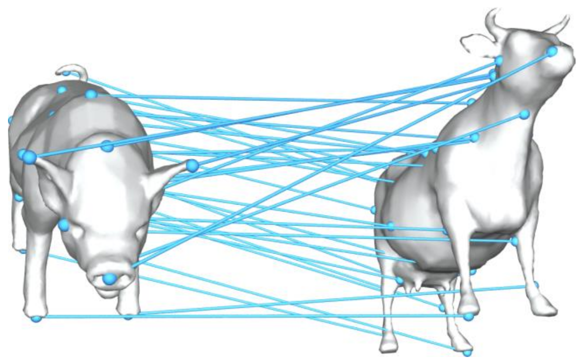
Additional data helps



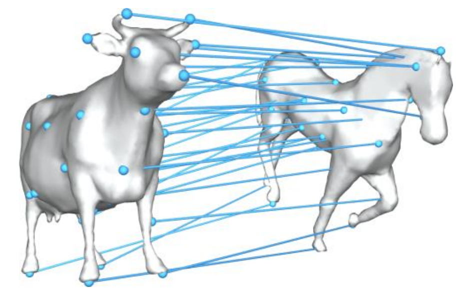
Additional data helps

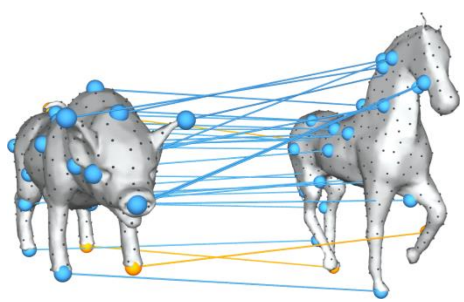
Blended intrinsic maps
[Kim et al. 11]





← Intermediate object →

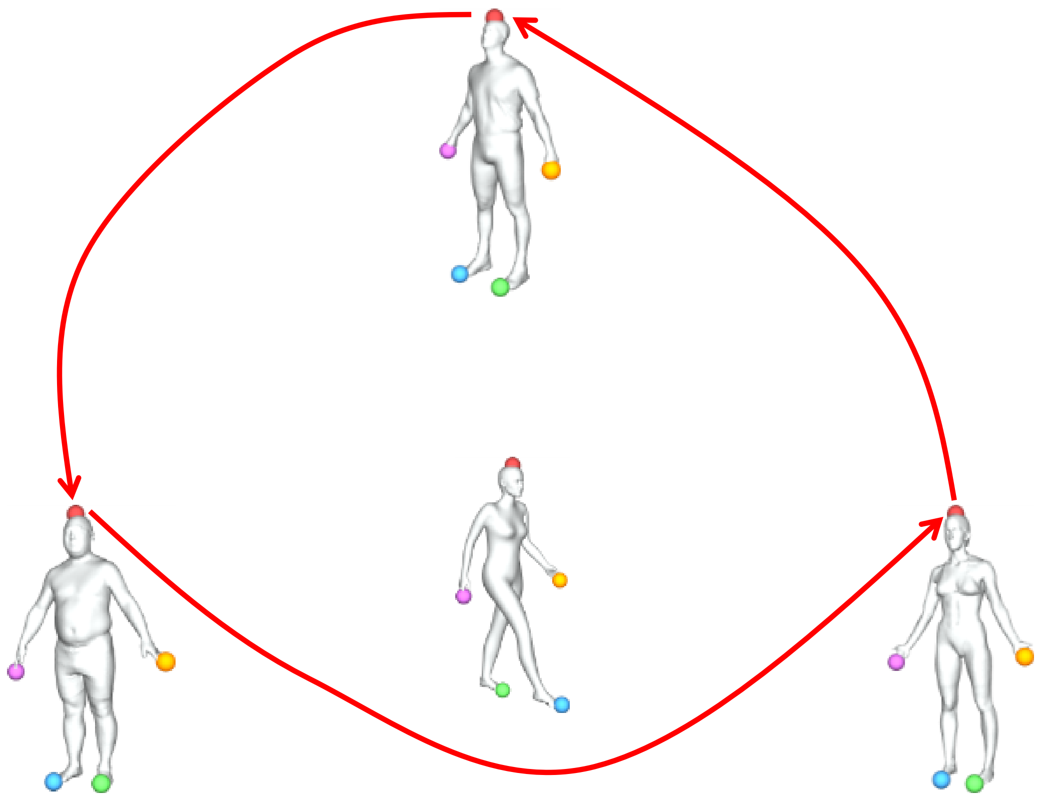




Composite

Cycle-consistency

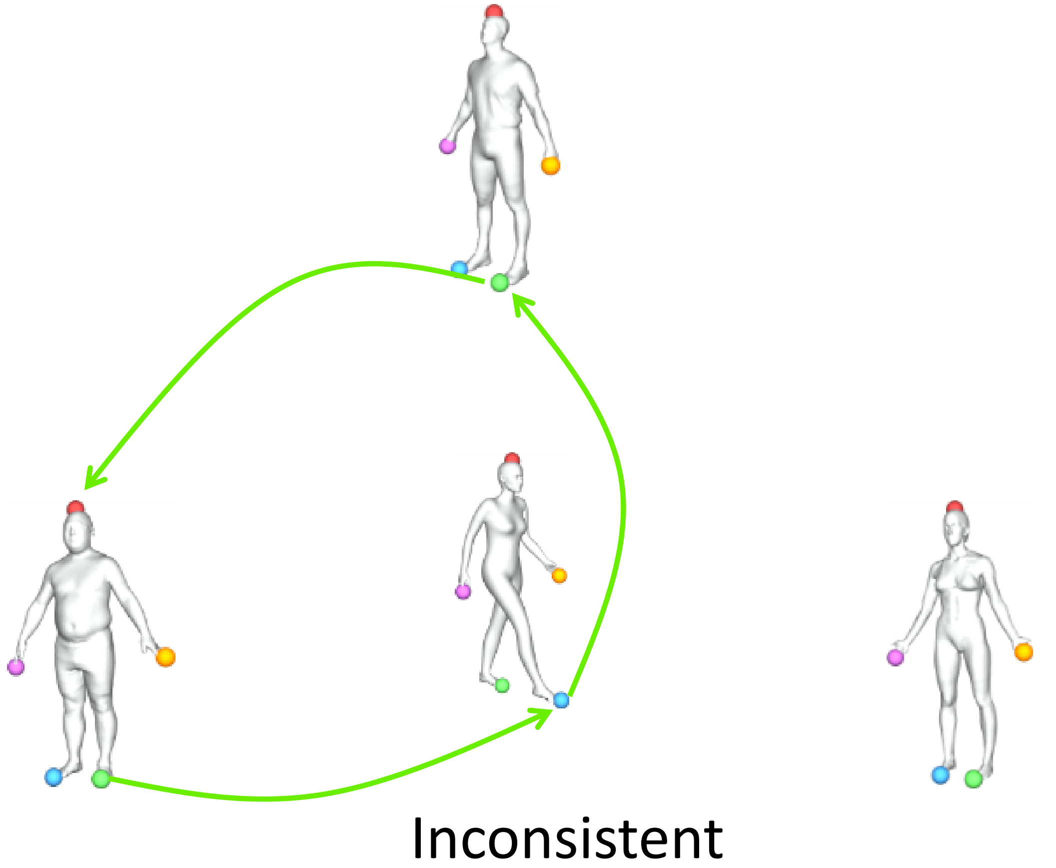
- Maps are consistent along cycles



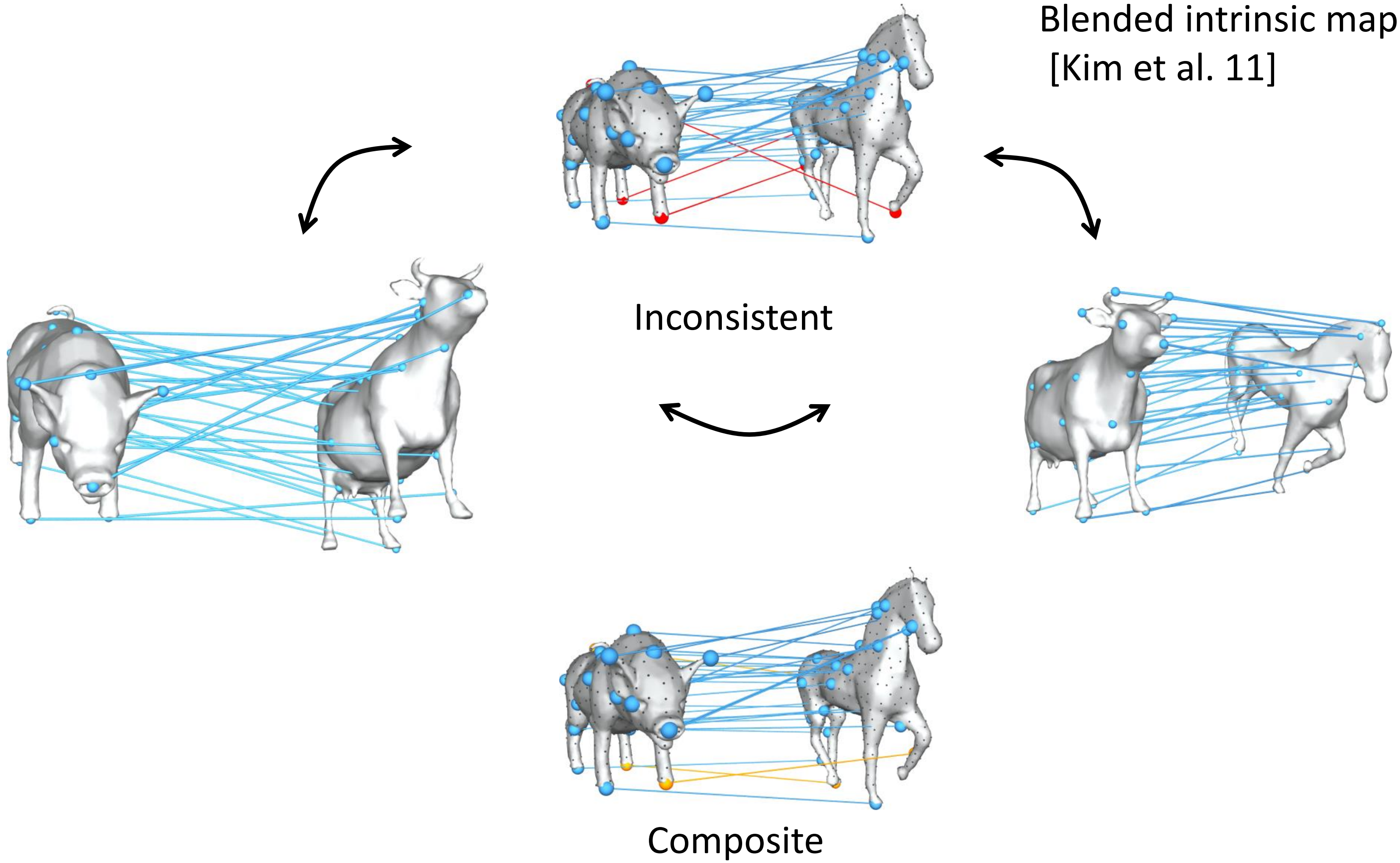
Consistent

Cycle-consistency

- Maps are consistent along cycles



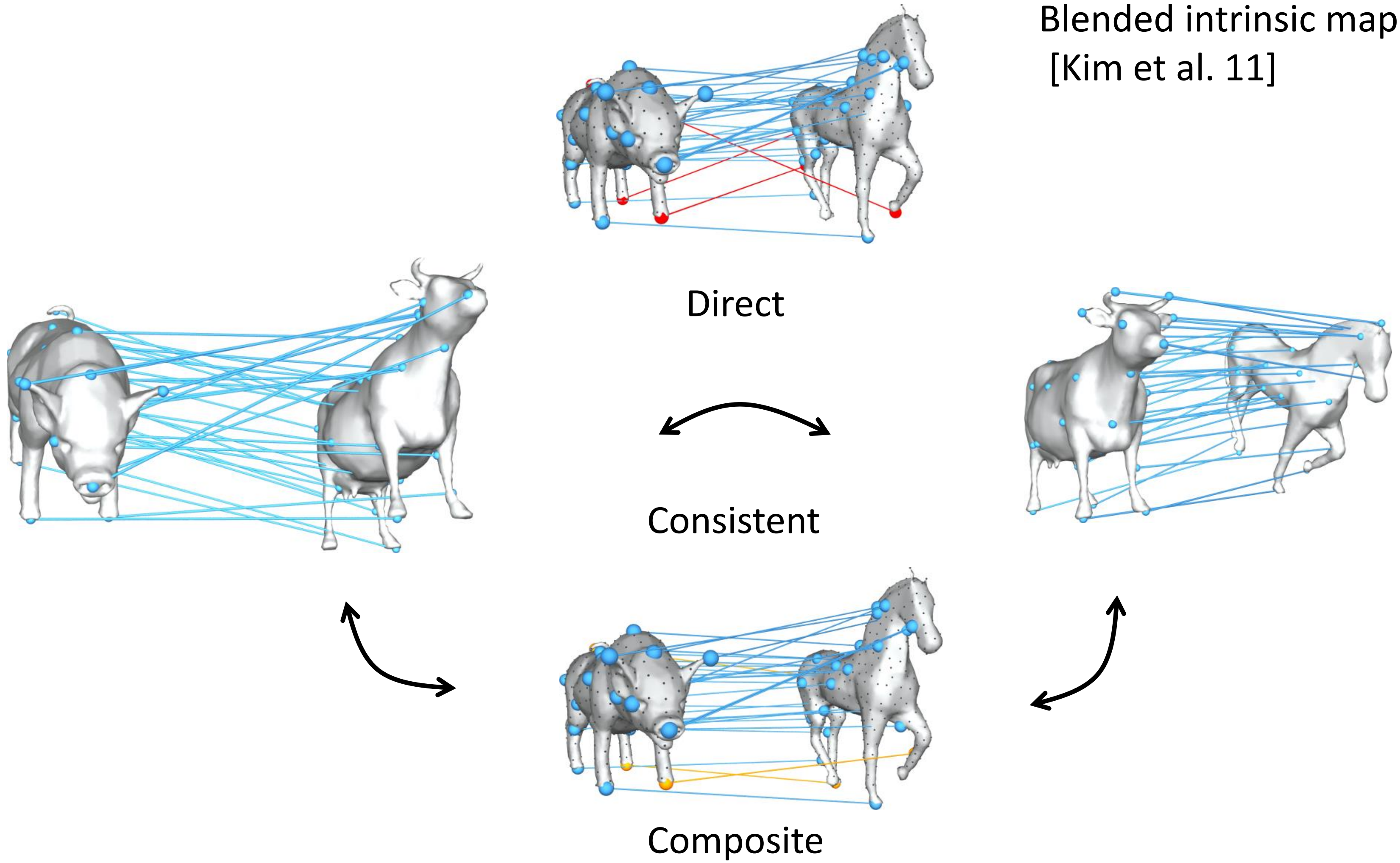
Cycle-consistency



Blended intrinsic maps
[Kim et al. 11]

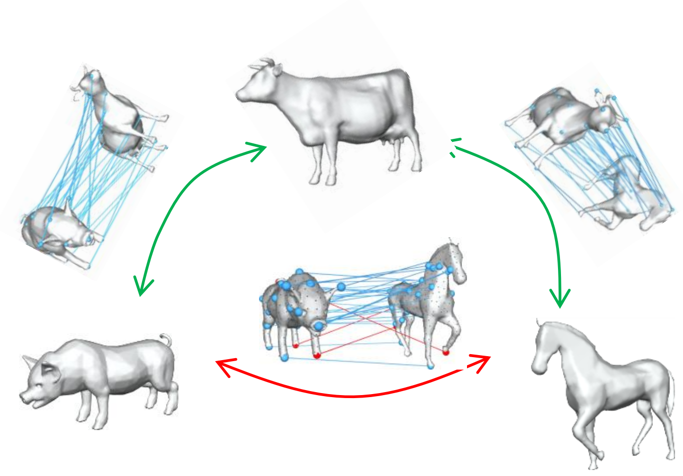
Cycle-consistency

Blended intrinsic maps
[Kim et al. 11]



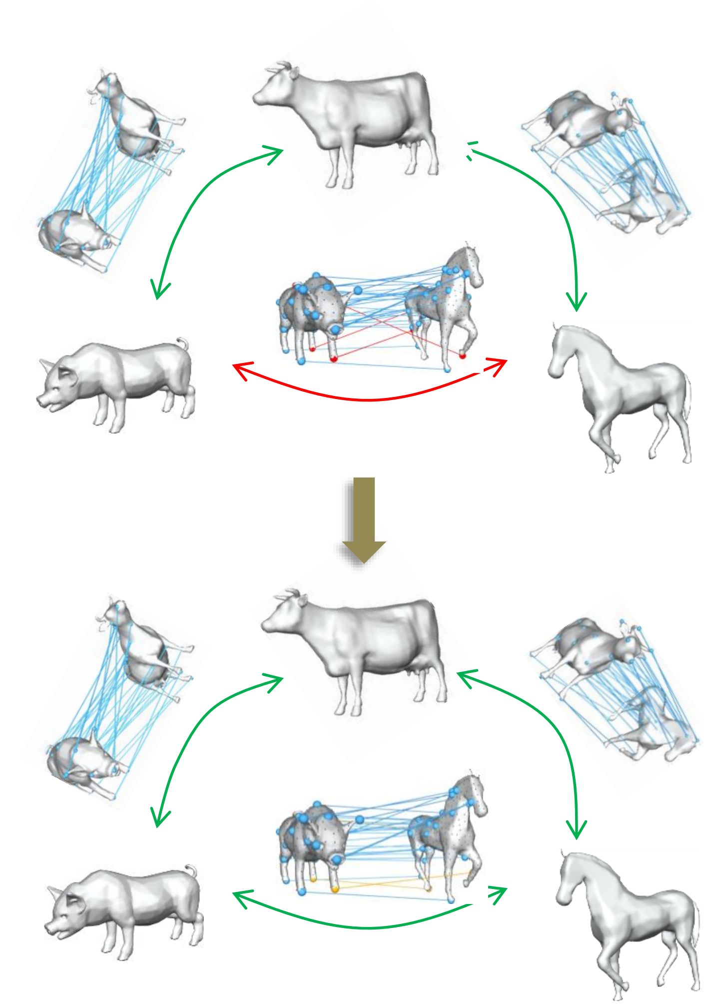
Joint matching formulation

- Input:
  - Shapes
  - Pair-wise maps
    (existing algorithms)



Joint matching formulation

- Input:
  - Shapes
  - Pair-wise maps
    (existing algorithms)



- Output:
  - Cycle-consistent
  - "Close" to the input maps

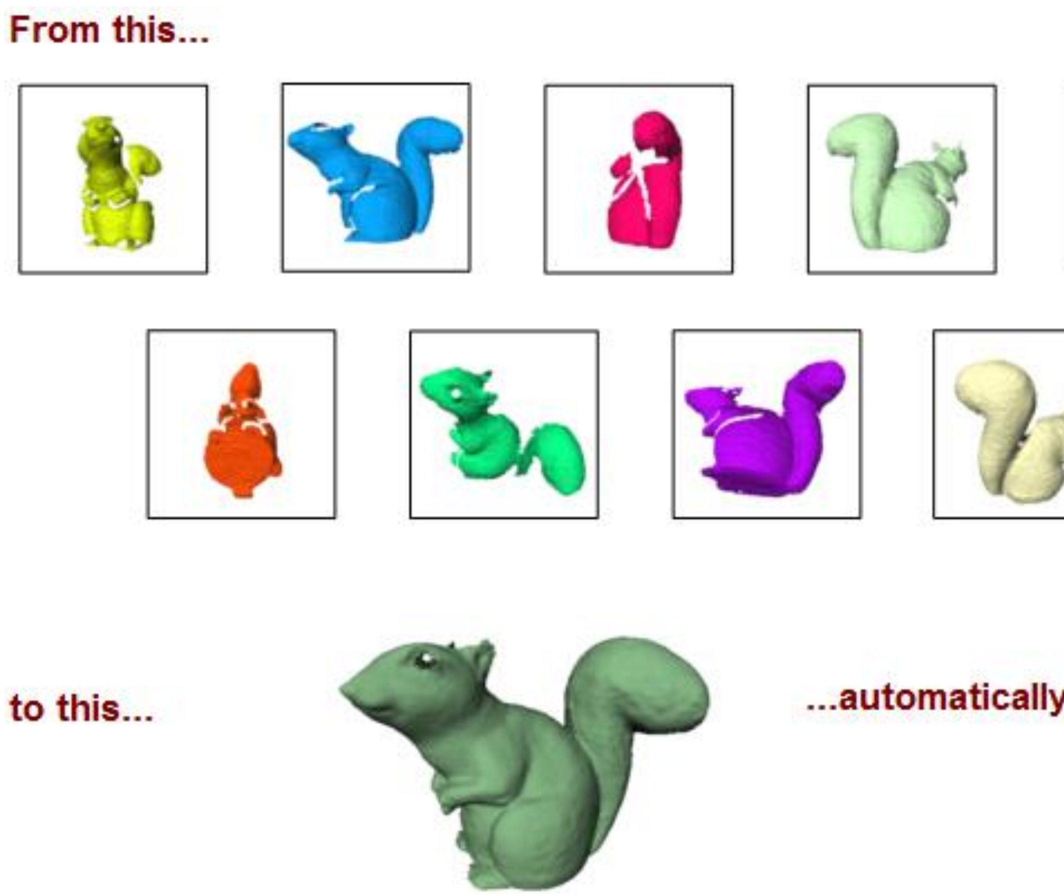
NP-complete [Huber 2002]

Outline

- Three existing approaches
  - Spanning tree based
  - Inconsistent cycle detection
  - Spectral techniques

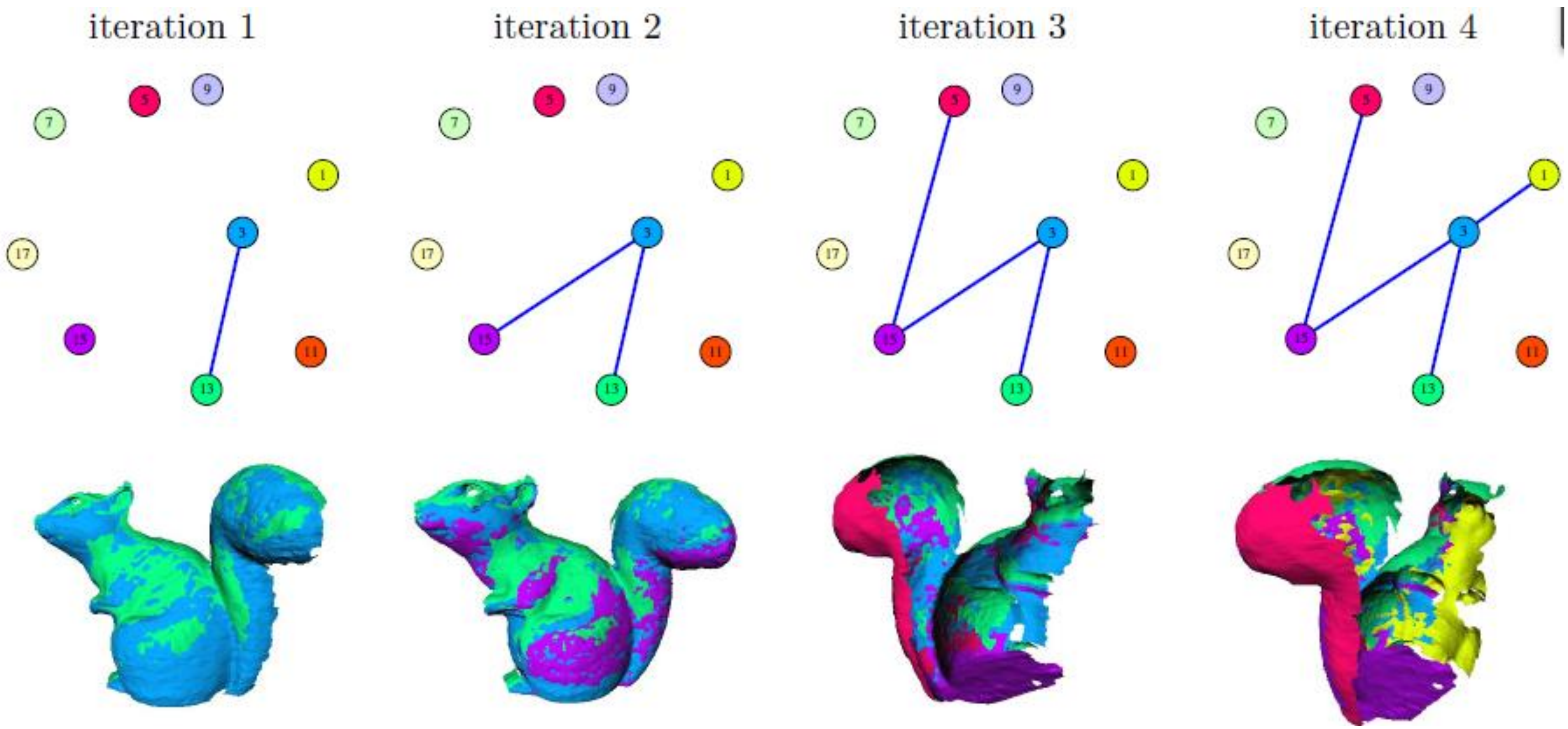
- Convex optimization framework

Spanning tree based

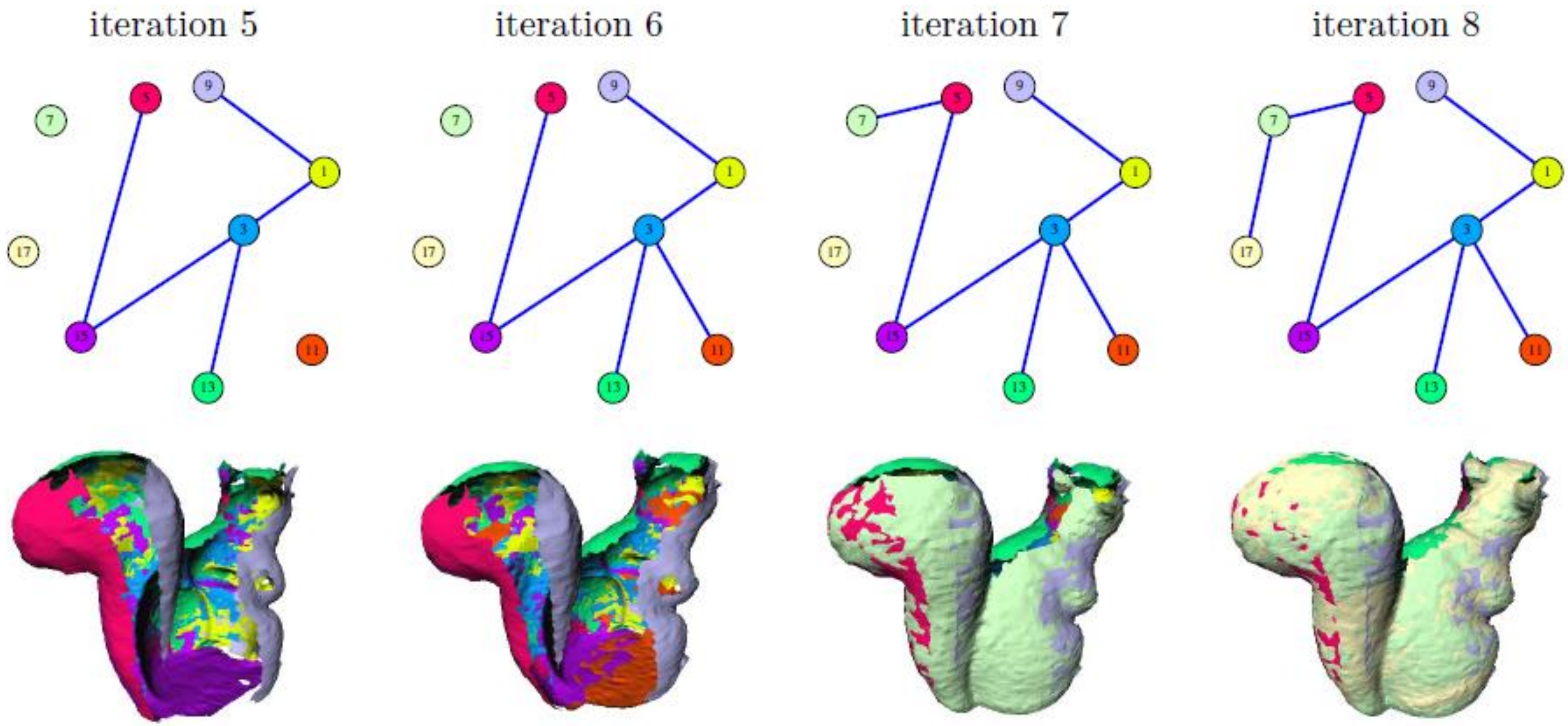


Automatic Three-dimensional Modeling from Reality, PhD thesis, D. Huber, Robotics Institute, Carnegie Mellon University, 2002

Spanning tree based

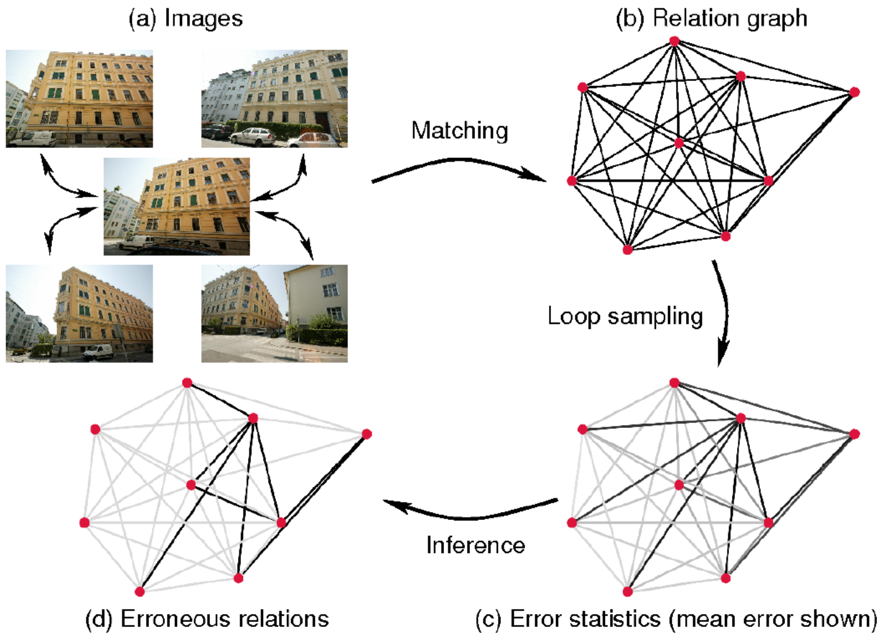


Spanning tree based



Issue: A single incorrect match can destroy everything

Detecting inconsistent cycles



large for inconsistent cycles

maximize $$\sum_L \rho_L x_L$$

subject to

$$x_L \geq x_e \; \forall e \in L,$$

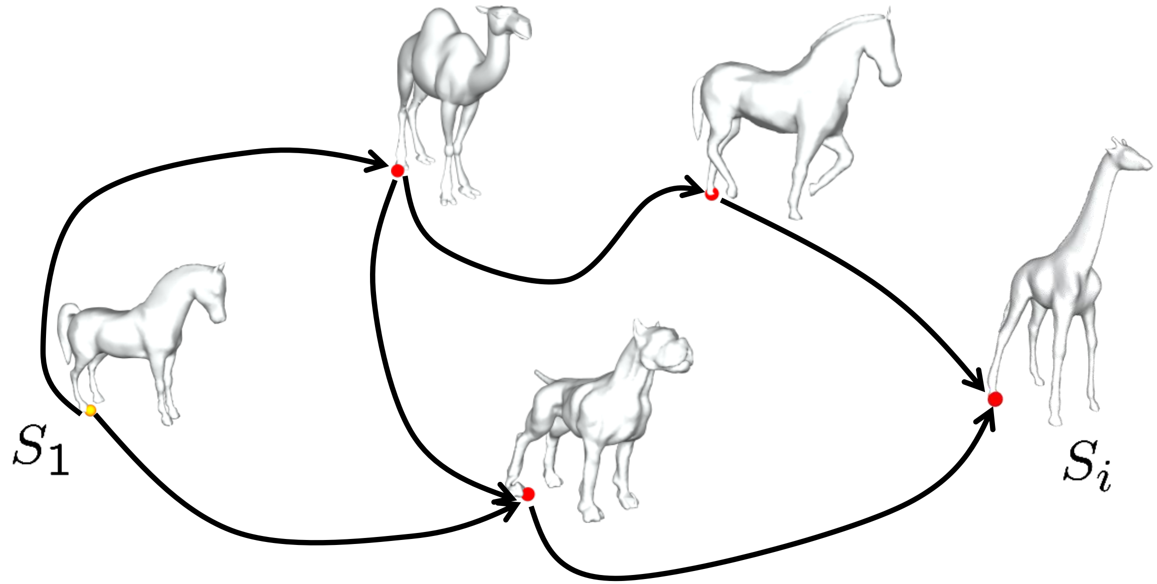
$$x_L \leq \sum_{e \in L} x_e,$$

$$x_L \in [0, 1],$$

$$x_e \in [0, 1].$$

Disambiguating visual relations using loop constraints, C. Zach, M. Klopscjotz, and M. POLLEFEYS, *CVPR'10*

Spectral techniques



$$C_{1i}^{output} = \sum_{k=1}^{k_{max}} \alpha_k C_{1i}^{(k)}$$

$$C_{1i}^{(k)} = \sum X_{ji}^{initial} C_{ij}^{(k-1)}$$

The resulting soft map

From paths of length *k*

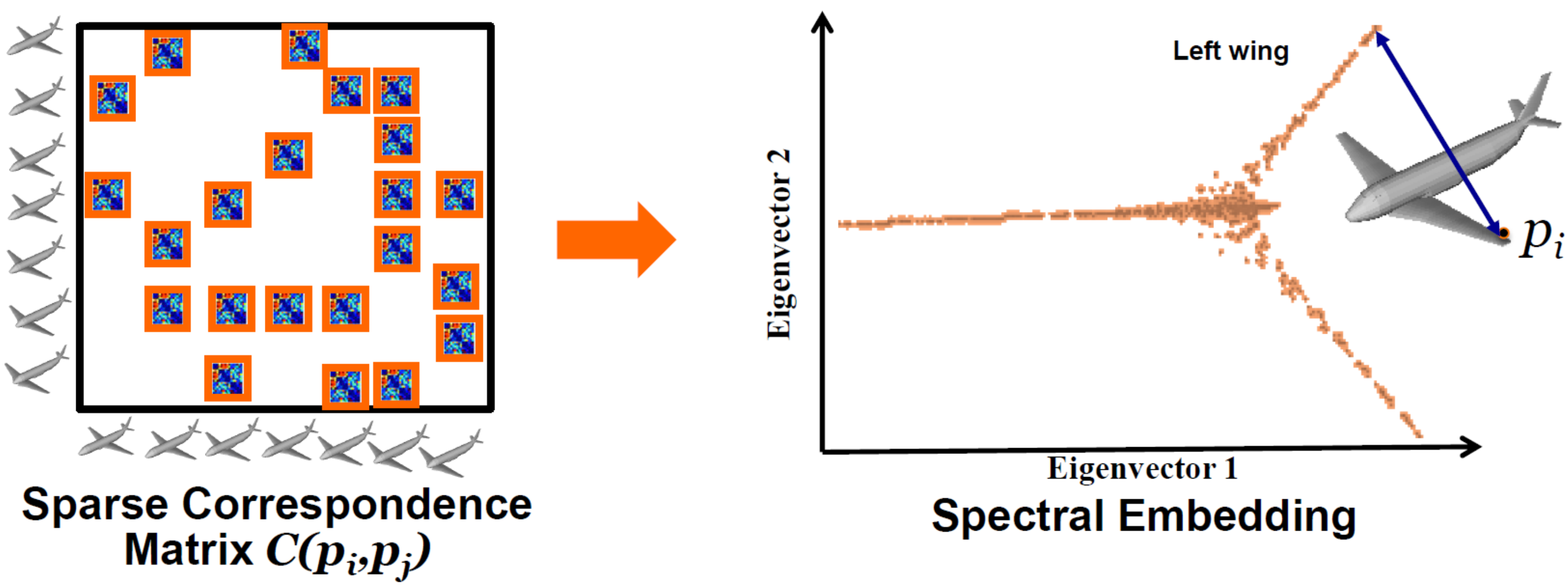
Diffusion

Multiplication and aggregation of mapping matrices

An Optimization Approach for Extracting and Encoding Consistent Maps in a Shape Collection, Q. Huang, G. Zhang, L. Gao, S. Hu, A. Bustcher, L. Guibas, SIGGRAPH ASIA'12

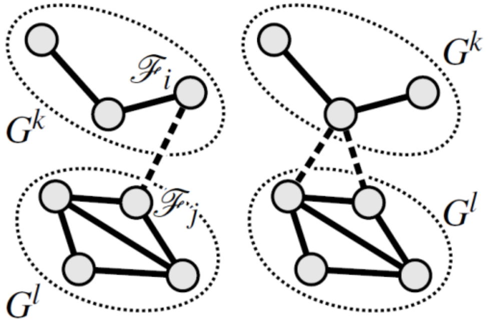
Spectral techniques

- Compute fuzzy correspondence based on diffusion distance in graph represented by

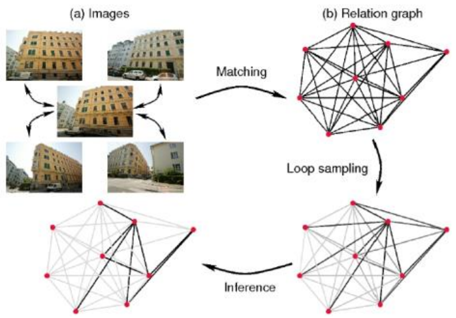


Exploring collections of 3d models using fuzzy correspondences, V. G. Kim, W. Li, N. Mitra, S. Diverdi, and T. Funkhouser, SIGGRAPH'12

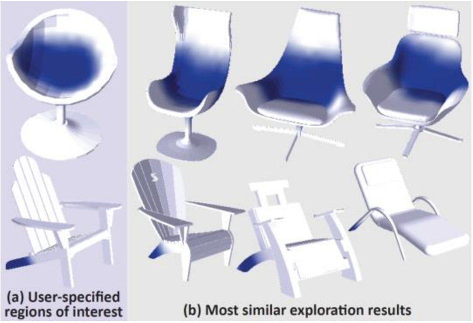
Approaches we have discussed



Spanning tree optimization
[Huber and Hebert 03]



Detecting Inconsistent Cycles
[Zach et al. 10, Nguyen et al. 11]



Spectral techniques
[Kim et al.12,
Pachauri et al.13]

- Cons:
  - --- Many parameters to set
  - --- Does it converge?
  - --- Run quickly?

Outline

- Three existing approaches
  - Spanning tree based
  - Inconsistent cycle detection
  - Spectral techniques

- Convex optimization framework

Convex optimization framework

Parameter-free!

Near-optimal!

Efficient!

MAP Estimation

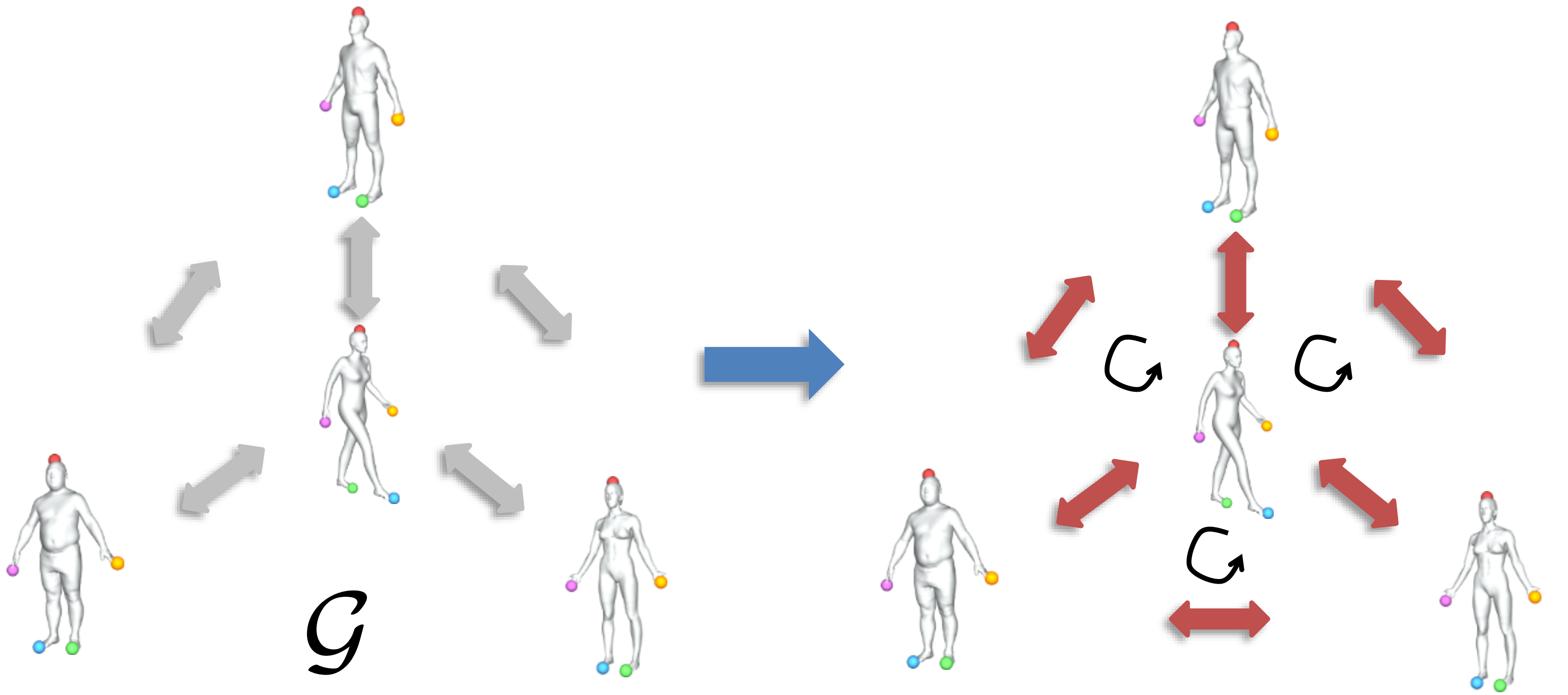
MAX-CUT

Compressed Sensing

Low-rank Matrix Recovery

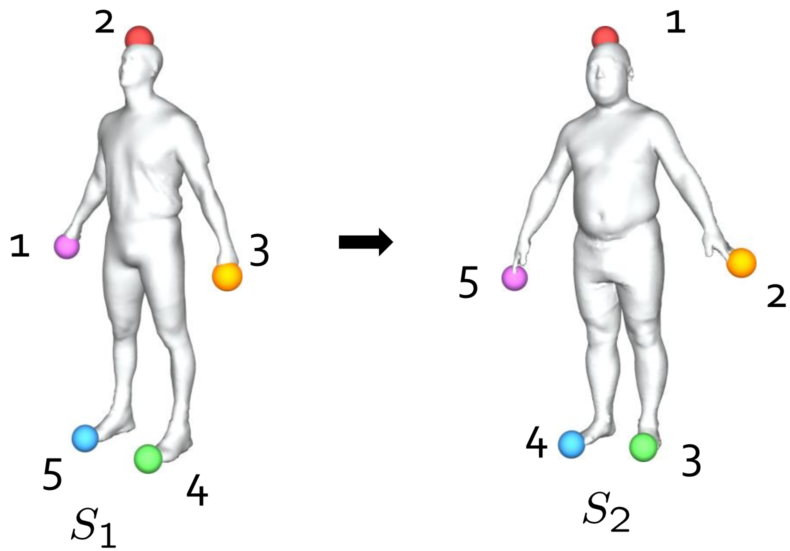
Super-Resolution

Basic setting



n objects, each object has *m* points

Matrix representation of maps

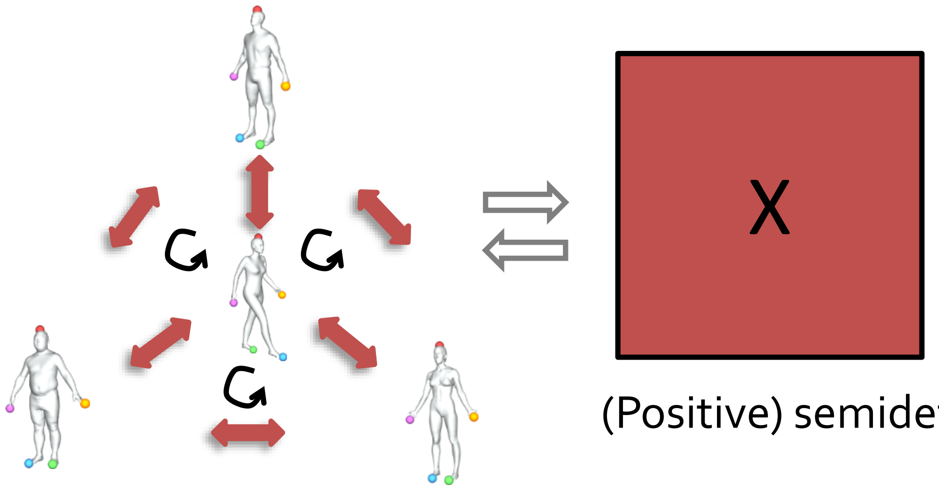


$$X_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{12}^T & I_m & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{(n-1),n} \\ X_{1n}^T & \vdots & X_{(n-1),n}^T & I_m \end{bmatrix}$$

- Diagonal blocks are identity matrices
- Off diagonal blocks are permutation matrices
- Symmetric

Cycle-consistency constraint

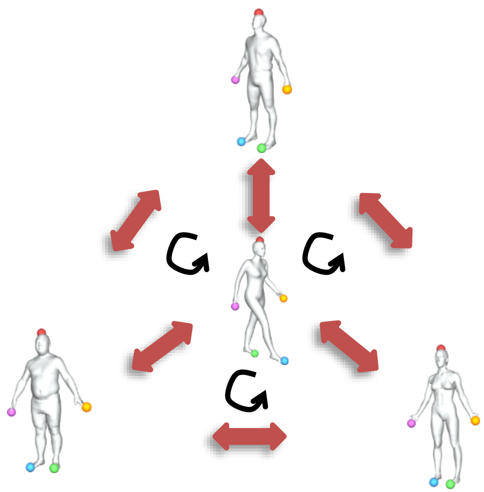


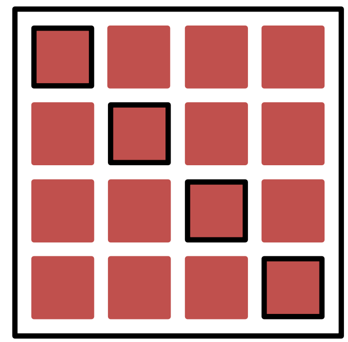
$$\succeq 0$$

(Positive) semidefiniteness

$$\mathbf{X}_{ij} = X_{j1}^T X_{i1} \Longleftrightarrow \mathbf{X} = \begin{bmatrix} \mathbf{I}_m \\ \vdots \\ \mathbf{X}_{n1}^T \end{bmatrix} \begin{bmatrix} \mathbf{I}_m & \cdots & \mathbf{X}_{n1} \end{bmatrix}$$

Parameterization



$\mathbf{X} =$ 

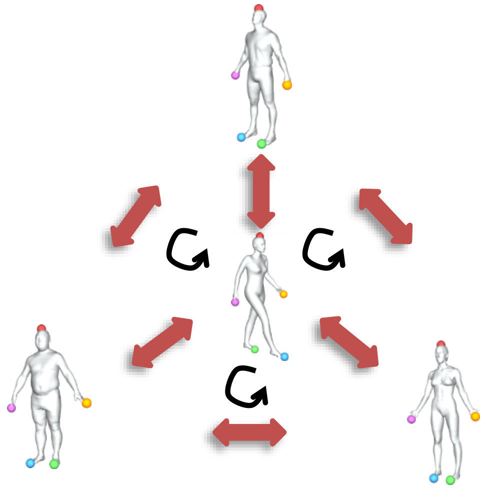
$$\mathbf{X}_{ii} = I_m, \quad 1 \leq i \leq n$$

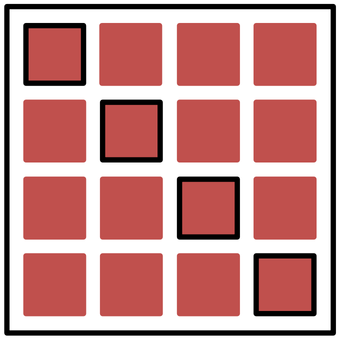
$$\mathbf{X}_{ij}\mathbf{1} = \mathbf{1}, \mathbf{X}_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \leq i < j \leq n$$

$$\mathbf{X} \succeq 0$$

$$\mathbf{X} \in \{0, 1\}^{nm}$$

Parametrizing maps



$$\mathbf{X} =$$ 

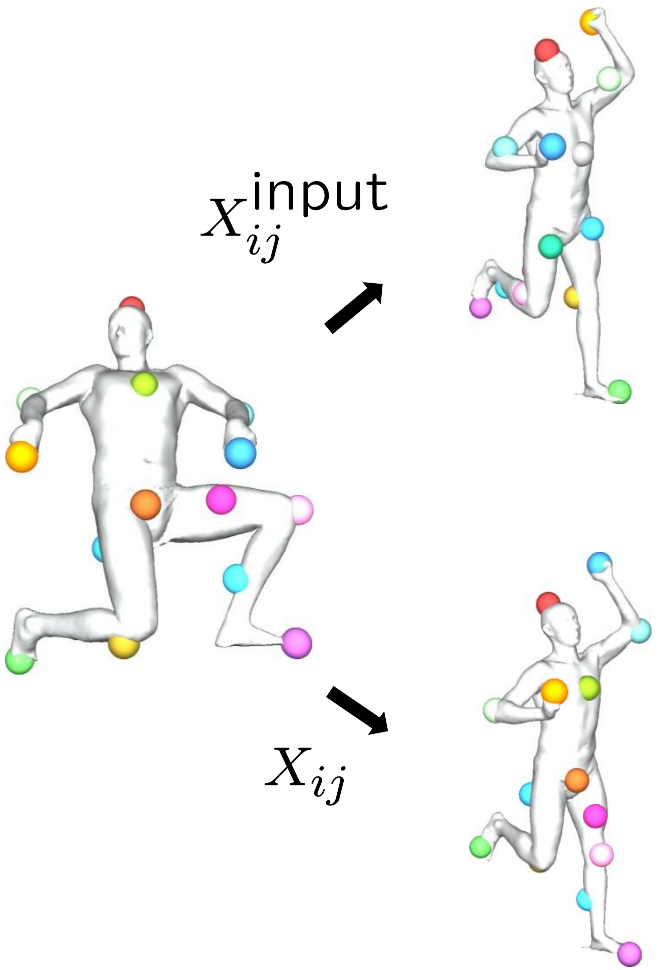
$$\mathbf{X}_{ii} = I_m, \quad 1 \leq i \leq n$$

$$\mathbf{X}_{ij}\mathbf{1} = \mathbf{1}, \mathbf{X}_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \leq i < j \leq n$$

$$\mathbf{X} \succeq 0$$

$$\mathbf{X} \geq 0$$

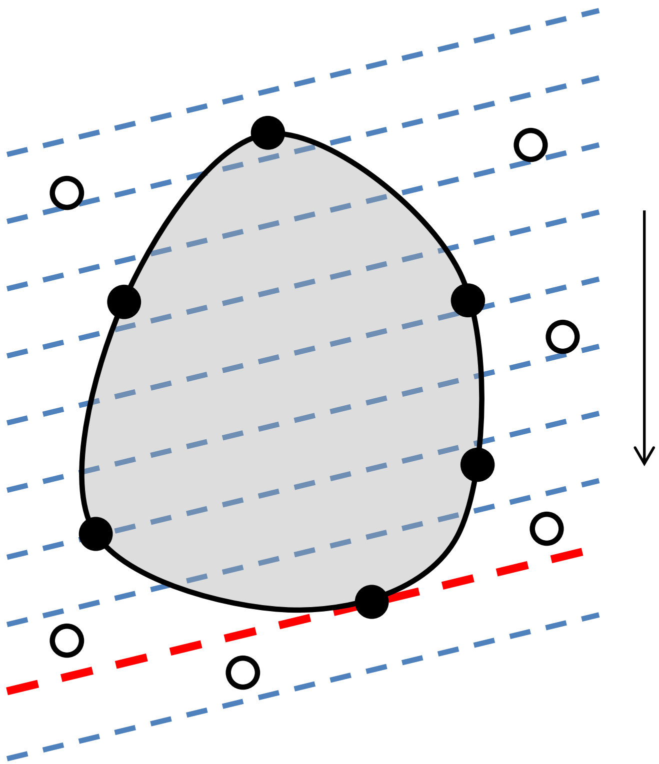
Objective function



L1-norm:

$$\sum_{(i,j)\in\mathcal{E}} \|\mathbf{X}_{ij}^{input} - \mathbf{X}_{ij}\|_1$$

Convex program



minimize $\sum_{(i,j)\in\mathcal{E}} \|\mathbf{X}_{ij}^{input} - \mathbf{X}_{ij}\|_1$

subject to $\mathbf{X}_{ii} = I_m, \quad 1 \leq i \leq n$

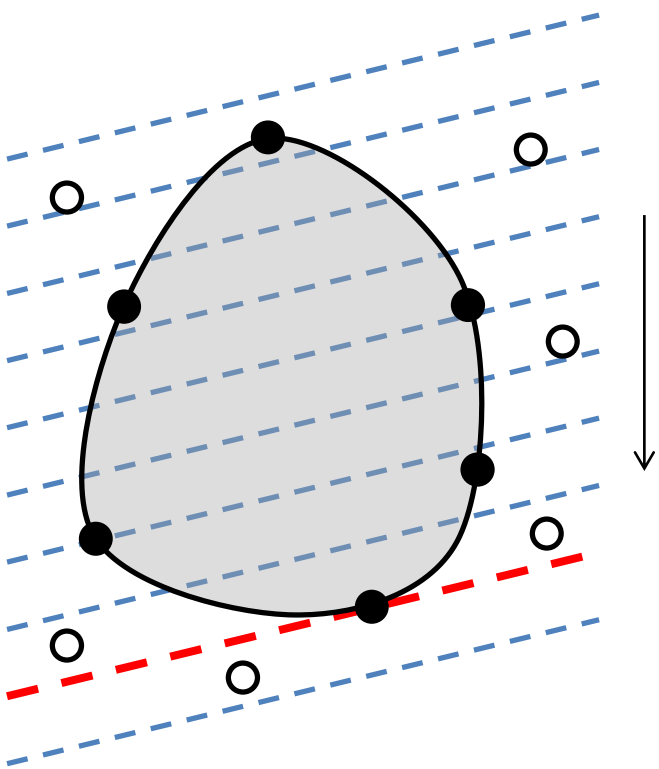
$$\mathbf{X}_{ij}\mathbf{1} = \mathbf{1}, \mathbf{X}_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \leq i < j \leq n$$

$$\mathbf{X} \succeq 0$$

$$\mathbf{X} \geq 0$$

ADMM [Boyd et al.11]

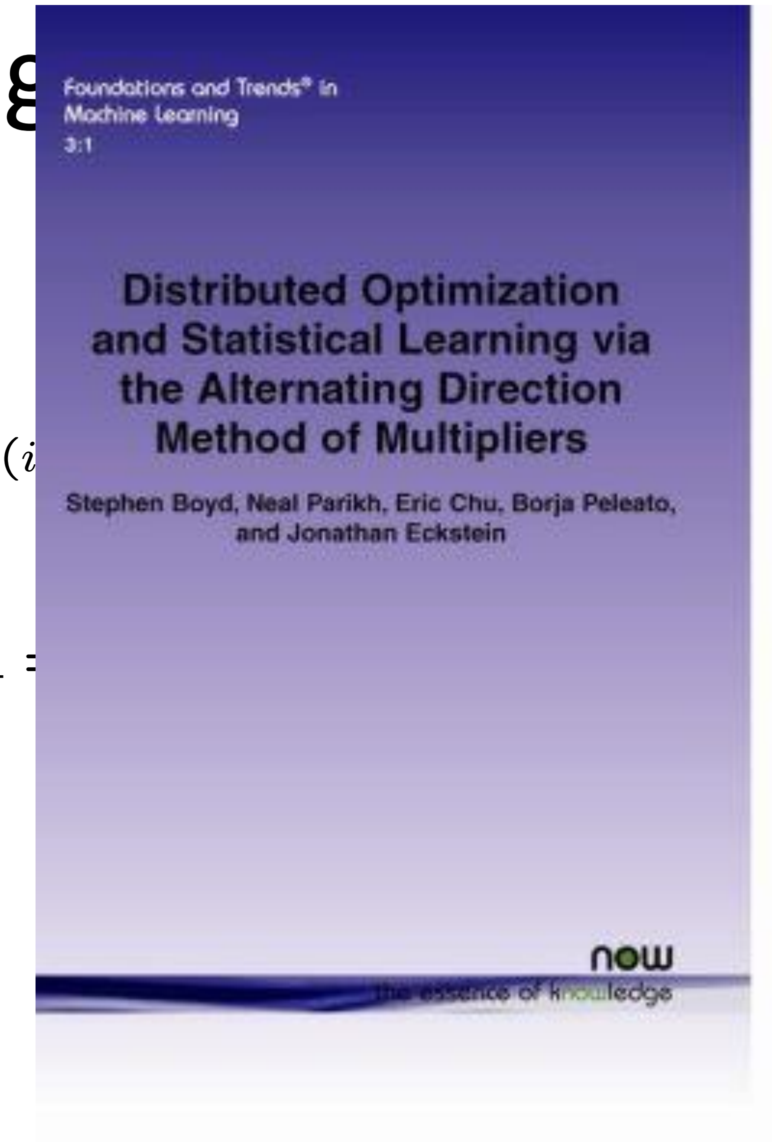
Convex prog



minimize $(i$

subject to

$$\mathbf{X}_{ij}\mathbf{1} =$$ i

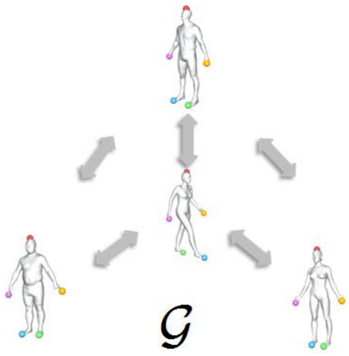


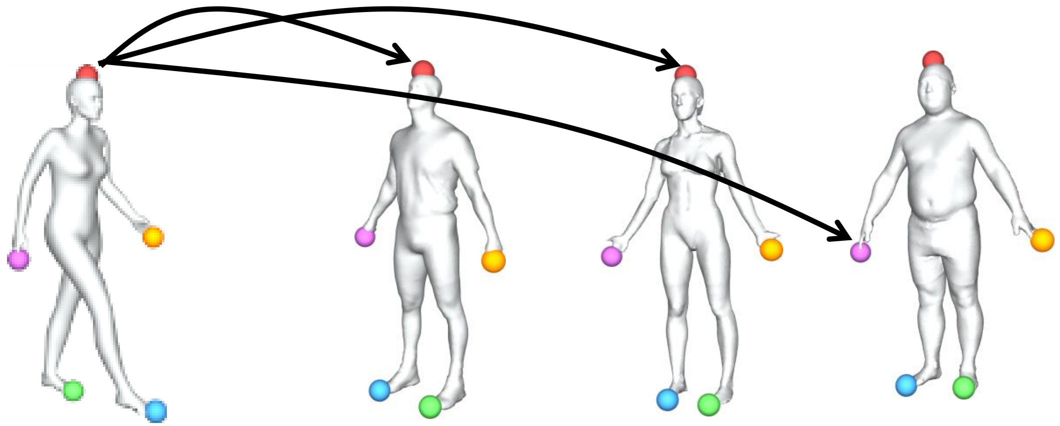
Deterministic guarantee

- *Exact recovery condition:*

  #incorrect corres. per point

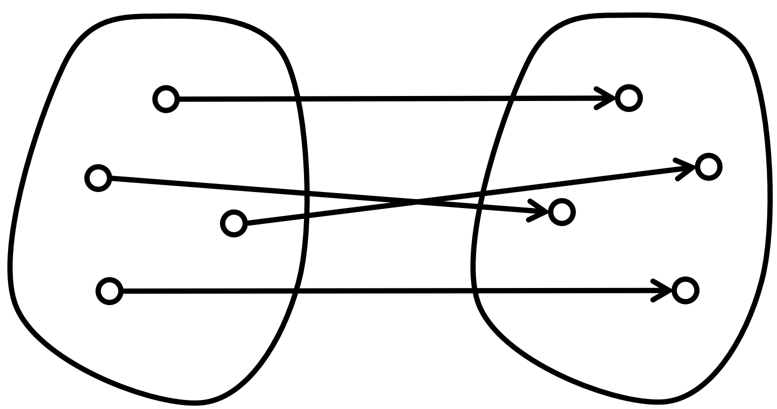
  < algebraic-connectivity(G)/4





Complete input graph G

- 25% incorrect correspondences
- Worst-case scenario
  - Two clusters of objects of equal size
  - Wrong correspondences between objects of different clusters only (50%)



Randomized setting

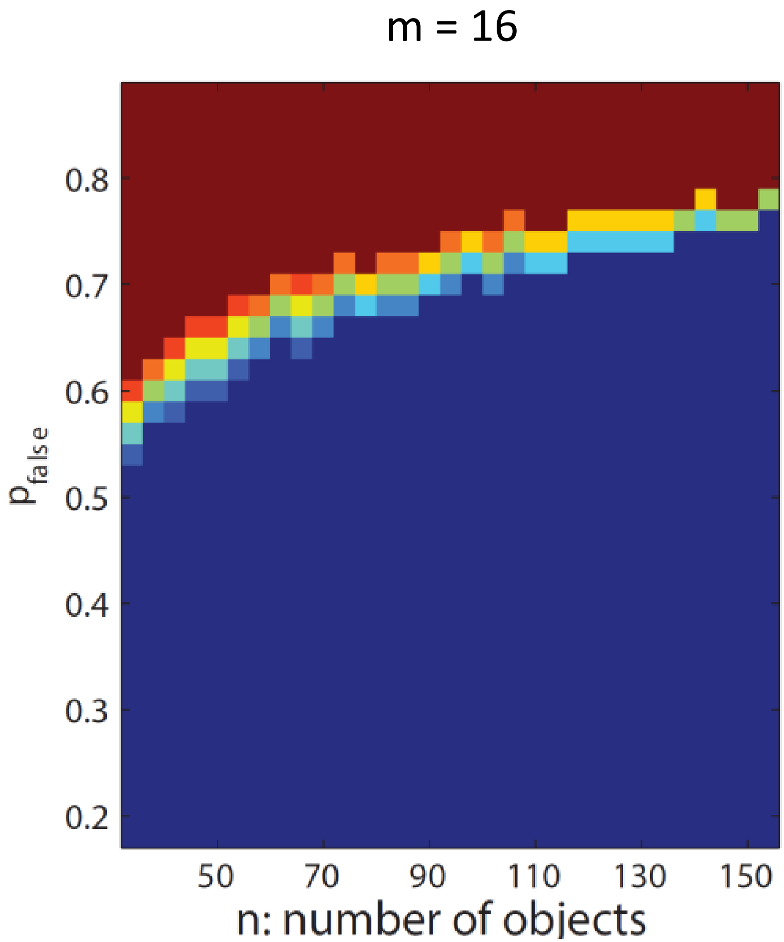
- Erdős–Rényi model *G(n, p_{obs}, p_{true}) :*
  - p_{obs} : the probability that two objects connect;
  - p_{true} : the probability that a pair-wise map is correct;
  - Incorrect maps are random permutations;

- *Theorem: The ground truth maps can be recovered w.h.p if*

$$p_{true} > c\frac{\log^2 n}{\sqrt{np_{obs}}}$$

Phase transition

Numerical optimization: ADMM [Wen et al. 10, Boyd et al. 11]

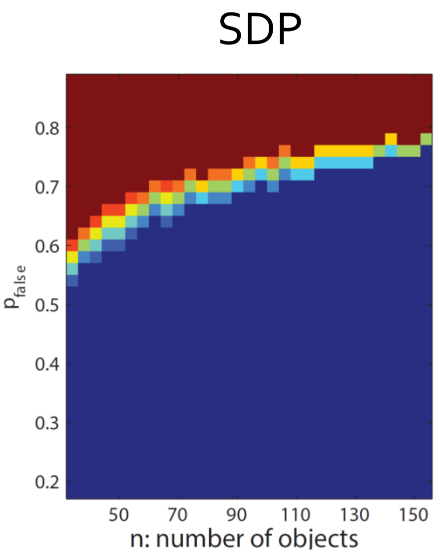


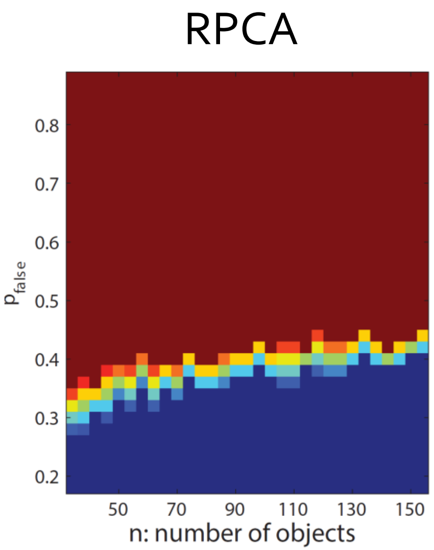
Red: never recovers

Blue: always recovers

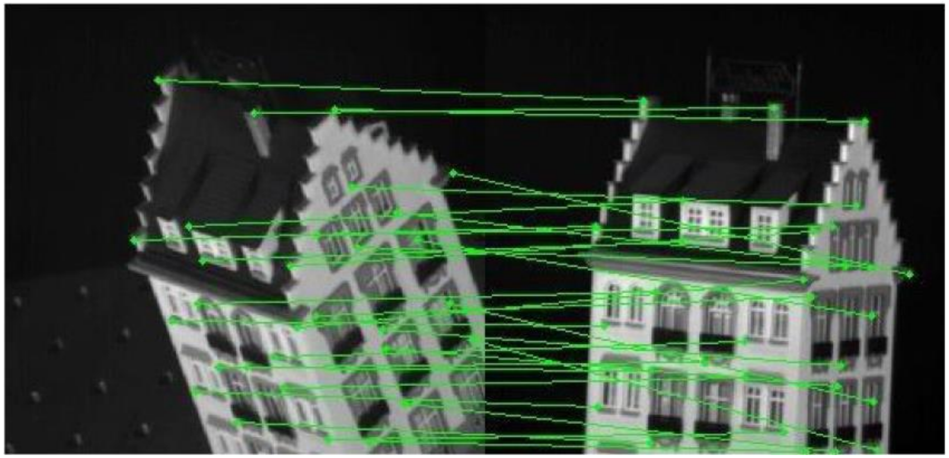
Versus RPCA [Candes et al. 11]

$$\mathbf{X} = \begin{bmatrix} \mathbf{I}_m \\ \vdots \\ \mathbf{X}_{1n}^T \end{bmatrix} \begin{bmatrix} \mathbf{I}_m & \cdots & \mathbf{X}_{1n} \end{bmatrix}$$





CMU hotel



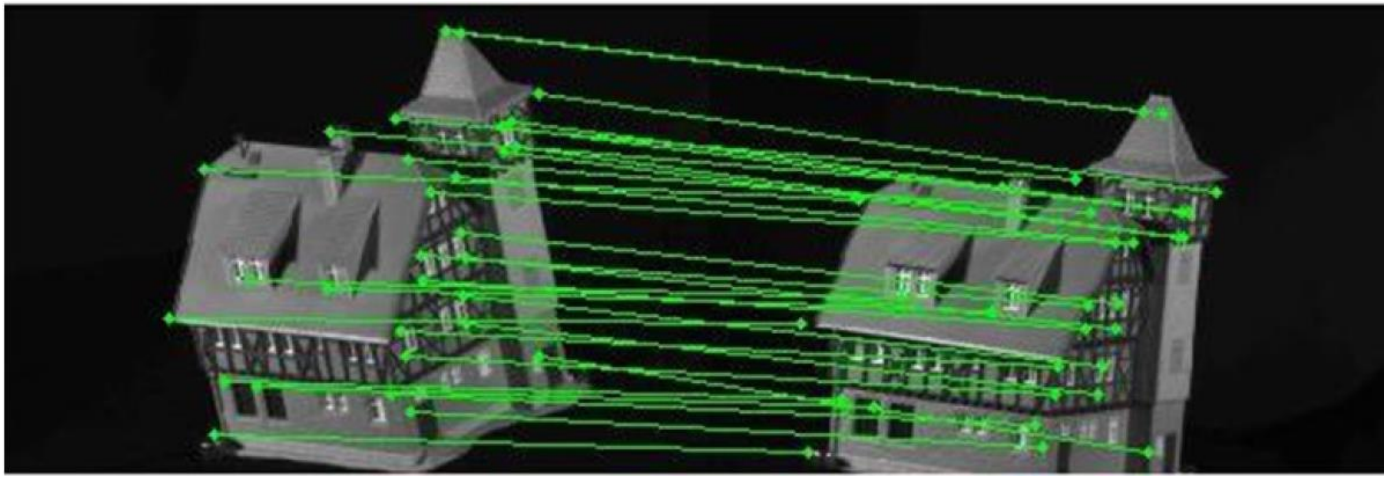
Input:

- 102 images (30 points per image)
- RANSAC [Fisher 81]
- Each image connects with 10 random images

SDP Running time: 6m19s (3.2GHZ, single core)

Input	SDP	RPCA	Leordeanu et al. 12
64.1%	100%	90.1%	94.8%

CMU house



Input:

110 images (30 points per image)

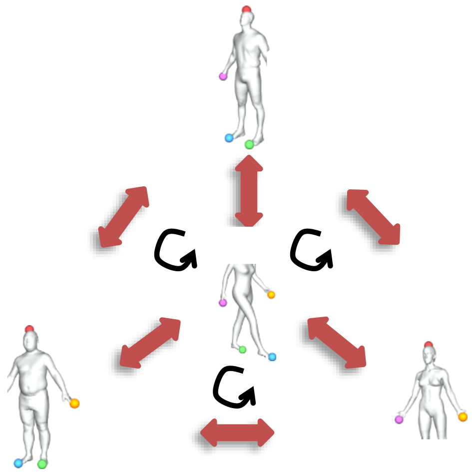
RANSAC [Fisher 81]

Each image connects with 10 random images

SDP Running time: 7m12s (3.2GHZ, single core)

Input	SDP	RPCA	Leordeanu et al. 12
68.2%	100%	92.2%	99.8%

Constraint set



$$X = \begin{bmatrix} I_{m_1} & X_{12} & \cdots & X_{1n} \\ X_{21} & I_{m_2} & \cdots & X_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & \cdots & \cdots & I_{m_n} \end{bmatrix}$$

minimize $\sum_{(i,j)\in\mathcal{E}} \|\mathbf{X}_{ij}^{input} - \mathbf{X}_{ij}\|_1$

$$\mathbf{X}_{ii} = I_m, \quad 1 \le i \le n$$

$$\begin{bmatrix} m & \mathbf{1}^T \\ \mathbf{1} & \mathbf{X} \end{bmatrix} \succeq 0$$

$$\mathbf{X} \ge 0$$

#Distinctive points

$$\mathbf{X}_{ii} = I_m, \quad 1 \le i \le n$$

$$\mathbf{X}_{ij}\mathbf{1} = \mathbf{1}, \mathbf{X}_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \le i < j \le n$$

$$\mathbf{X} \succeq 0$$

$$\mathbf{X} \ge 0$$

SDP relaxation of MAP [kumar et al. 09]

Partial similarity

Step I: Estimate m (#distinctive points)

-- Gap in the spectrum of the input map matrix

(In the same spirit as [Pachauri et al. 13])

Step II:

minimize $\sum_{(i,j)\in\mathcal{E}} \|\mathbf{X}_{ij}^{input} - \mathbf{X}_{ij}\|_1$

subject to $\mathbf{X}_{ii} = I_{m_i}, \quad 1 \leq i \leq n$

$$\begin{bmatrix} m & \mathbf{1}^T \\ \mathbf{1} & \mathbf{X} \end{bmatrix} \succeq 0$$

$$\mathbf{X} \geq 0$$

Random model

- Extended model: an universe of m elements:
  - For each set S_i, each point s is included in S_i with probability p_{set};
    - p_{obs} : the probability that two objects connect
    - p_{true} : the probability that a pair-wise map is correct
    - Incorrect maps satisfy $\mathbb{E}\boldsymbol{X}_{ij}^{\text{in}} = \mathbf{1} \cdot \mathbf{1}^\top / m.$

Similar theoretical guarantee

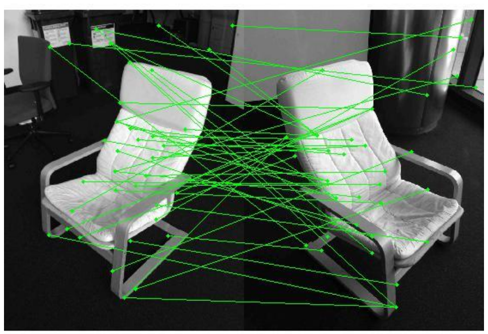
- *Theorem: The size of the universe* ***m*** *and the ground truth maps can be recovered w.h.p if*

$$p_{true} > C_1 \frac{\log^2 n}{\sqrt{np_{obs}} p_{set}^2}$$

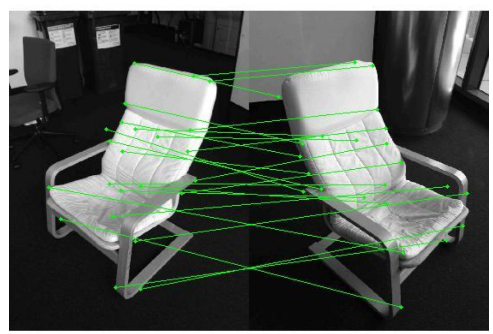
Chair

16 images
Clustered SIFT features
+ RANSAC (60-120 points per image)

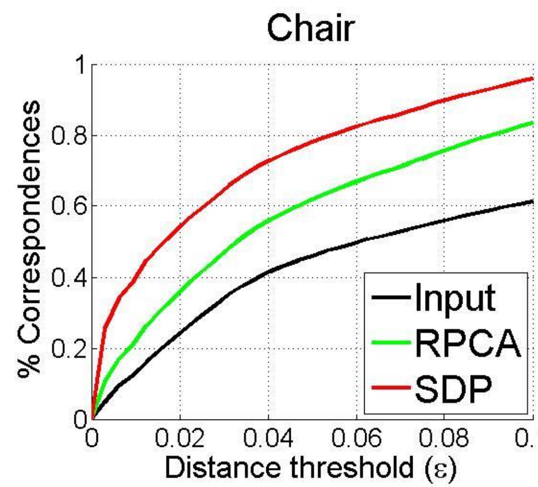
SDP Running time: 2m19s (3.2GHZ, single core)



Input



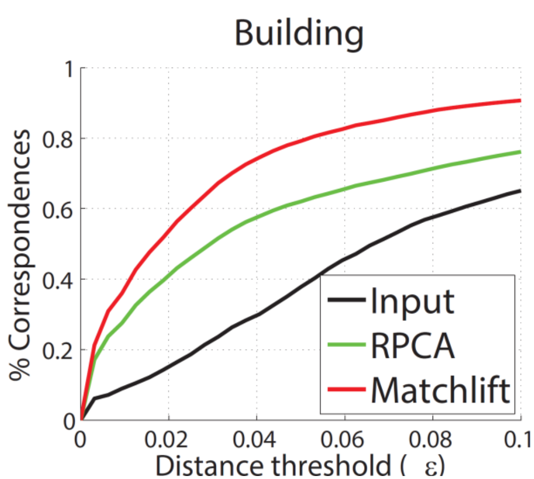
output



Building

Input

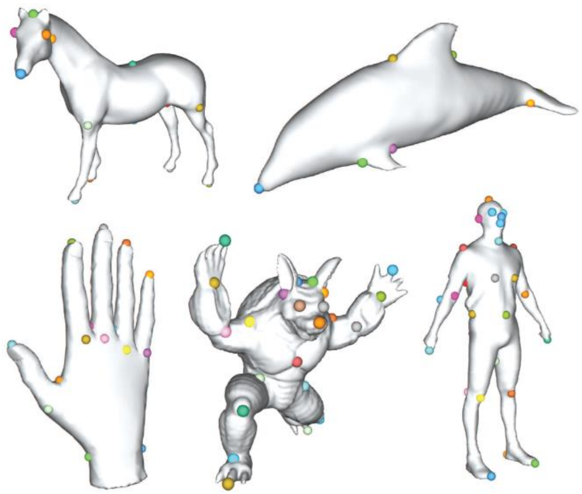
Output



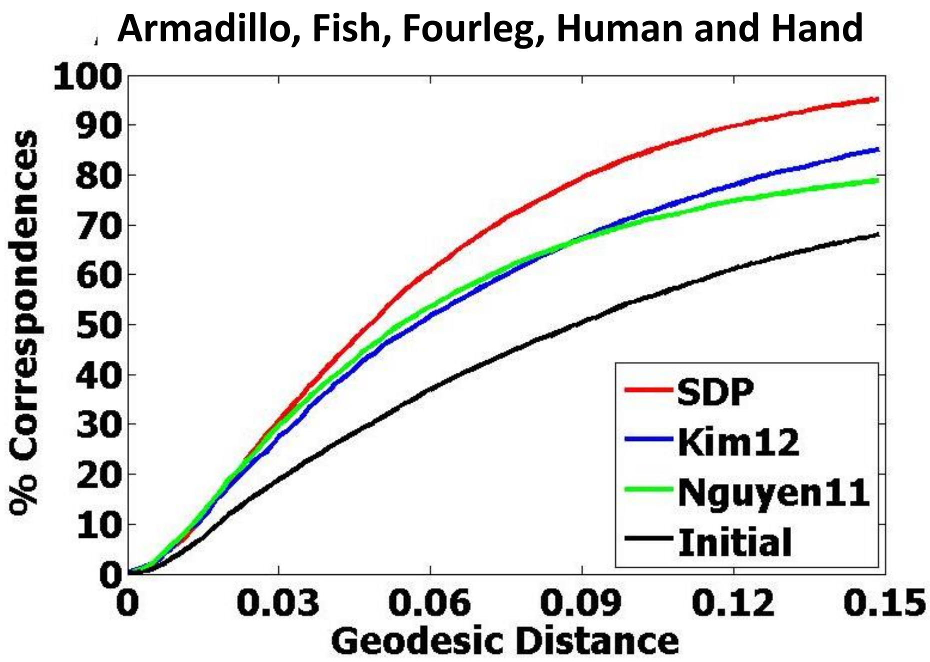
16 images
Clustered SIFT features
+ RANSAC (60-120 points per image)

SDP Running time: 5m07s (3.2GHZ, single core)

3D Benchmark



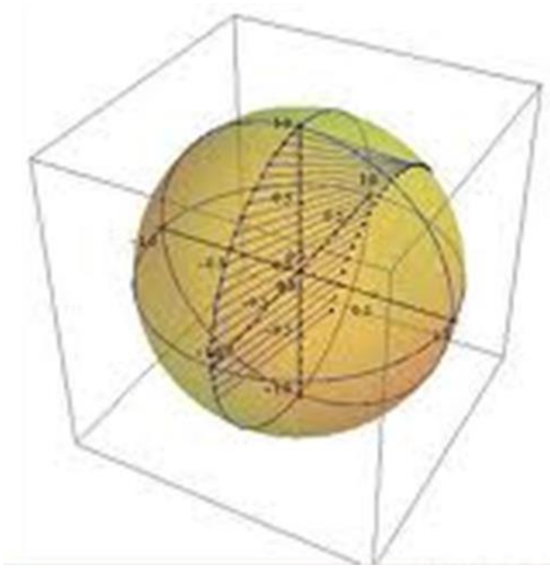
SHREC07-UnSym



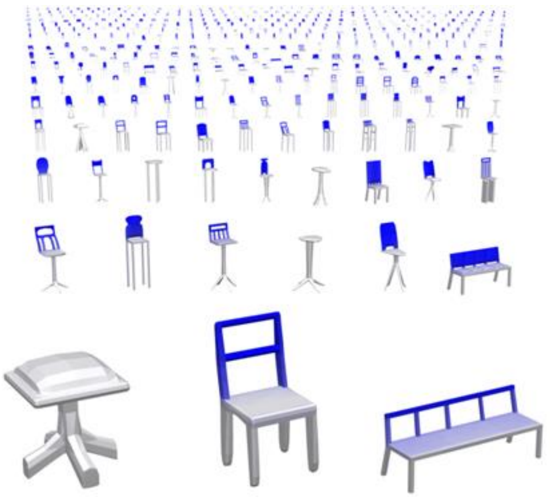
20 objects, 128 points per object

Blended intrinsic maps as input
[Kim et al. 11]

Next lecture



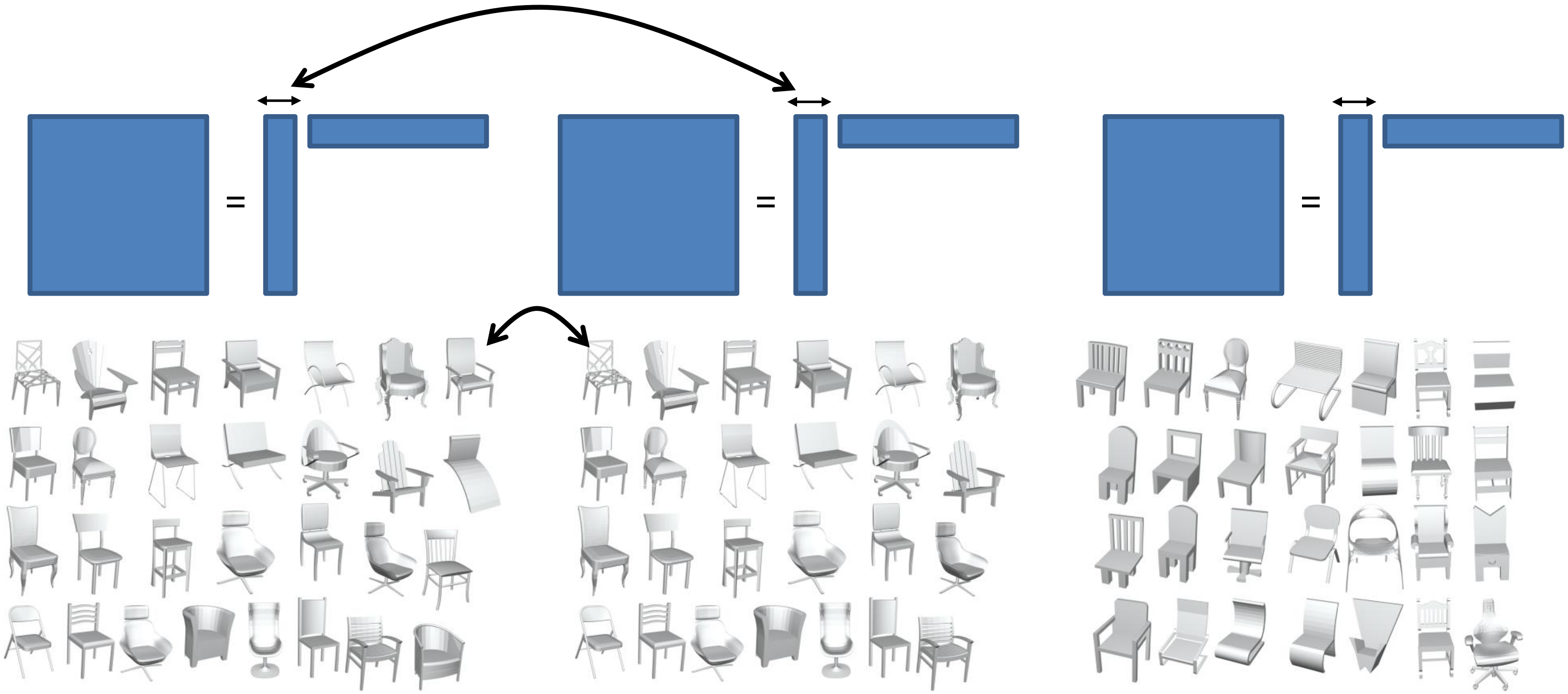
Rotations
[Wang and Singer'13]



Functional maps
[Huang et al. 14]

Next lecture

Super-objects



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- An optimization approach to improving collections of shape maps, A. Nguyen, M. Ben-Chen, K. Welnicka, Y. Ye, and L. Guibas, SGP '11
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