CS 395T
Numerical Optimization for Graphics and AI (3D Vision)

Qixing Huang

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[Ma et al. 05]

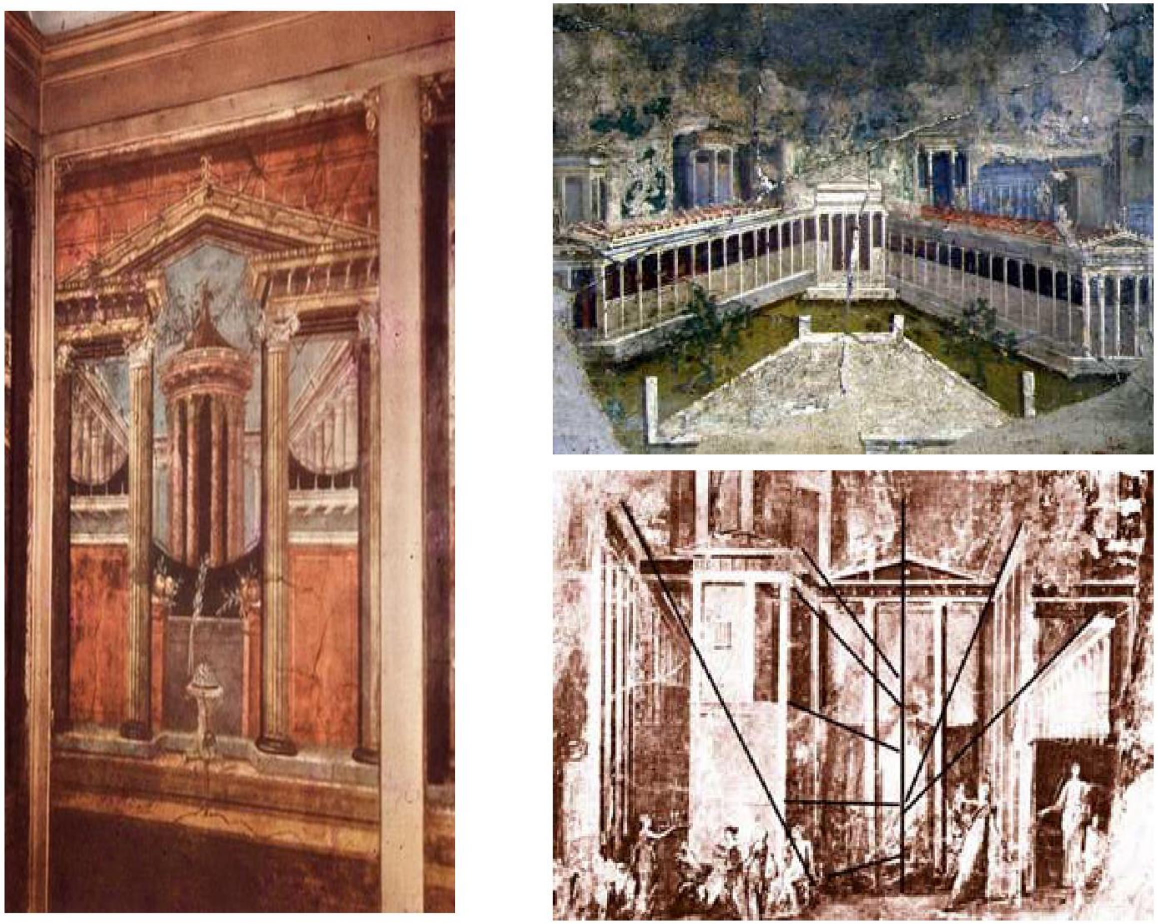
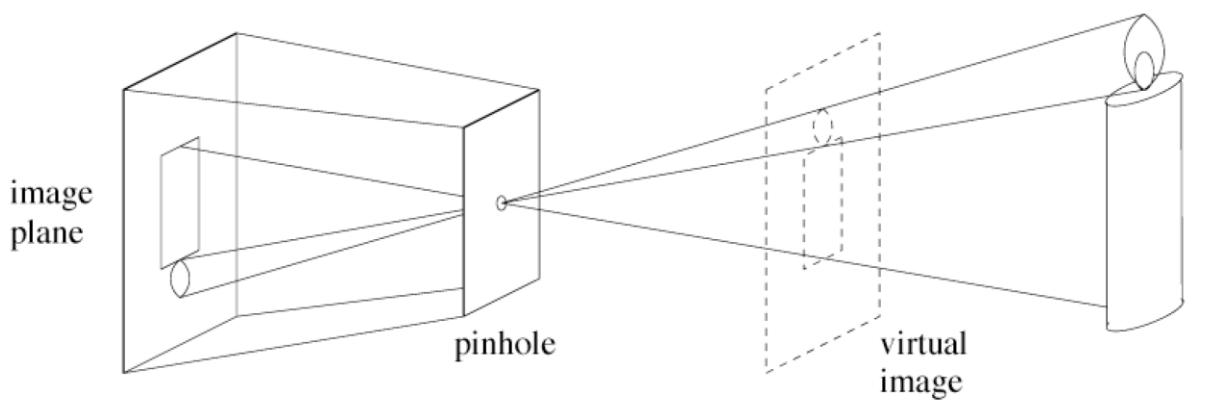


Figure 3.1. Frescoes from the first century B.C. in Pompeii. More (left) or less (right) correct perspective projection is visible in the paintings. The skill was lost during the middle ages, and it did not reappear in paintings until fifteen centuries later, in the early renaissance.

Pinhole camera

- The dominant image formation model in computer vision
- A pinhole camera is a box in which one wall has a small hole
- Exactly one ray from each point in the scene passes through the pinhole and hits the wall opposite to it
- The inversion of the image is corrected for by considering a virtual image on the opposite side of the pinhole



Mathematical model under this idealized camera

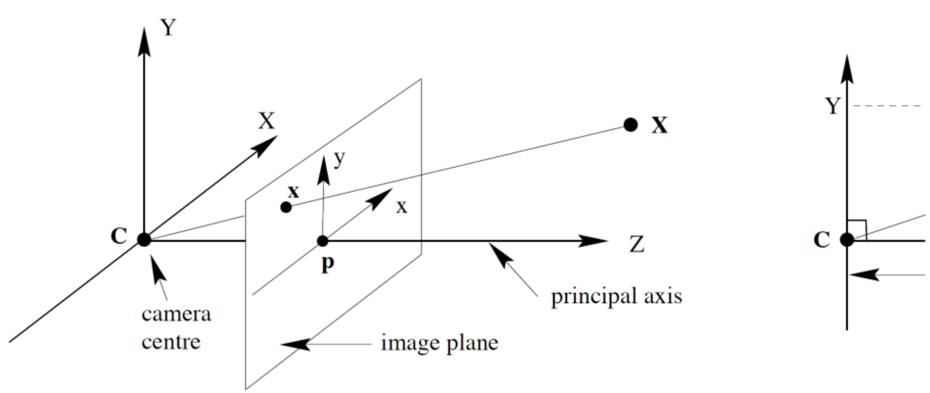
- It is clear that the camera is given by a perspective projection that maps the 3D space to a 2D plane

$$(X, Y, Z)^\top \xrightarrow{Projection} (x, y)^\top$$

- The equations of perspective projections are given by

$$x = f\frac{X}{Z} \qquad y = f\frac{Y}{Z}$$

f is the focal length of the camera, i.e., the distance between the image plane and the pinhole



Homogeneous coordinates

- The representation of the image point **x** = (x,y) is referred to as the inhomogeneous representation of the point **x**

- The homogeneous representation of a point x is given by **x** = (x, y, 1). In fact, the homogeneous representation of a point maps it to an entire class of set of points:

  $(x, y) \leftrightarrow (\lambda x, \lambda y, \lambda), \qquad \forall \lambda \neq 0$ In particular, $(x/z, y/z) \leftrightarrow (x, y, z)$

- Homogeneous coordinates encode the invariance of all points along a line and its projection

Examples

- The equation of a line *ax+by+cz = 0* can be rewritten using homogeneous coordinates

$$\boldsymbol{x}^\top \boldsymbol{l} = 0, \quad \text{where} \quad \boldsymbol{l} = (a, b, c)^\top$$

- The general conic in 3 dimensions is given by

$$ax^2 + bxy + cy^2 + dxz + eyz + fz^2 = 0$$

which can be written using 2D homogeneous coordinates as

$$\boldsymbol{x}^\top C \boldsymbol{x} = 0 \quad \text{where} \quad C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

Points at infinity

- In R^2, all pairs of lines intersect except for the ones that are parallel
- In P^2, all pairs of lines intersect, and parallel lines intersect in points of infinity and these points have the form $(x,y,0)^\top$
- Consider the two lines given by

$$\begin{aligned} \boldsymbol{l}_1 &= (a_1, b_1, c_1)^\top \\ \boldsymbol{l}_2 &= (a_2, b_2, c_2)^\top \end{aligned}$$

- The intersection of these two lines is given by

$$\boldsymbol{x} = \boldsymbol{l}_1 \times \boldsymbol{l}_2$$

Intersection of parallel lines

- Given a line $\boldsymbol{l}_1 = (a, b, c)^\top$, a line parallel to it is given by $\boldsymbol{l}_2 = (a, b, c')^\top$

- The intersection is now given by

$$\begin{aligned} \boldsymbol{l}_1 \times \boldsymbol{l}_2 &= (bc' - cb, ac - ac', 0)^\top \\ &= (c - c')(b, -a, 0)^\top \\ &\sim (b, -a, 0)^\top \end{aligned}$$

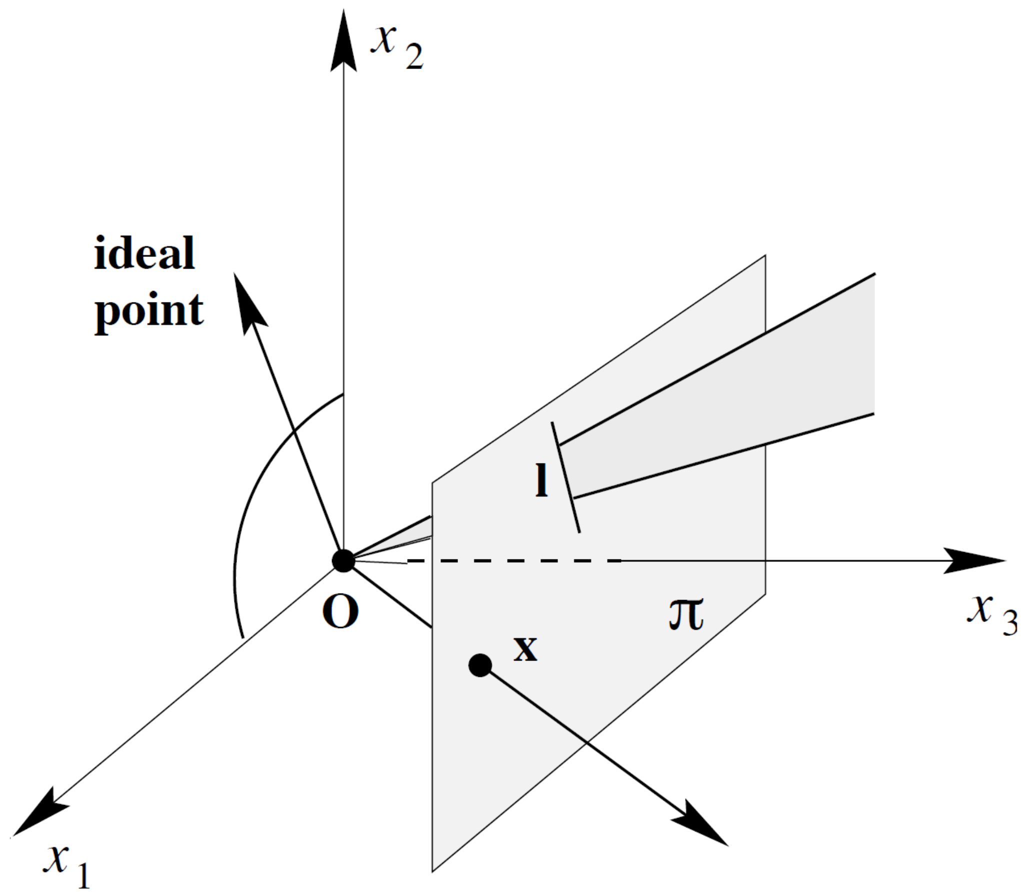
Duality

- In *P*^{2}, points and lines are dual of each other

- The point of intersection of two lines is their cross product. Likewise, the line passing through any two points is given by their cross product $\boldsymbol{l} = \boldsymbol{x}_1 \times \boldsymbol{x}_2$
- The definition of points at infinity leads us to the definition of the line at infinity $\boldsymbol{l}_\infty$.
- Consider two points at infinity $\boldsymbol{x}_1 = (x_1, y_1, 0)^\top$ and $\boldsymbol{x}_2 = (x_2, y_2, 0)^\top$
- The line passing through these two points is given by

$$\begin{aligned}
\boldsymbol{l}_\infty &= \boldsymbol{x}_1 \times \boldsymbol{x}_2 \\
&= (0, 0, x_1 y_2 - y_1 x_2)^2 \\
&\sim (0, 0, 1)^\top
\end{aligned}$$

A model for P^2 in R^3



Hartley&Zisserman

Intrinsic/Extrinsic Parameters of a Camera

- The following equation maps the real world point X_0 in *homogeneous coordinates to its projection x' also in homogeneous coordinates*

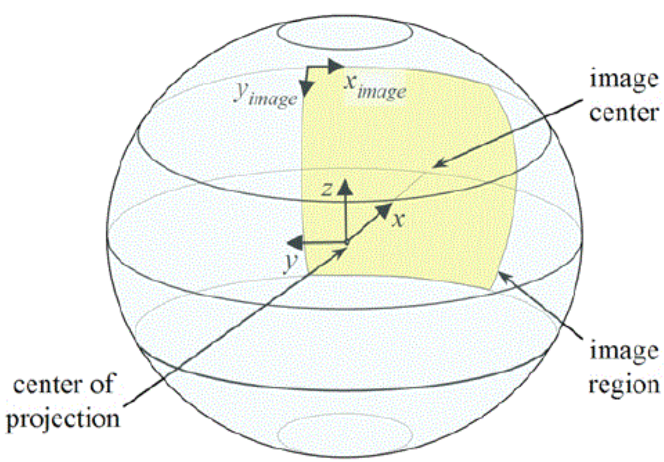
$$\underbrace{\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}}_{\boldsymbol{x}'} = \underbrace{\begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\Pi_0} \underbrace{\begin{bmatrix} R & \boldsymbol{T} \\ \mathbf{0}^\top & 1 \end{bmatrix}}_{g} \underbrace{\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{bmatrix}}_{\boldsymbol{X}_0}$$

Intrinsic parameters canonical projection matrix Extrinsic parameters

Spherical projection

- This choice is partly motivated by retina shapes often encountered in biological systems

$$\pi_s : \mathbb{R}^3 \to \mathbb{S}^2, \quad \mathbf{X} \mapsto \mathbf{X} = \frac{\mathbf{X}}{\|\mathbf{X}\|}$$



Questions?