Slide Credit: Mirela Ben-Chen

Mesh and Mesh Simplification

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Mesh DataStructures

Data Structures

- What should be stored?
  - Geometry: 3D coordinates
  - Attributes
    - e.g. normal, color, texture coordinate
    - Per vertex, per face, per edge
  - Connectivity
    - Adjacency relationships

Data Structures

- What should it support?
  - Rendering
  - Geometry queries
    - What are the vertices of face #2?
    - Is vertex A adjacent to vertex H ?
    - Which faces are adjacent to face #1?
  - Modifications
    - Remove/add a vertex/face
    - Vertex split, edge collapse

Data Structures

- How good is a data structure?
  - Time to construct (preprocessing)
  - Time to answer a query
  - Time to perform an operation
  - Space complexity
  - Redundancy

Mesh Data Structures

- Face Set
- Shared Vertex
- Half Edge
- Face Based Connectivity
- Edge Based Connectivity
- Adjacency Matrix
- Corner Table

Face Set

TRIANGLES		
Vertex coord.	Vertex coord.	Vertex coord.
[10 20 30]	[40 5 20]	[10 4 3]
	. . .	

- Simple
- STL File
- No connectivity
- Redundancy

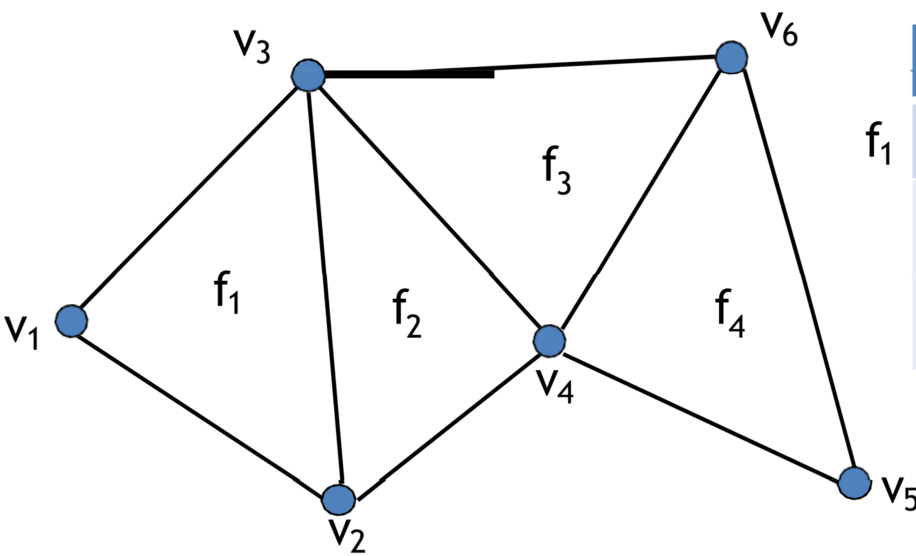
Shared Vertex

TRIANGLES		
Vertex Index	Vertex Index	Vertex Index
2	1	3
	. . .	

VERTICES
Vertex Coord.
[40 5 20]
[10 20 30]
[10 4 3]
. . .

- Connectivity
- No neighborhood

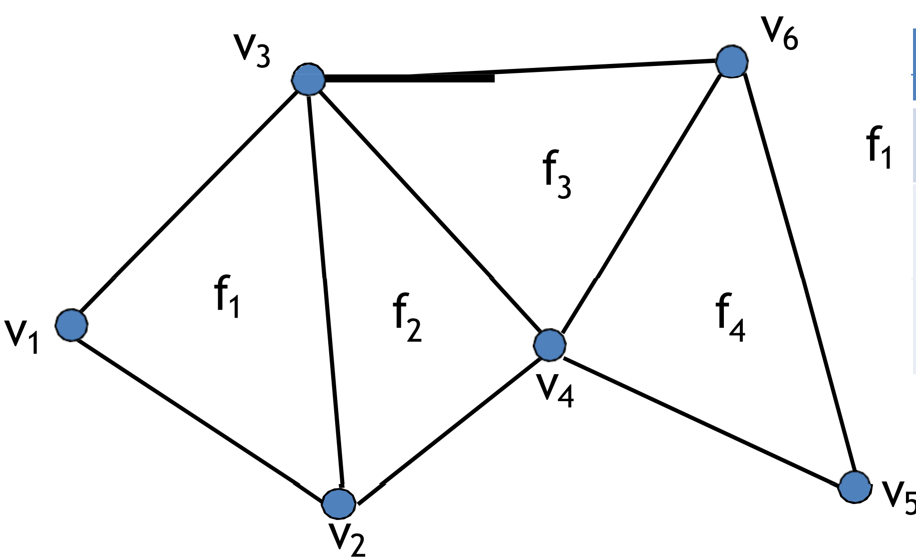
Shared Vertex



	TRIANGLES		
f_1	2	3	1
		.	

	VERTICES
v_1	[20 100]
v_2	[19 200]
v_3	[14 150]
	.
	.
	.

Shared Vertex

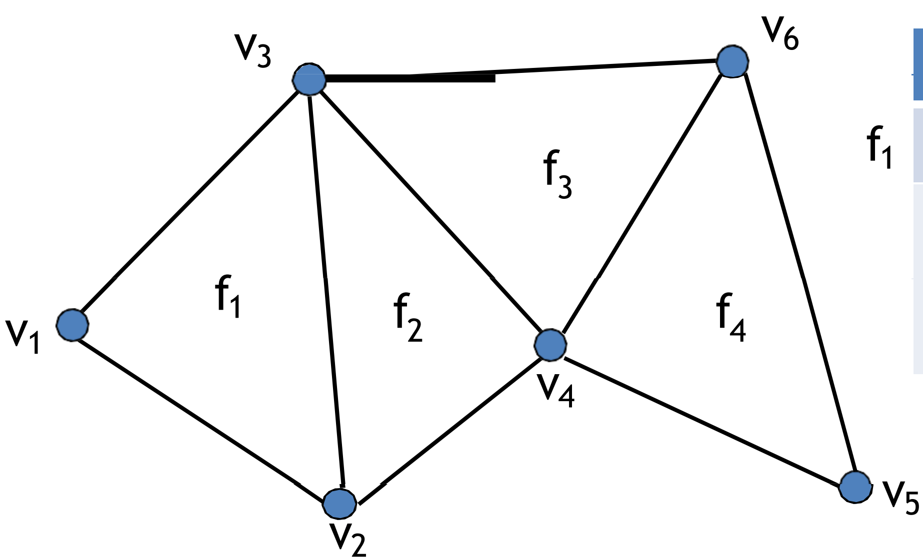


	TRIANGLES		
f_1	2	3	1
		.	

	VERTICES
v_1	[20 100]
v_2	[19 200]
v_3	[14 150]
	.
	.
	.

- What are the vertices of face f_1?
  - *O*(1) - first triplet from face list

Shared Vertex



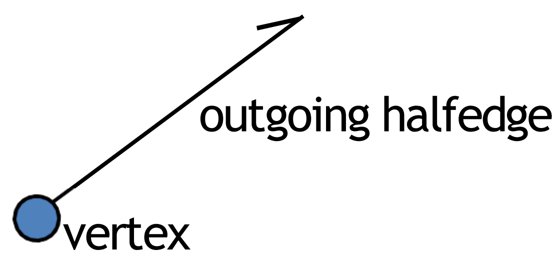
	TRIANGLES		
f1	2	3	1
		.	

	VERTICES
v1	[20 100]
v2	[19 200]
v3	[14 150]
	.
	.
	.

- **Are vertices v_1 and v_5 adjacent?**
  - **Requires a full pass over all faces**

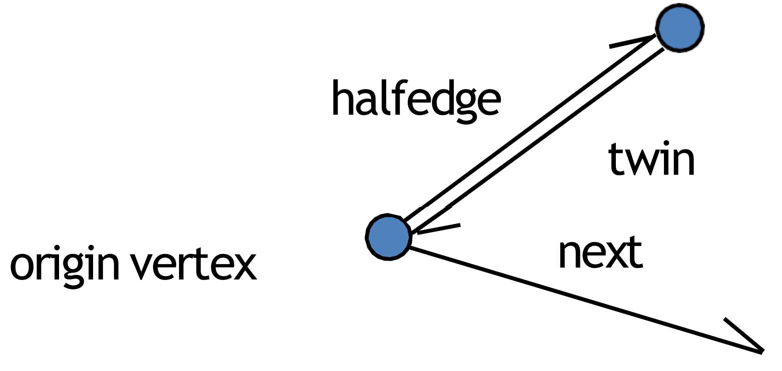
Half Edge Data Structure

- Vertex stores
  - Position
  - 1 outgoing halfedge



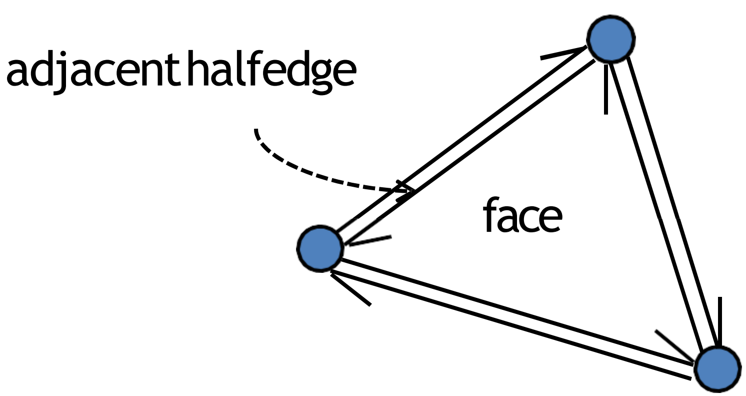
Half Edge Data Structure

- Halfedge stores
  - 1 origin vertex index
  - 1 incident face index
  - next, prev, twin halfedge indices



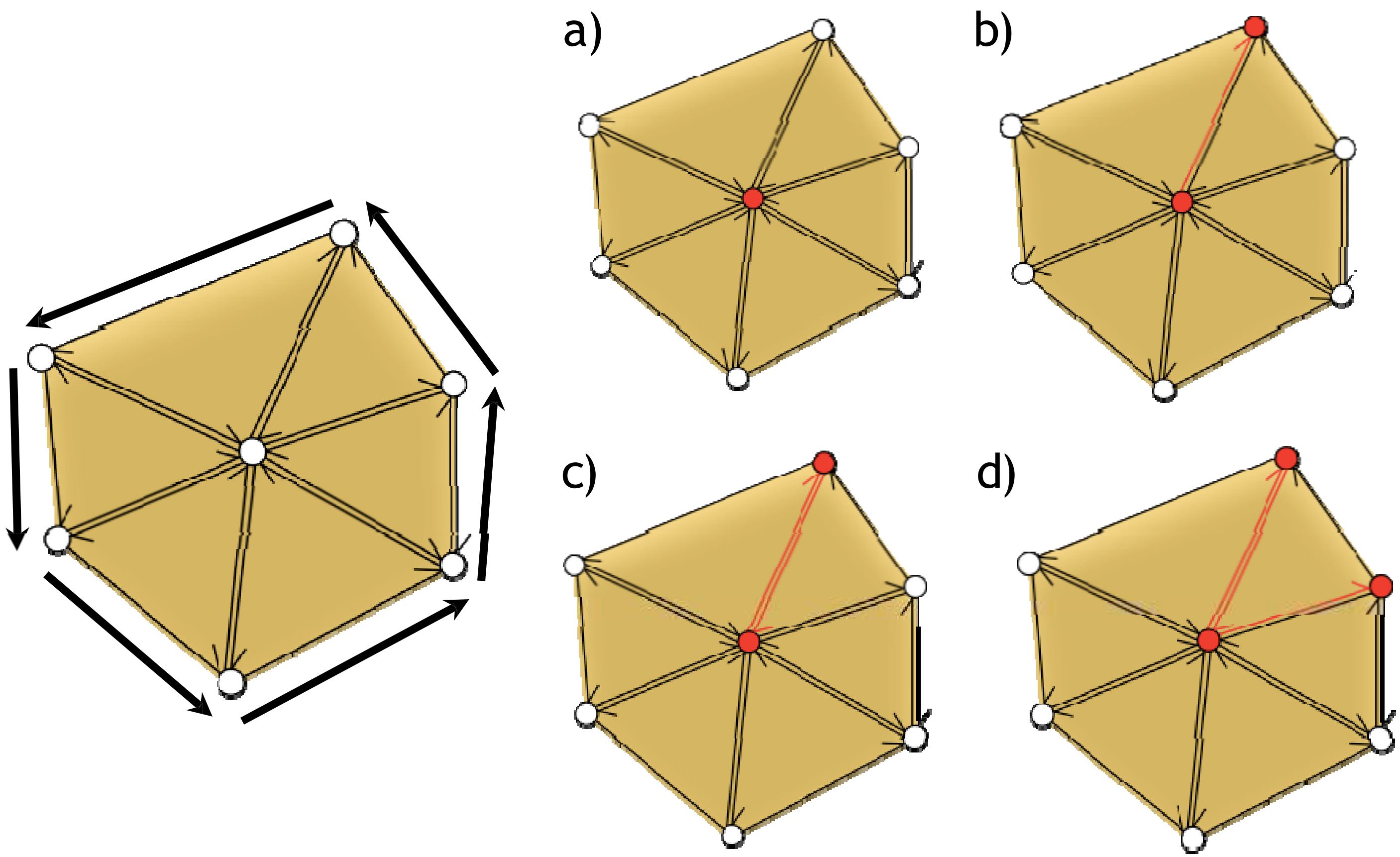
Half Edge Data Structure

- Face stores
  - 1 adjacent halfedge index



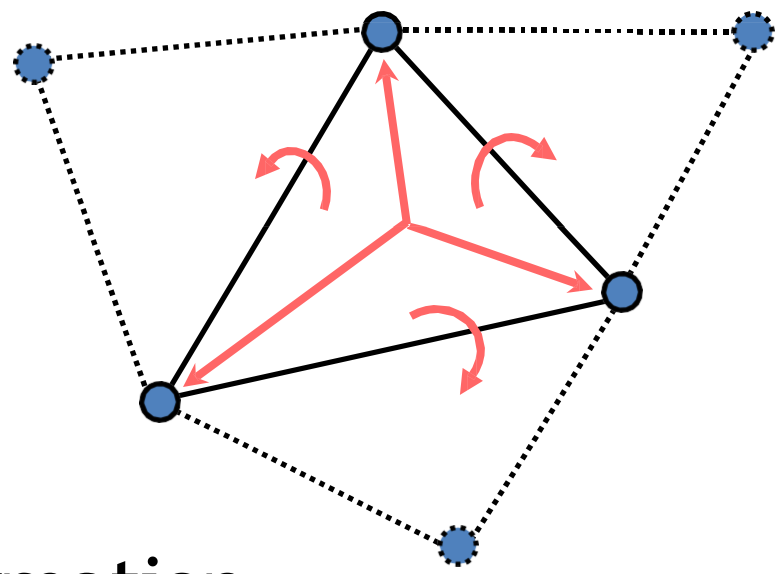
Half Edge Data Structure

- Neighborhood Traversal



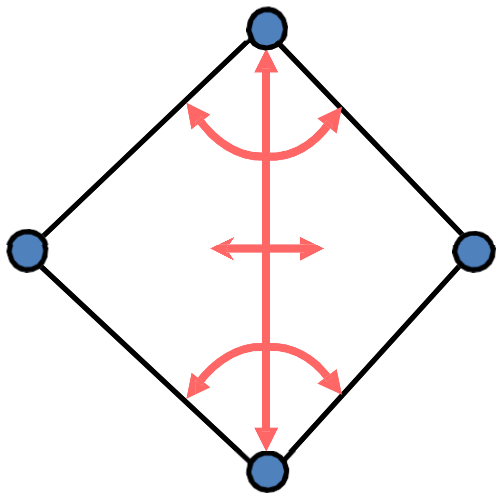
Face Based Connectivity

- Vertex:
  - position
  - 1 adjacent face index
- Face:
  - 3 vertex indices
  - 3 neighboring face indices
- No (explicit) edge information

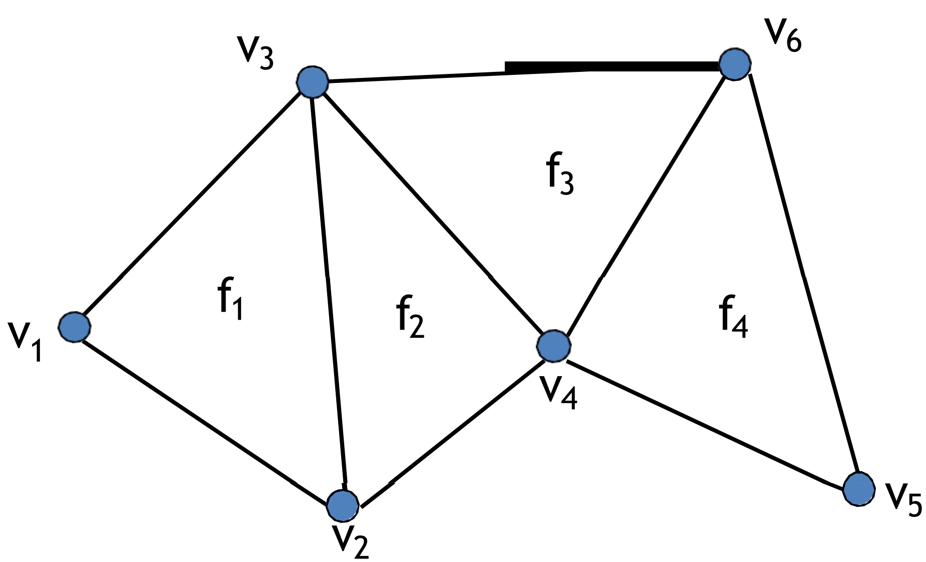


Edge Based Connectivity

- Vertex
  - position
  - 1 adjacent edge index
- Edge
  - 2 vertex indices
  - 2 neighboring face indices
  - 4 edges
- Face
  - 1 edge index
- No edge orientation information



Adjacency Matrix



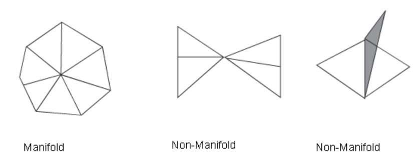
A =

	v_1	v_2	v_3	v_4	v_5	v_6
v_1		1	1			
v_2	1		1	1		
v_3	1	1		1		1
v_4		1	1		1	1
v_5				1		1
v_6			1	1	1	

- Adjacency Matrix "A"
- If there is an edge between v_i & v_j then A_{ij}= 1

Adjacency Matrix

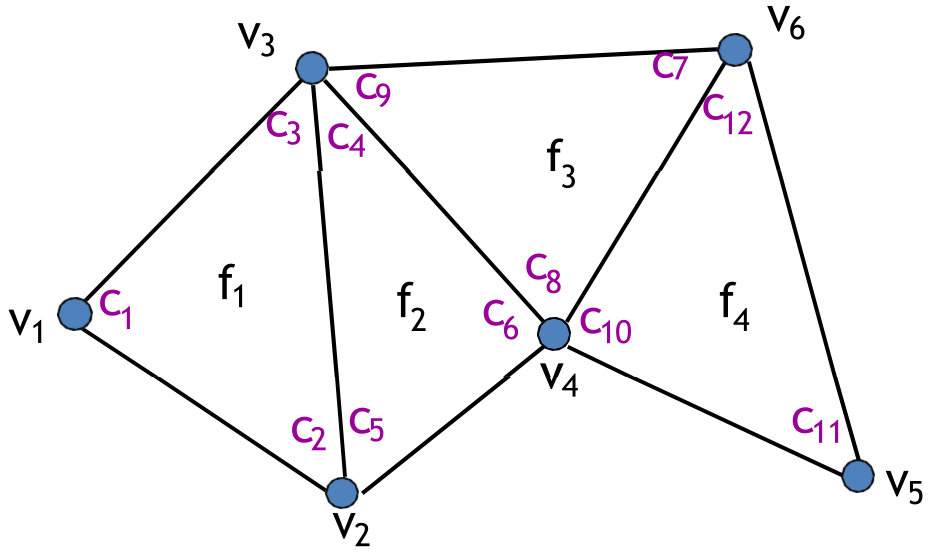
- Symmetric for undirected simple graphs
- $(A^n)_{ij}$ = # paths of length n from v_i to v_j
- Pros:



  Can represent non-manifold meshes
- Cons:
  - No connection between a vertex and its adjacent faces

Corner Table

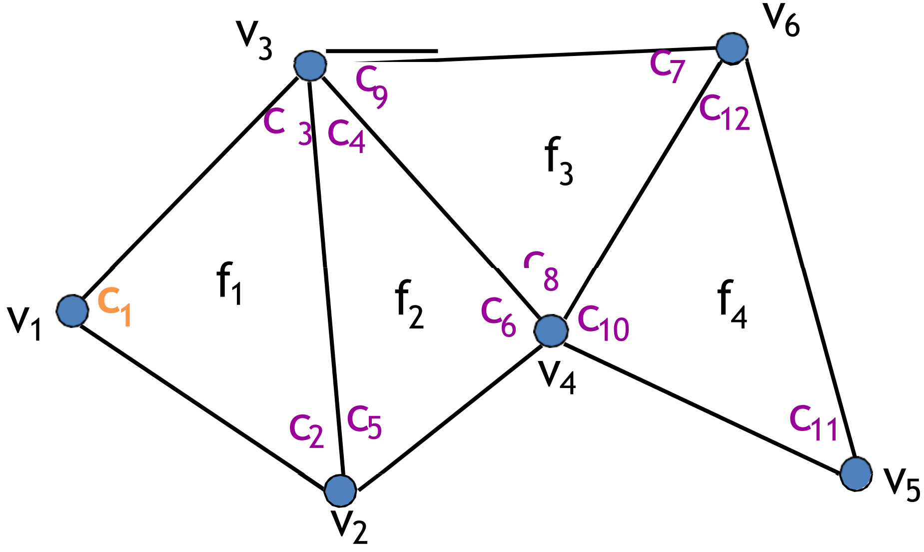
- Corner is a vertex with one of its indicent triangles



Corner Table

- Corner is a vertex with one of its indicent triangles

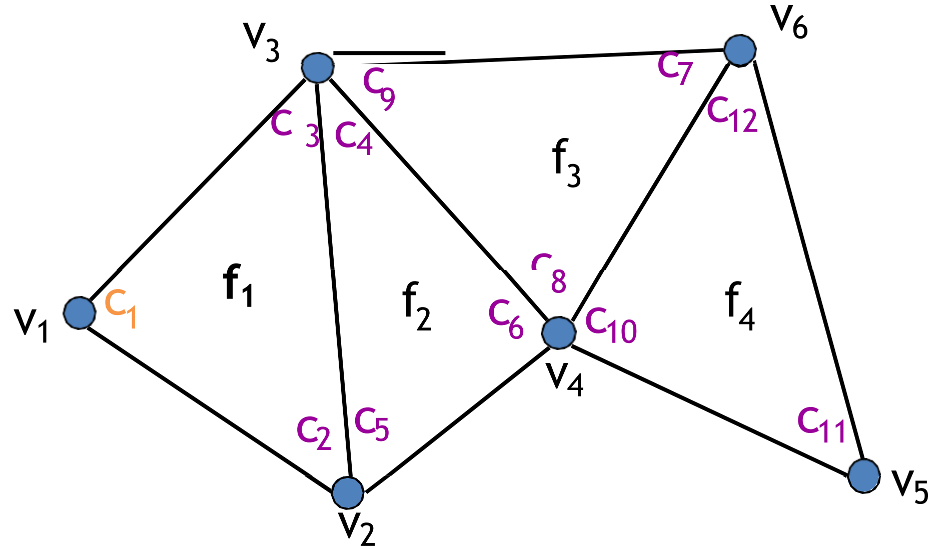
Corner - c



Corner Table

- Corner is a vertex with one of its indicent triangles

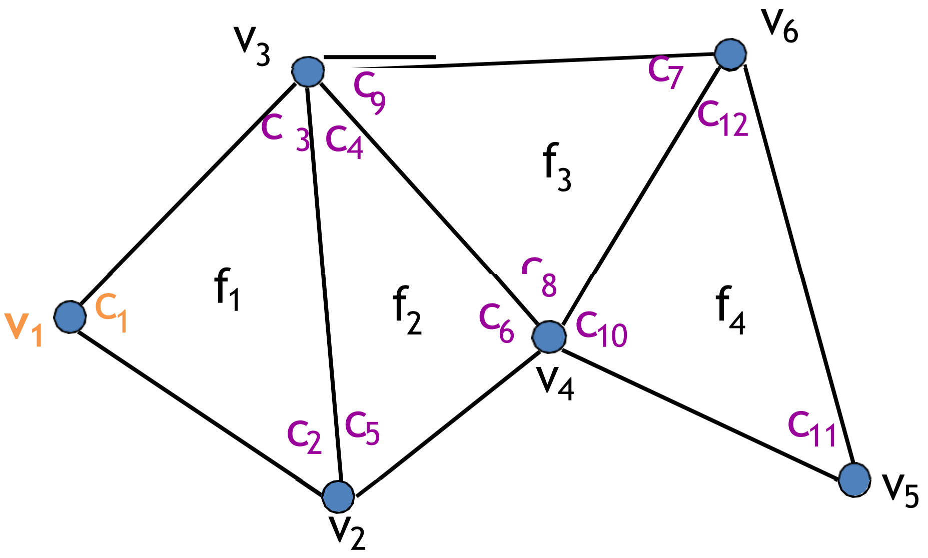
Corner - c
Triangle - c.t



Corner Table

- Corner is a vertex with one of its indicent triangles

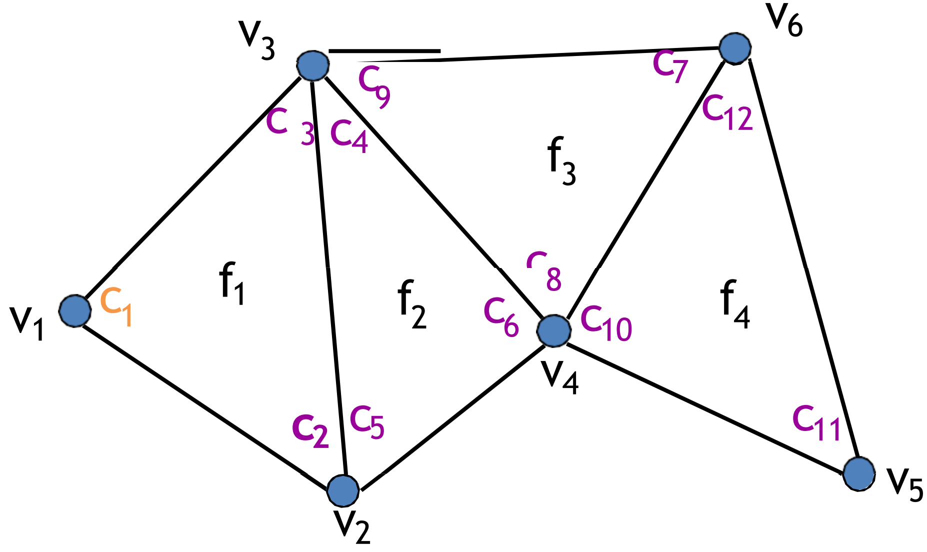
Corner - c
Triangle - c.t
Vertex - c.v



Corner Table

- Corner is a vertex with one of its indicent triangles

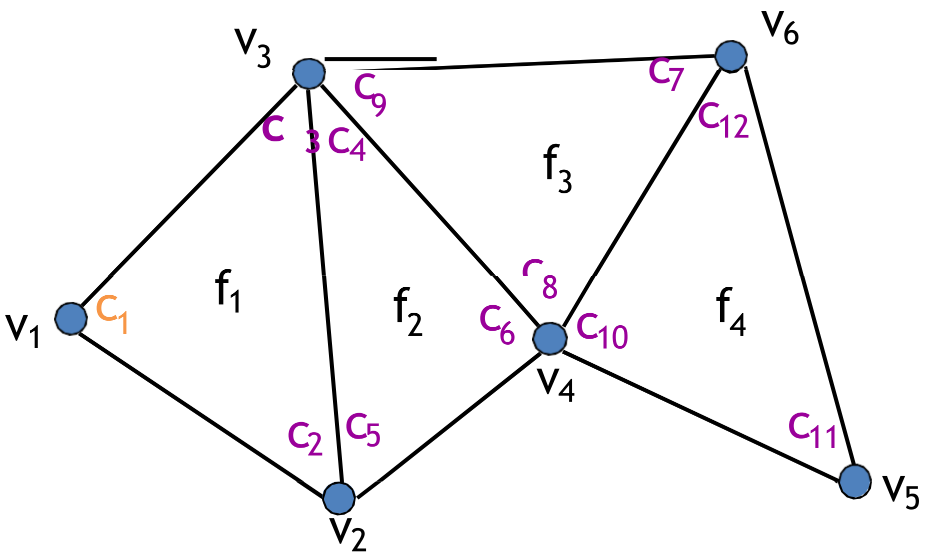
Corner - c
Triangle - c.t
Vertex - c.v
Next corner in c.t (ccw) - c.n



Corner Table

- Corner is a vertex with one of its indicent triangles

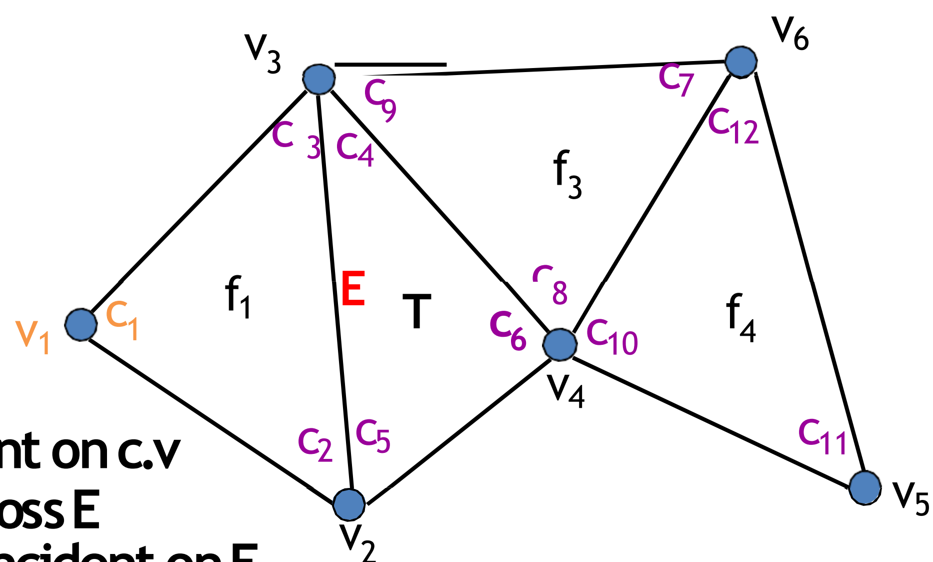
Corner - c
Triangle - c.t
Vertex - c.v
Next corner in c.t (ccw) - c.n
Previous corner - c.p (== c.n.n)



Corner Table

- Corner is a vertex with one of its indicent triangles

  Corner - c
  Triangle - c.t
  Vertex - c.v
  Next corner in c.t (ccw) - c.n
  Previous corner - c.p (== c.n.n)
  Corner opposite c - c.o
  - Edge E opposite c not incident on c.v
  - Triangle T adjacent to c.t across E
  - c.o.v vertex of T that is not incident on E



Corner Table

- Corner is a vertex with one of its indicent triangles

Corner - c
Triangle - c.t
Vertex - c.v
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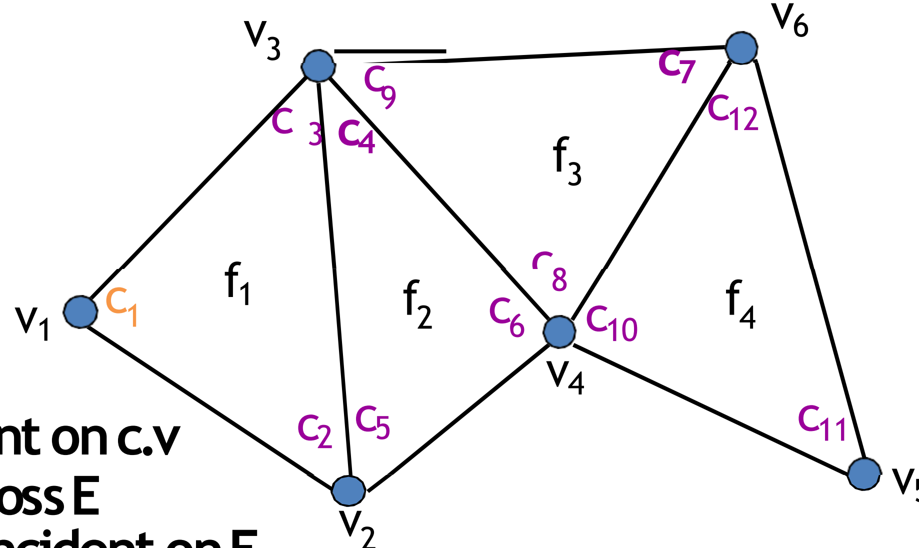
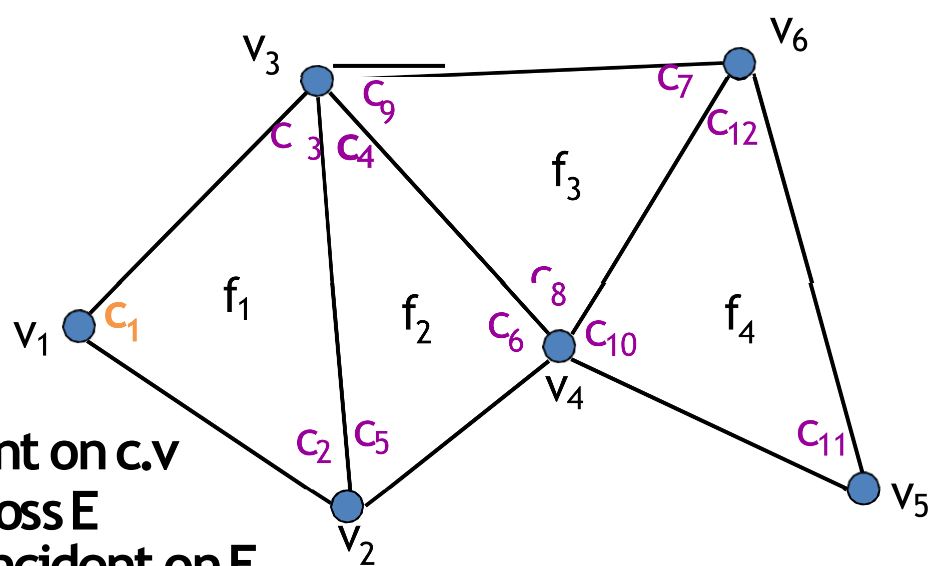
Corner opposite c - c.o
  Edge E opposite c not incident on c.v
  Triangle T adjacent to c.t across E
  c.o.v vertex of T that is not incident on E
Right corner - c.r - corner opposite c.n (== c.n.o)



Corner Table

- Corner is a vertex with one of its indicent triangles

Corner - c
Triangle - c.t
Vertex - c.v
Next corner in c.t (ccw) - c.n
Previous corner - c.p (== c.n.n)
Corner opposite c - c.o
    Edge E opposite c not incident on c.v
    Triangle T adjacent to c.t across E
    c.o.v vertex of T that is not incident on E
Right corner - c.r - corner opposite c.n (== c.n.o)
Left corner - c.l (== c.p.o == c.n.n.o)



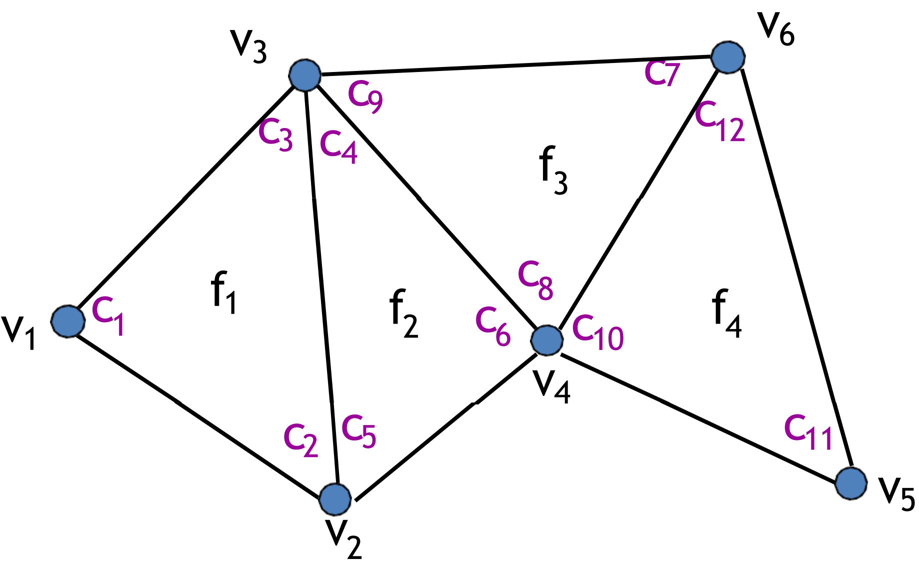
Corner Table

- Corner is a vertex with one of its indicent triangles

  Corner - c
  Triangle - c.t
  Vertex - c.v
  Next corner in c.t (ccw) - c.n
  Previous corner - c.p (== c.n.n)
  Corner opposite c - c.o
    Edge E opposite c not incident on c.v
    Triangle T adjacent to c.t across E
    c.o.v vertex of T that is not incident on E
  Right corner - c.r - corner opposite c.n (== c.n.o)
  Left corner - c.l (== c.p.o == c.n.n.o)

Corner Table

- Corner is a vertex with one of its indicent triangles



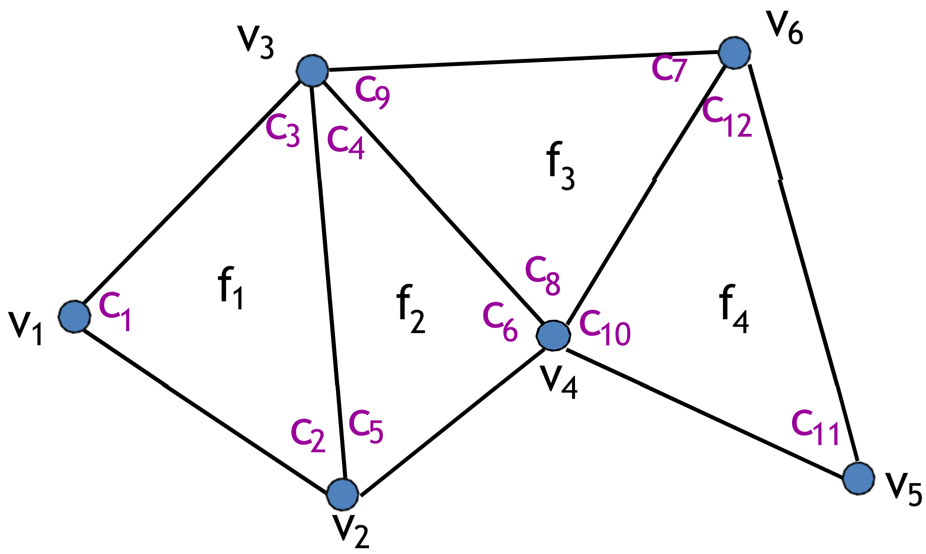
corner	c.v	c.t	c.n	c.p	c.o	c.r	c.l
c_1	v_1	f_1	c_2	c_3	c_6		
c_2	v_2	f_1	c_3	c_1			c_6
c_3	v_3	f_1	c_1	c_2		c_6	
c_4	v_3	f_2	c_5	c_6		c_7	c_1
c_5	v_2	f_2	c_6	c_4	c_7	c_1	
c_6	v_4	f_2	c_4	c_5	c_1		c_7

Corner Table

- Store:
  - Corner table
  - For each vertex – a list of all its corners
- Corner number *j**3-2, *j**3-1 and *j**3 match face number *j*

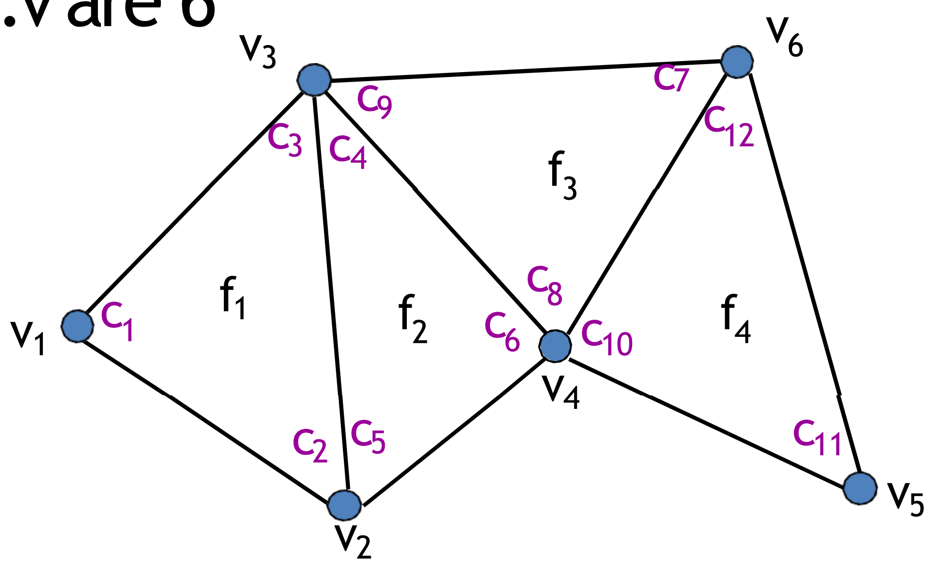
Corner Table

- What are the vertices of face #3?
  - Check c.v of corners 9, 8, 7



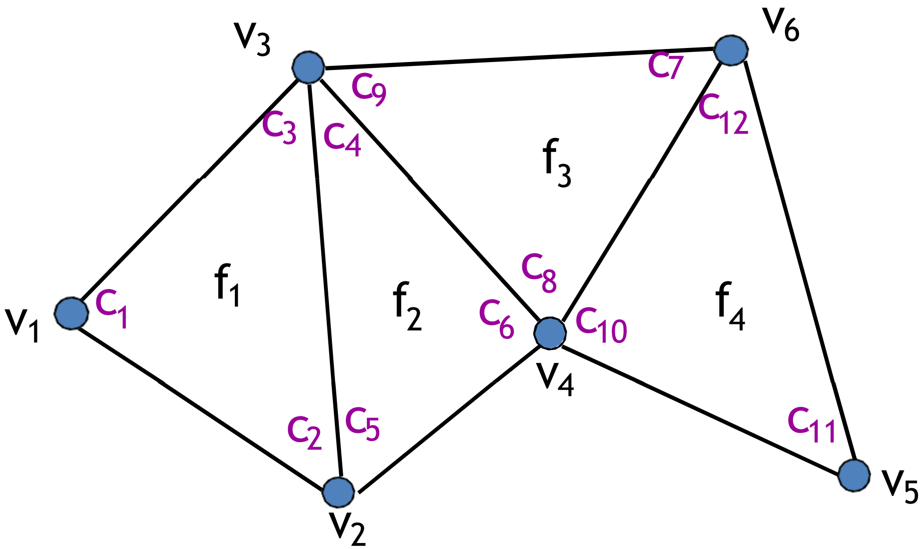
Corner Table

- Are vertices 2 and 6 adjacent?
  - Scan all corners of vertex 2, check if c.p.v or c.n.v are 6



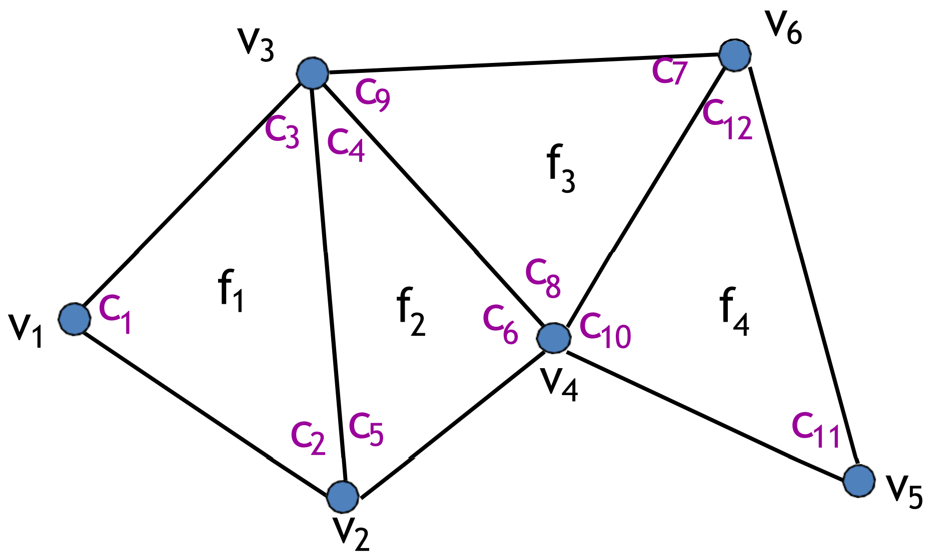
Corner Table

- Which faces are adjacent to vertex 3?
  - Check c.t of all corners of vertex 3



Corner Table

- One ring neighbors of vertex v_4?
  - Get the corners c_6 c_8 c_{10} of this vertex
  - Go to c_i.n.v and c_i.p.v for i = 6, 8, 10.
  - Remove duplicates



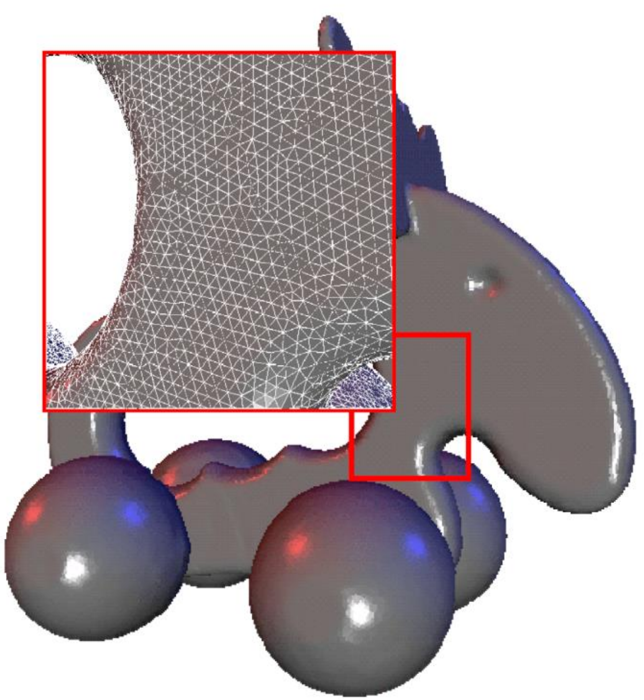
Corner Table

- Pros:
  - All queries in O(1) time
  - Most operations are O(1)
  - Convenient for rendering
- Cons:
  - Only triangular, manifold meshes
  - Redundancy

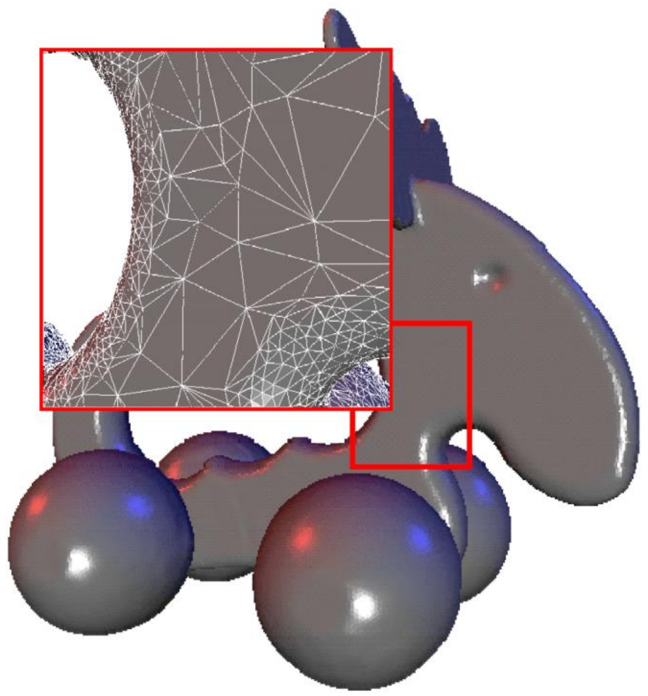
Multiple Simplification

Applications

- Oversampled 3D scan data



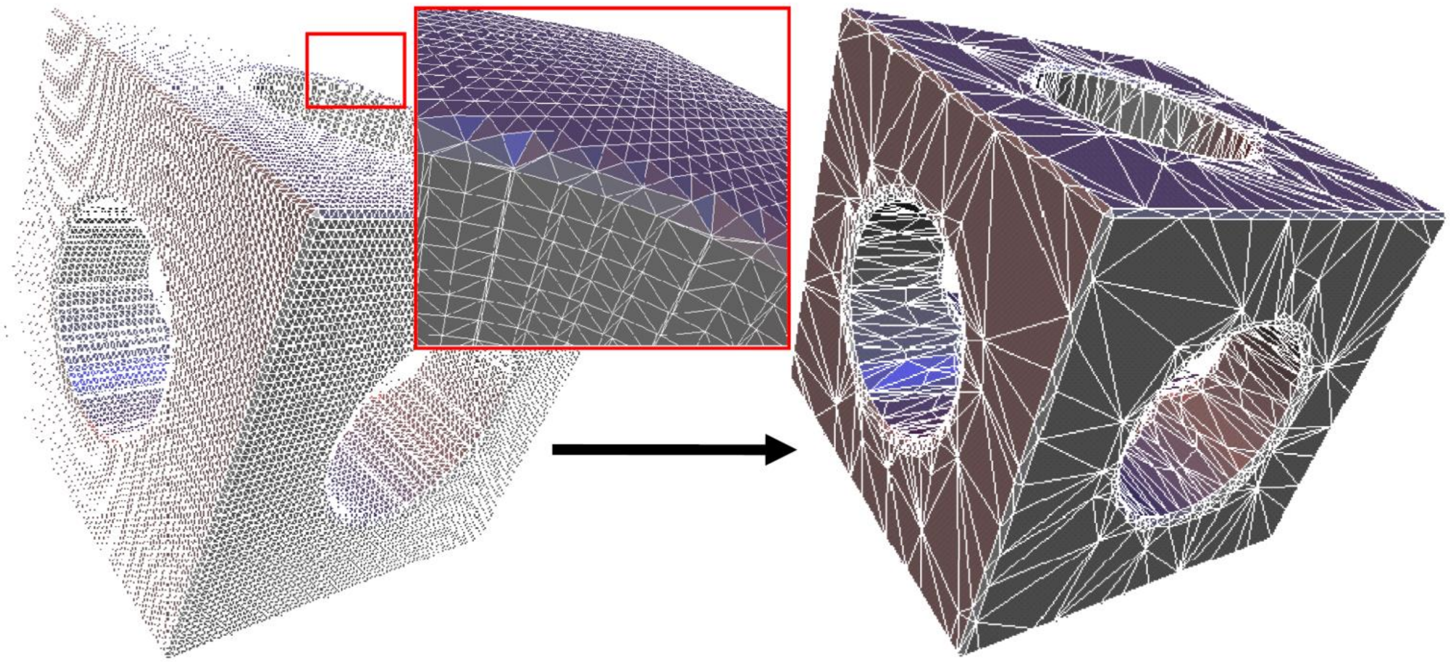
~150k triangles



~80k triangles

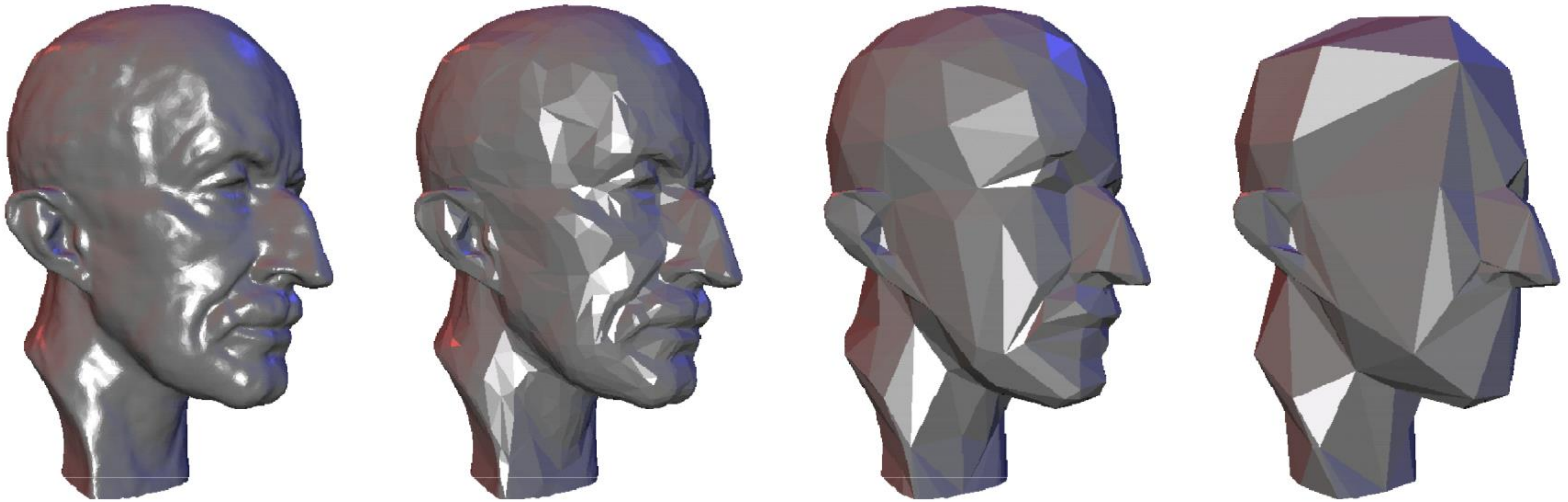
Applications

- Overtessellation: E.g. iso-surface extraction



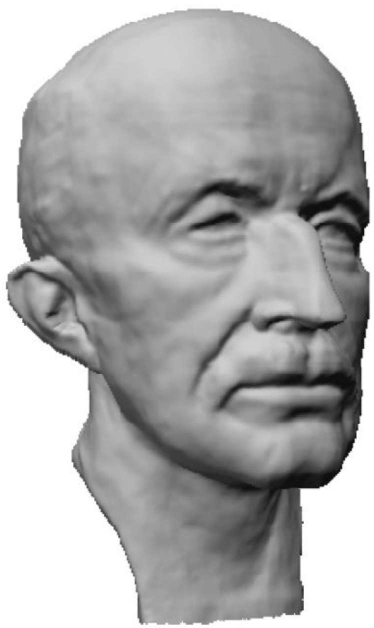
Applications

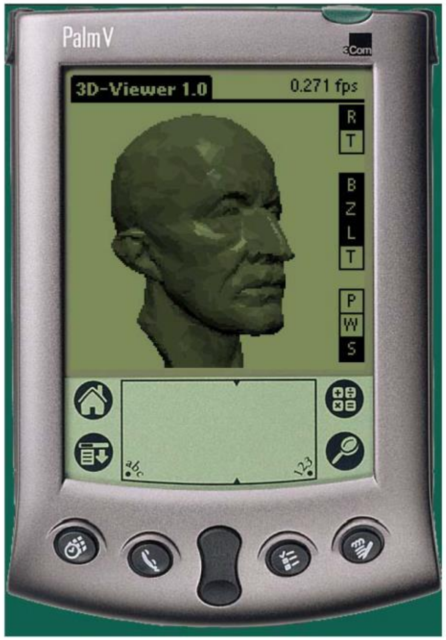
- Multi-resolution hierarchies for
  - efficient geometry processing
  - level-of-detail (LOD) rendering



Applications

- Adaptation to hardware capabilities

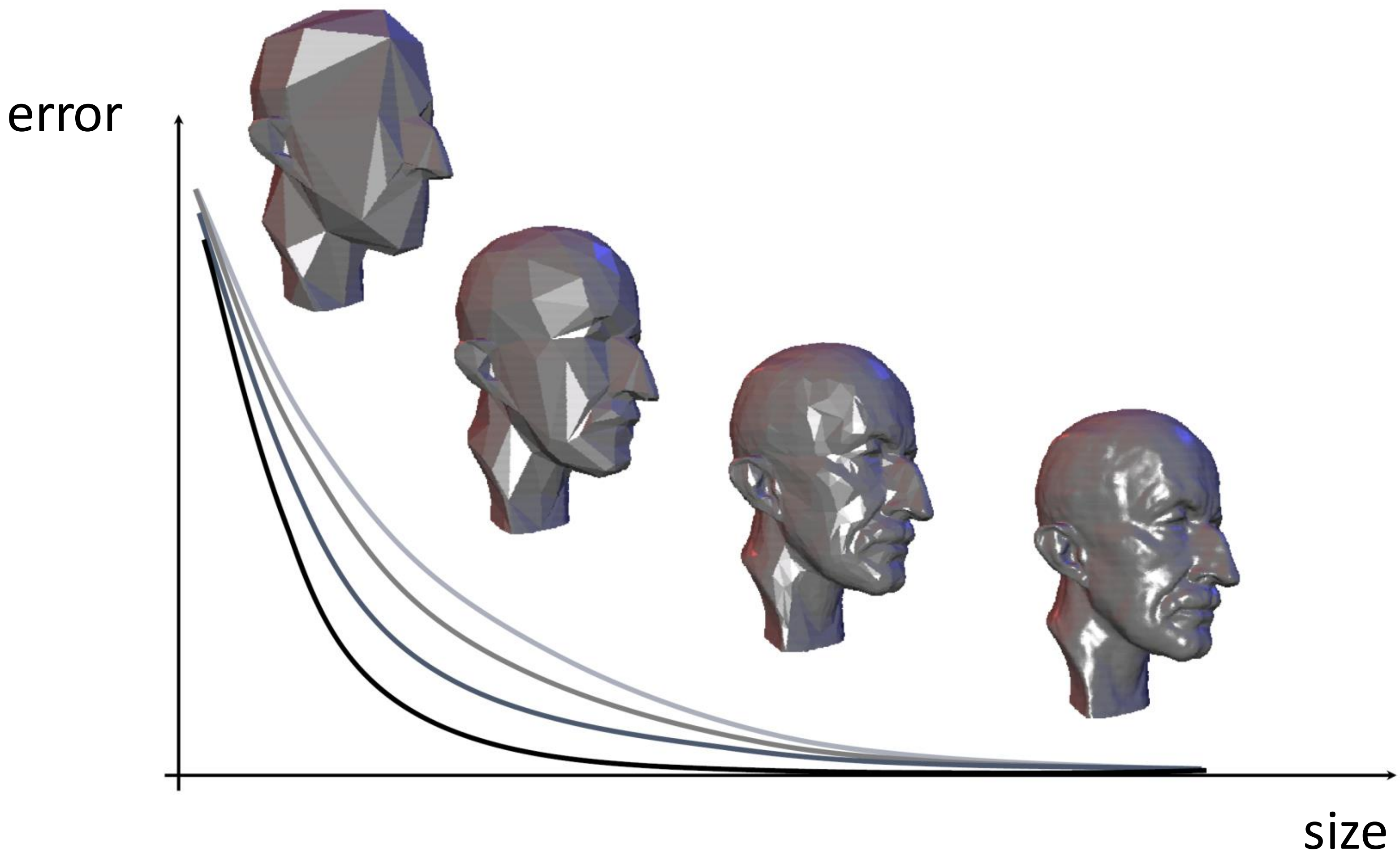




1999

2012

Size-Quality Tradeoff



Problem Statement

- Given: $M = (V,F)$
- Find: $M' = (V',F')$ such that
  - $|V'| = n < |V|$ and $d(M,M')$ is minimal, or
  - $d(M,M') < eps$ and $|V'|$ is minimal

- Respect additional fairness criteria
  - Normal deviation, triangle shape, scalar attributes, etc.

Mesh Decimation Methods

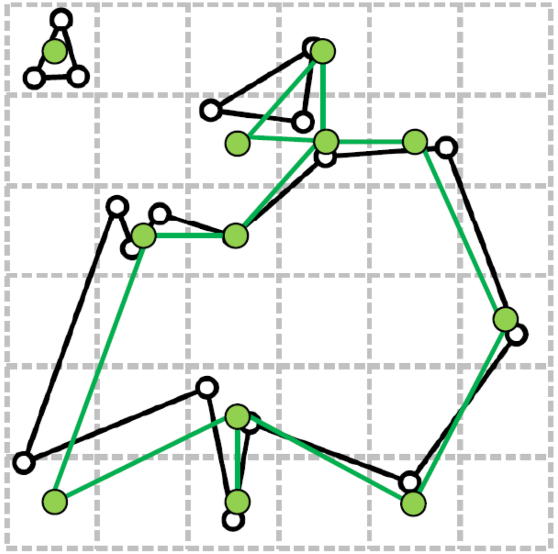
- **Vertex clustering**
- **Incremental decimation**
- Remeshing

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes

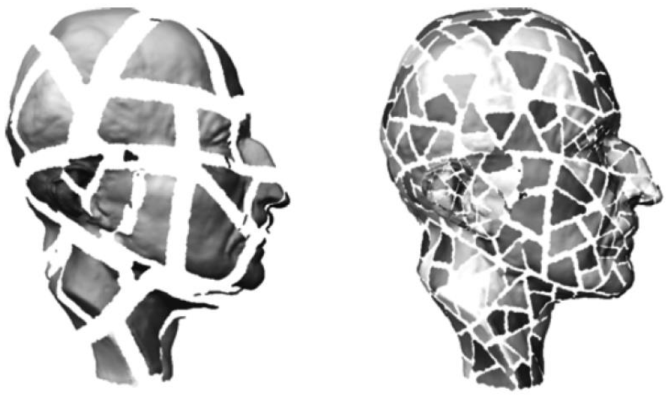
Vertex Clustering

- Cluster Generation
  - Uniform 3D grid
  - Map vertices to cluster cells
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

- Cluster Generation
  - Hierarchical approach
  - Top-down or bottom-up



- Computing a representative
- Mesh generation
- Topology changes

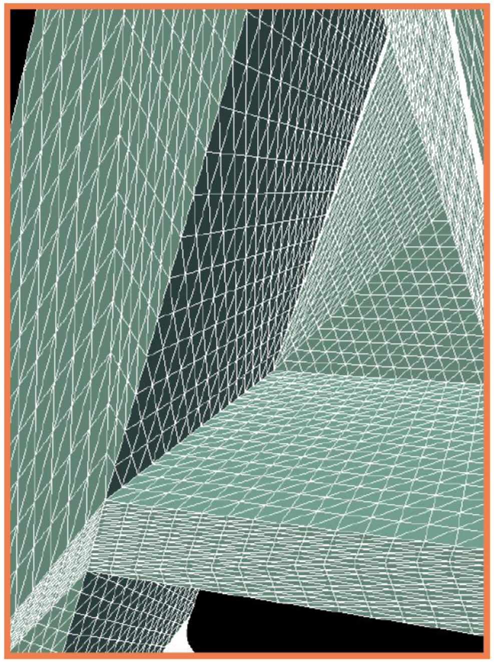
Vertex Clustering

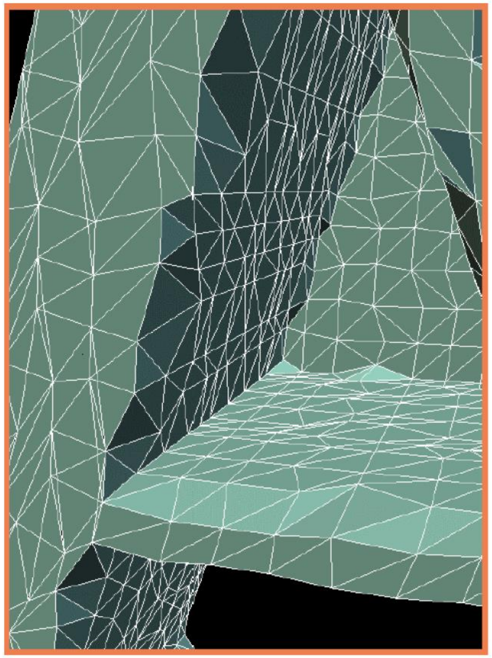
- Cluster Generation

- Computing a representative
  - Average/median vertex position
  - Error quadrics

- Mesh generation
- Topology changes

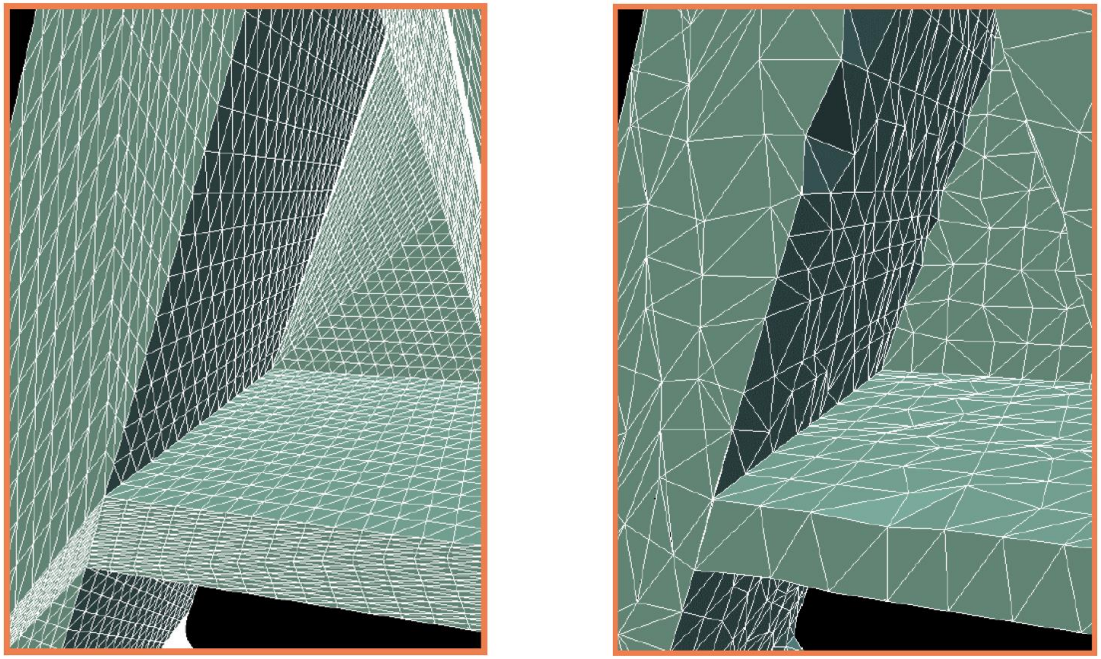
Computing a Representative





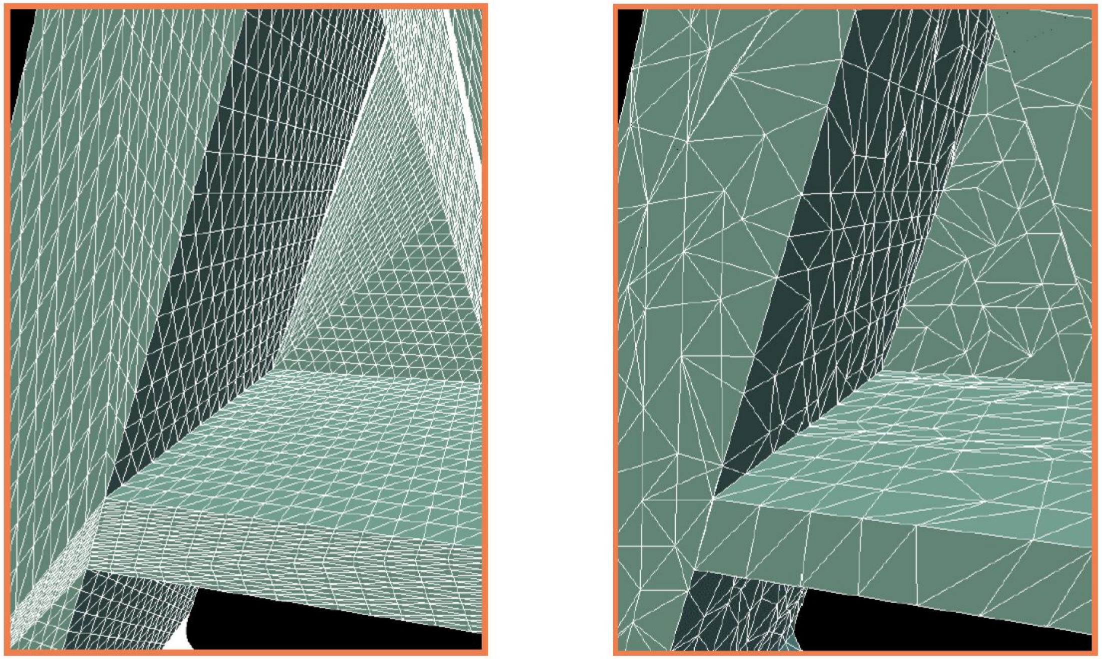
Average vertex position

Computing a Representative



Median vertex position

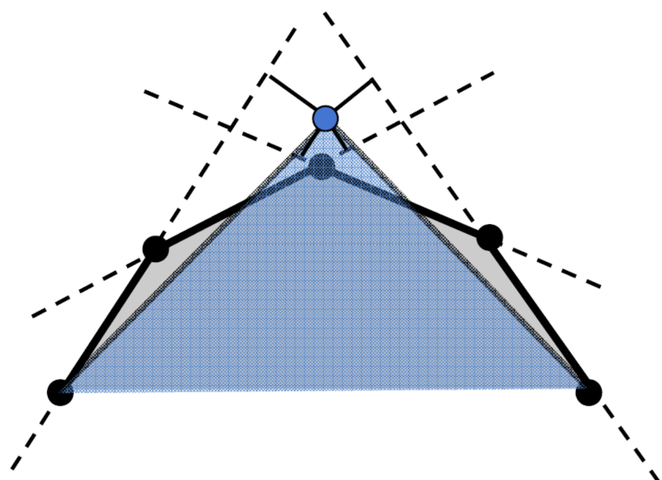
Computing a Representative



Error quadrics

Error Quadrics

- Patch is expected to be piecewise flat
- Minimize distance to neighboring triangles' planes



Error Quadrics

- Squared distance of point *p* to plane *q*

$$p = (x, y, z, 1)^T, \quad q = (a, b, c, d)^T$$

$$dist(q, p)^2 = (q^T p)^2 = p^T (q q^T) p =: p^T Q_q p$$

$$Q_q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Error Quadrics

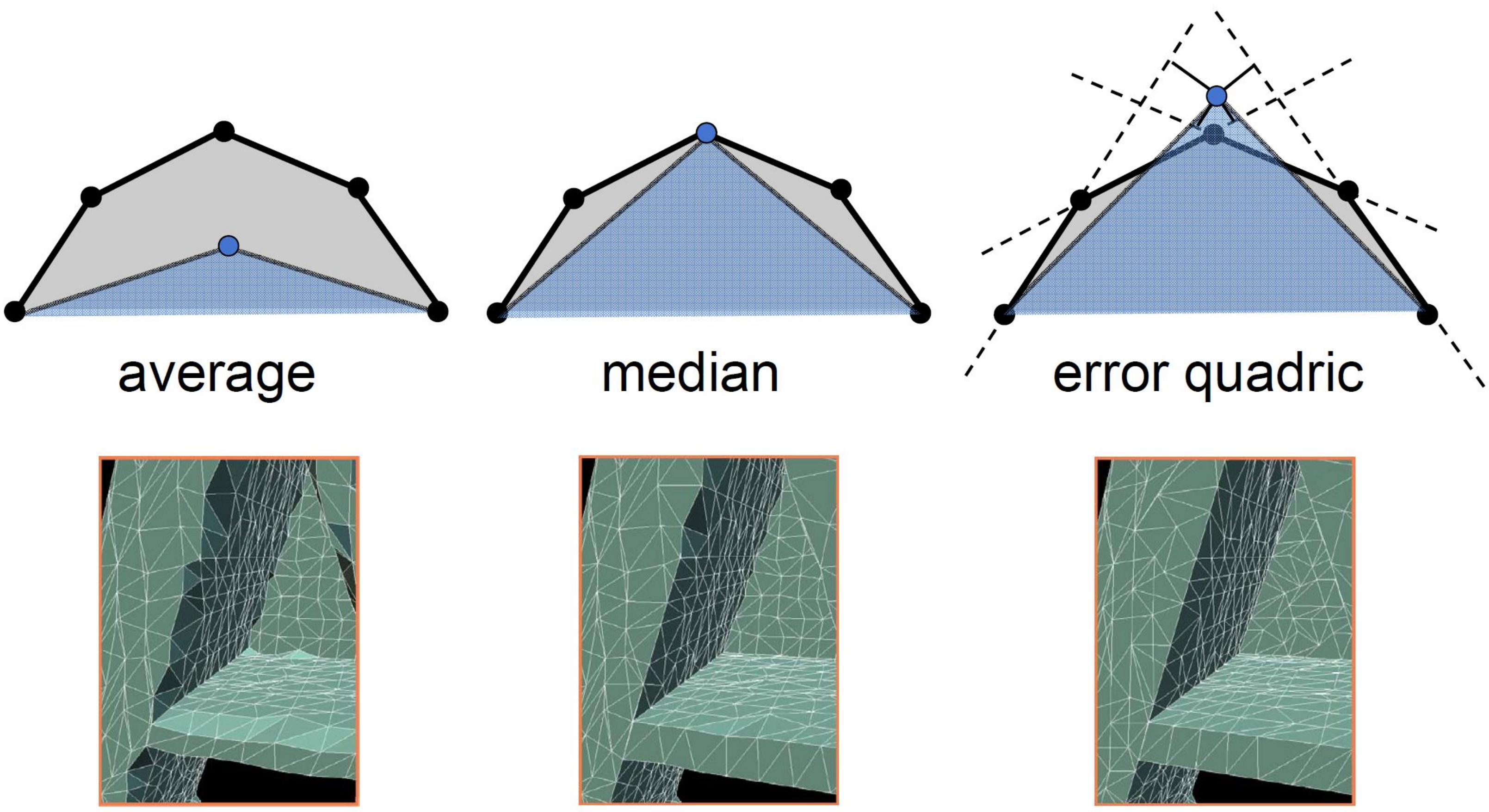
- Sum distances to planes q_i of vertex' neighboring triangles

$$\sum_i dist(q_i, p)^2 = \sum_i p^T Q_{q_i} p = p^T \left(\sum_i Q_{q_i} \right) p =: p^T Q_p p$$

- Point p* that minimizes the error satisfies:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} p^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Comparison



Vertex Clustering

- Cluster Generation
- Computing a representative

- Mesh generation
  - Clusters p<-> $\{p_0,\ldots, p_n\}$, q<-> $\{q_0,\ldots, q_m\}$

- Topology changes

Vertex Clustering

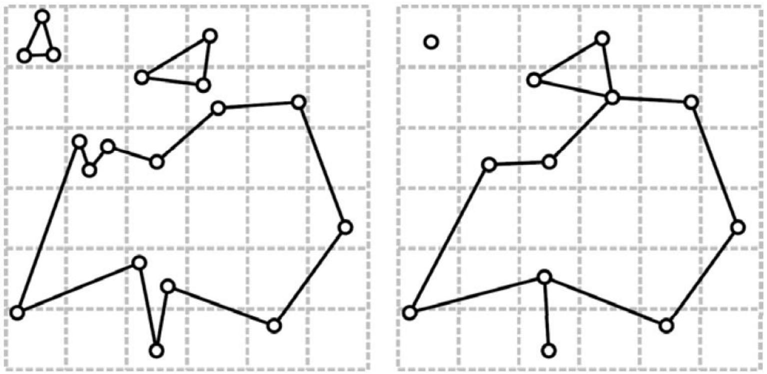
- Cluster Generation
- Computing a representative

- Mesh generation
  - Clusters p<-> $\{p_0,\ldots, p_n\}$, q<-> $\{q_0,\ldots, q_m\}$
  - Connect (p,q) if there was an edge (p_i, q_i)
- Topology changes

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation

- Topology changes
  - If different sheets pass through one cell
  - Can be non-manifold



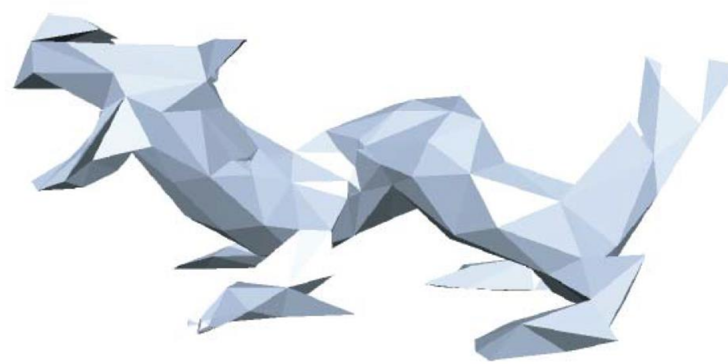
Incremental Decimation

Incremental Decimation

500K

50K

5K



0.5K

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

General Setup

- Repeat:
  - Pick mesh region
  - Apply decimation operator

- Until no further reduction possible

Greedy Optimization

- For each region
  - evaluate quality after decimation
  - enqeue(quality, region)

- Repeat:
  - get best mesh region from queue
  - apply decimation operator
  - update queue
- Until no further reduction possible

Global Error Control

- For each region
  - evaluate quality after decimation
  - enqeue(quality, region)

- Repeat:
  - get best mesh region from queue
  - If error < eps
    - Apply decimation operator
    - Update queue
- Until no further reduction possible

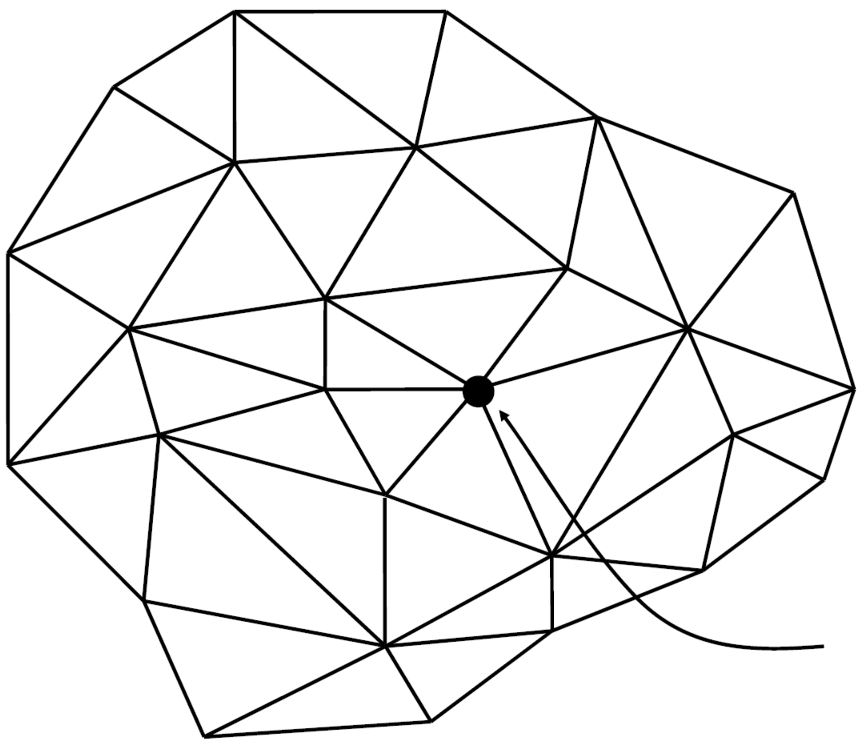
Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

Decimation Operators

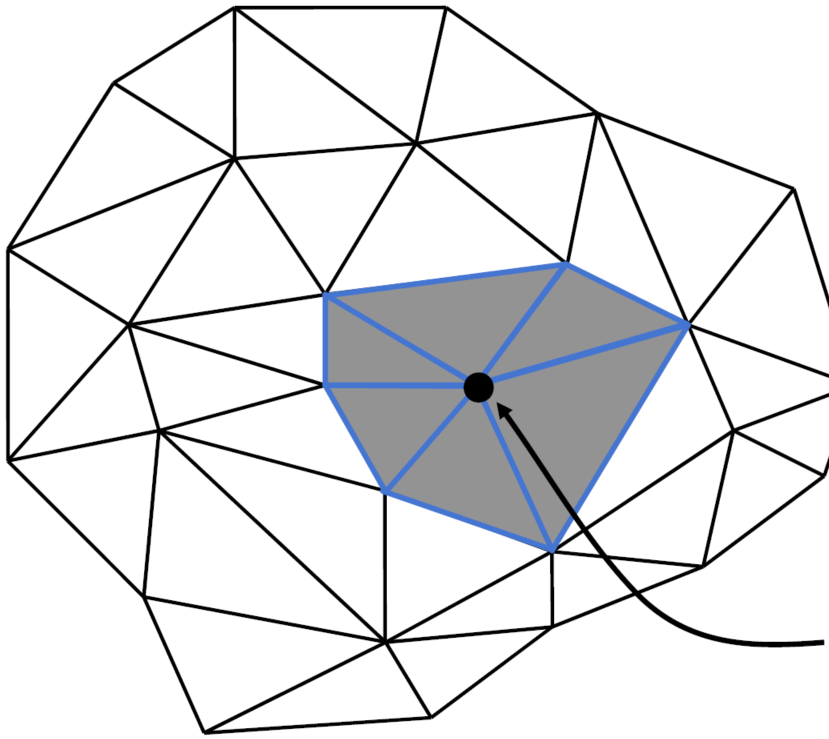
- What is a “region”?
- What are the DOF for re-triangulation?
- Classification
  - Topology-changing vs. topology-preserving
  - Subsampling vs. filtering
  - Inverse operation -> progressive meshes [Hoppe et al….]

Vertex Removal



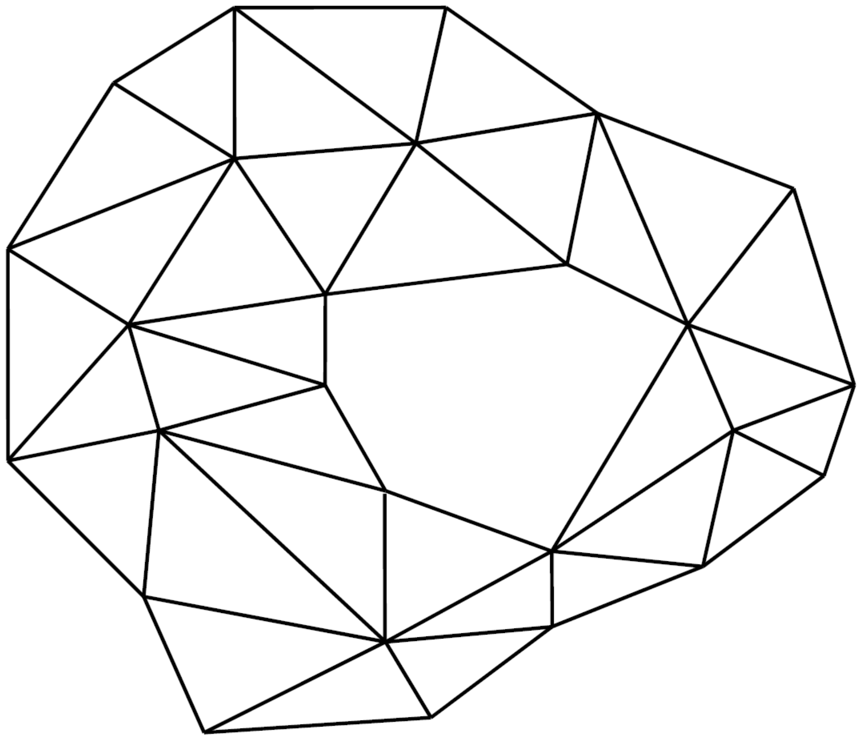
Select a vertex to
be eliminated

Vertex Removal



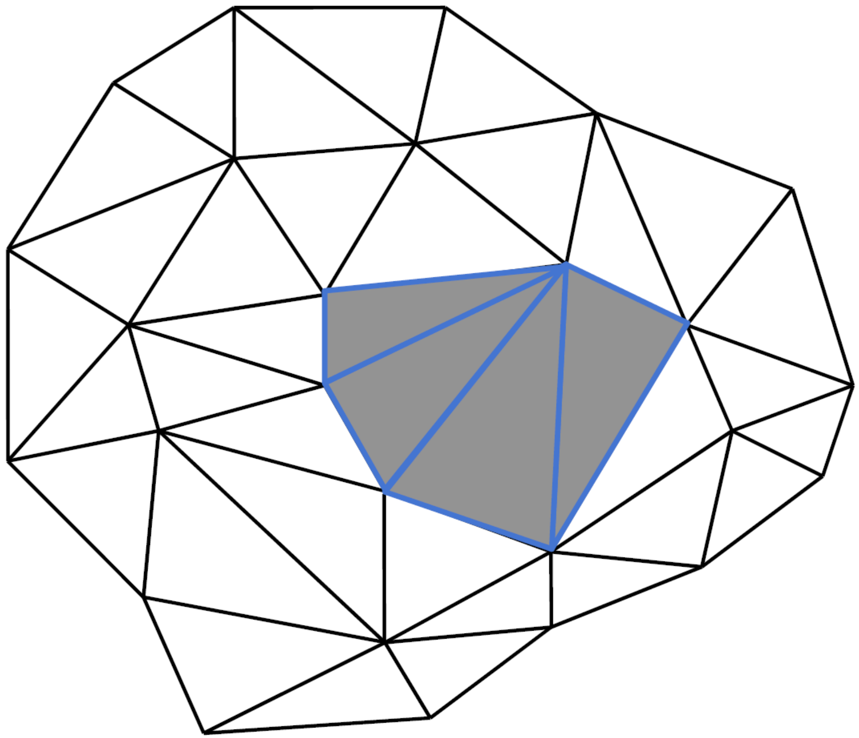
Select all triangles sharing this vertex

Vertex Removal



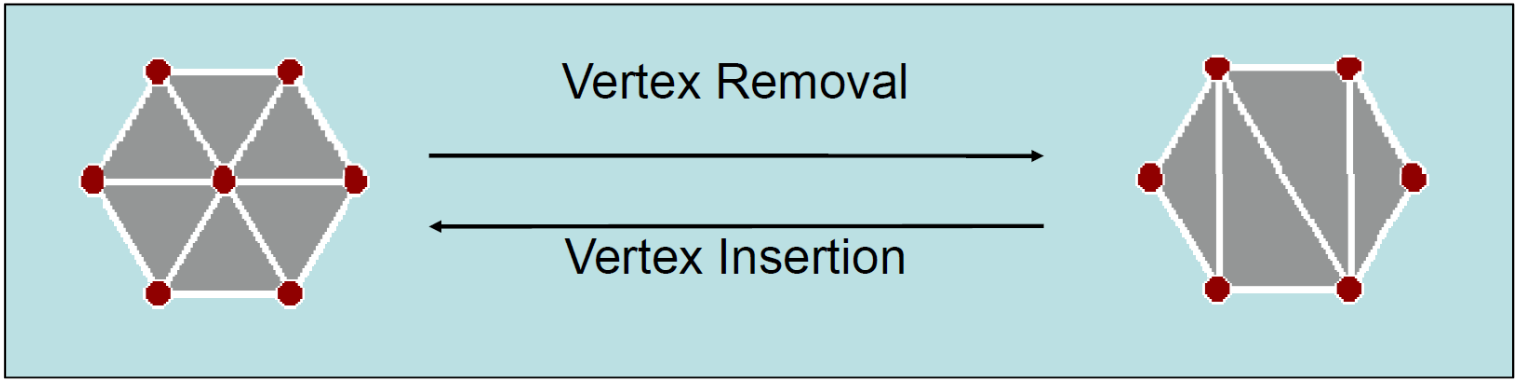
Remove the selected triangles, creating the hole

Vertex Removal



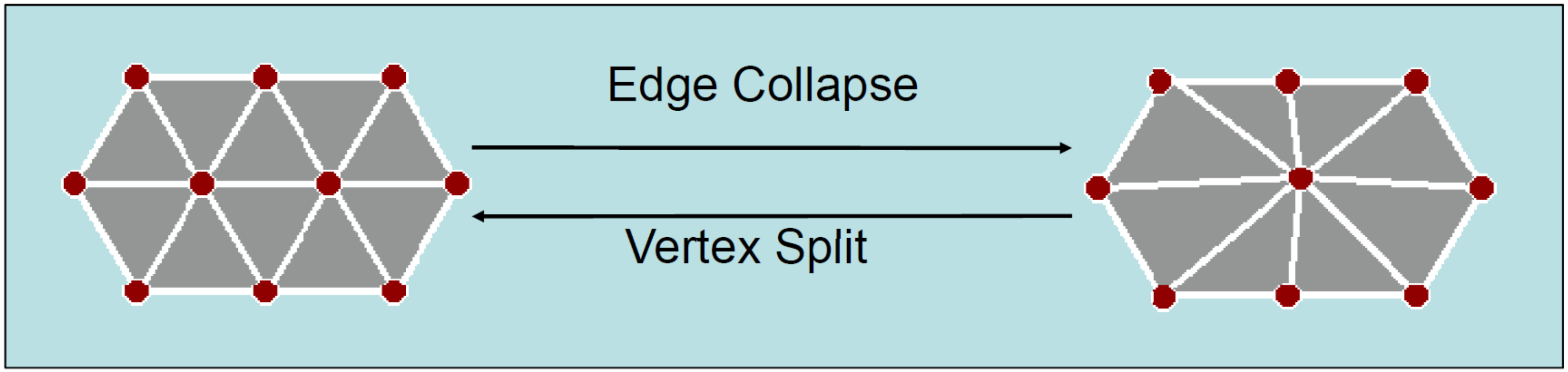
Fill the hole with new triangles

Decimation Operators



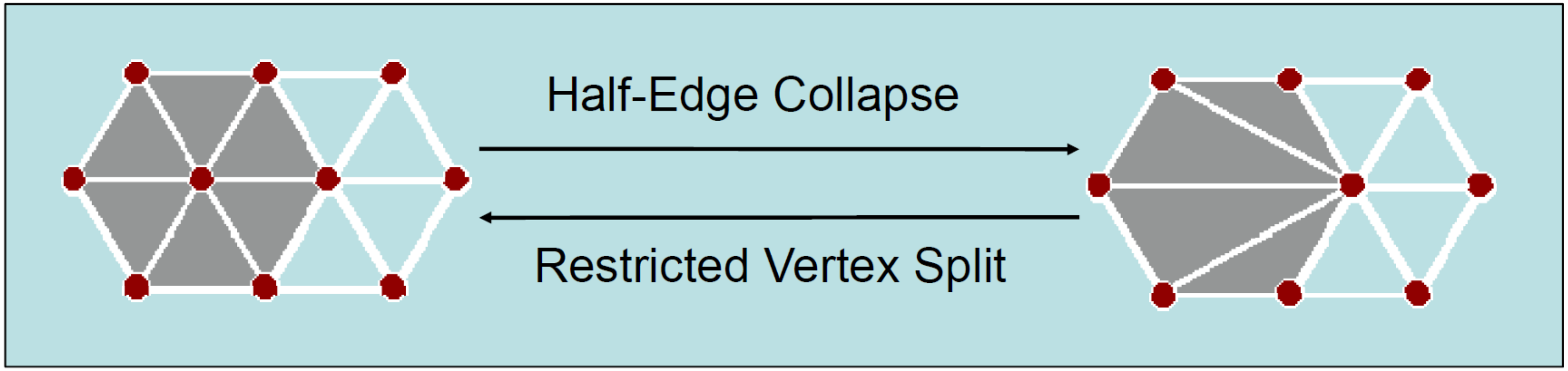
- Remove vertex
- Re-triangulate hole
  - Combinatorial degrees of freedom

Decimation Operators



- Merge two adjacent vertices
- Define new vertex position
  - Continuous degrees of freedom

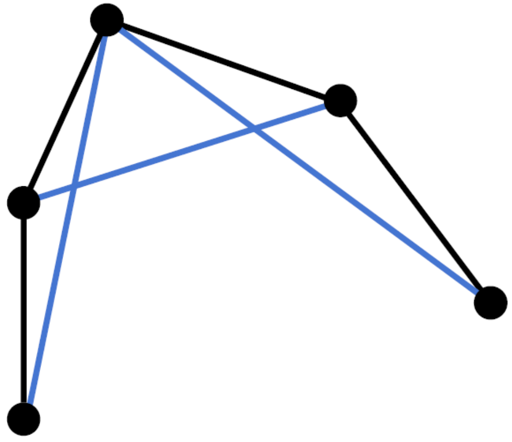
Decimation Operators



- Collapse edge into one end point
  - Special case of vertex removal
  - Special case of edge collapse
- No degrees of freedom

Fairness Criteria

- Rate quality of decimation operation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - …



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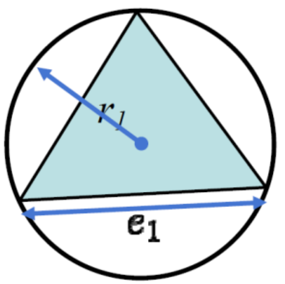
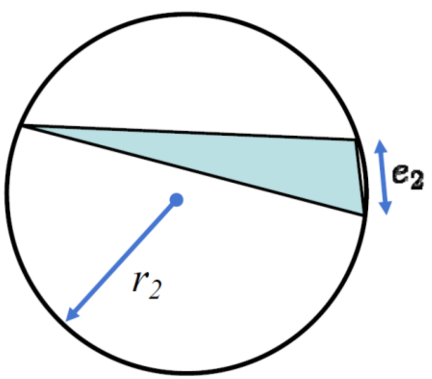


$$\frac{r_1}{e_1} < \frac{r_2}{e_2}$$

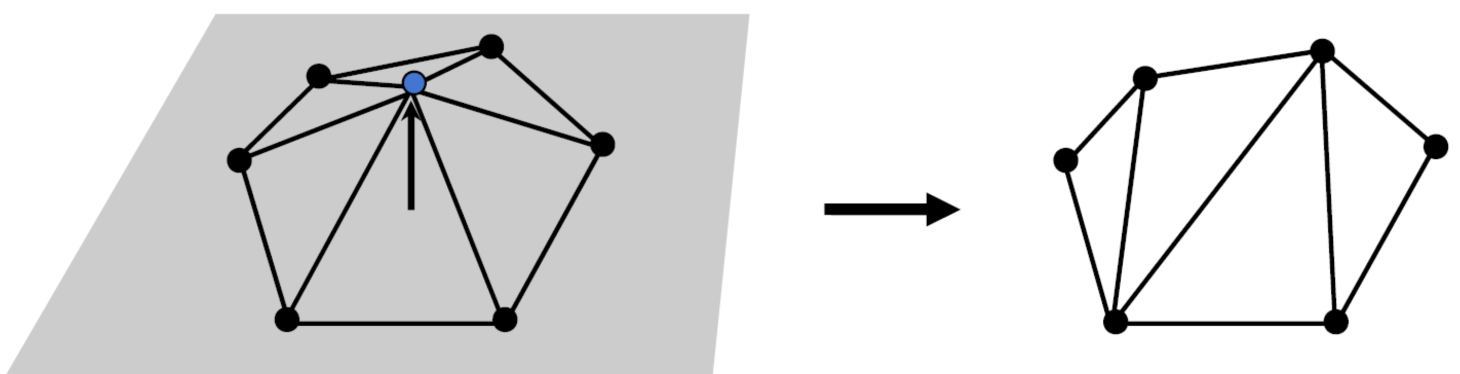


Incremental Decimation

- General Setup
- Decimation operators
- **Error metrics**
- Fairness criteria

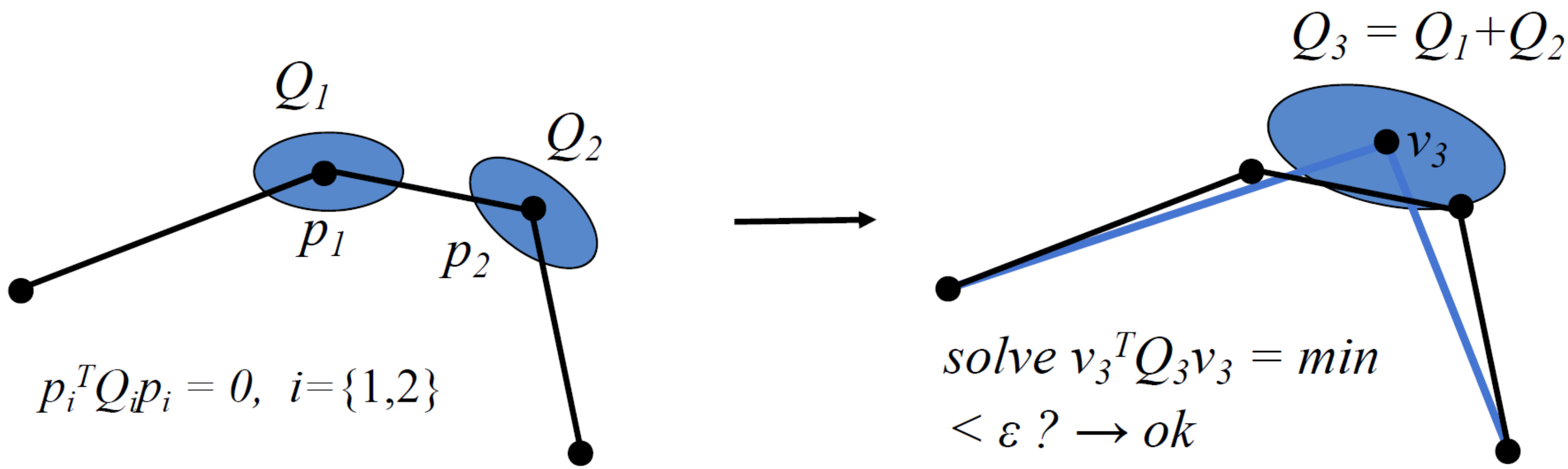
Local Error Metrics

- Local distance to mesh
  - Compute average plane
  - No comparison to *original* geometry



Global Error Metrics

- Error quadrics
  - Squared distance to planes at vertex
  - No bound on true error



Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

Fairness Criteria

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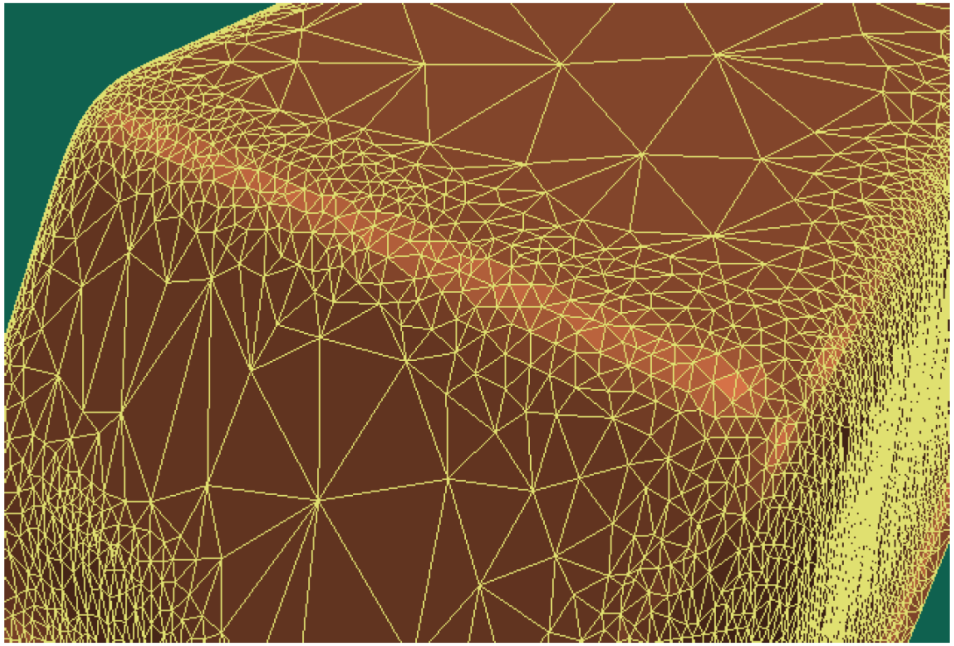

Fairness Criteria

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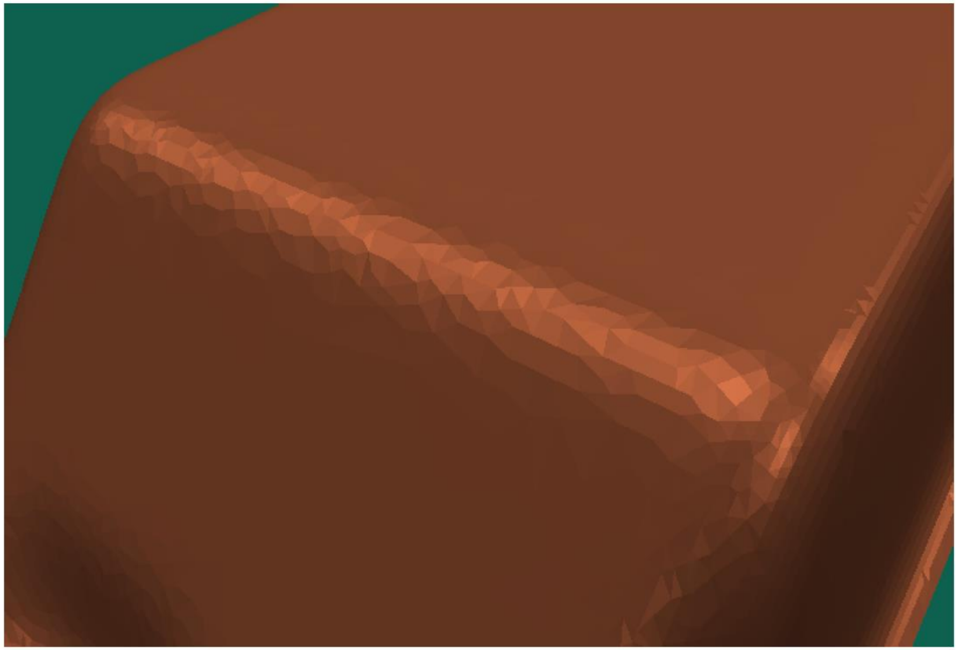

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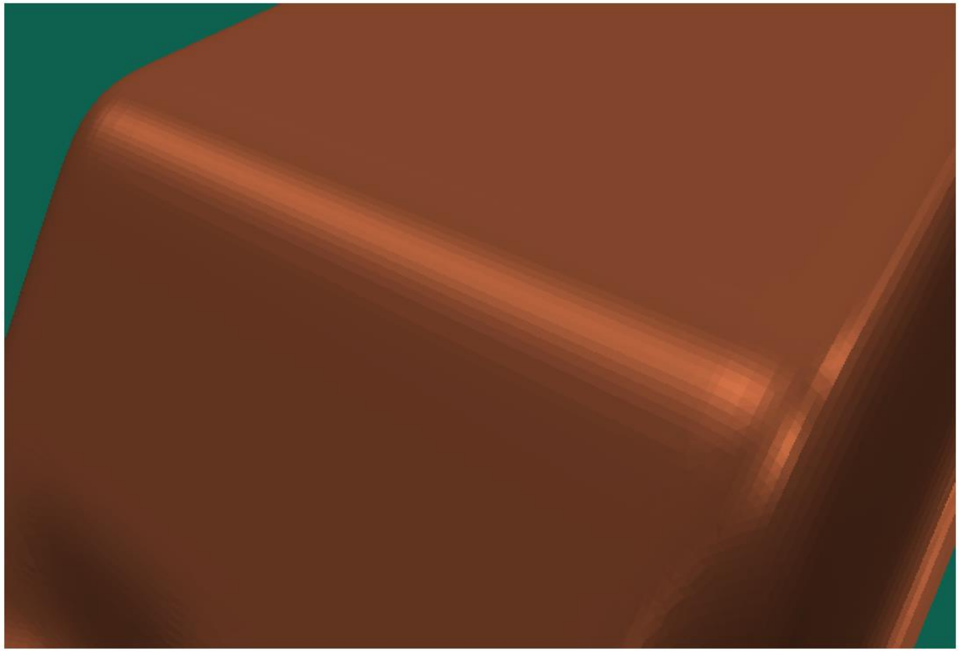

Fairness Criteria

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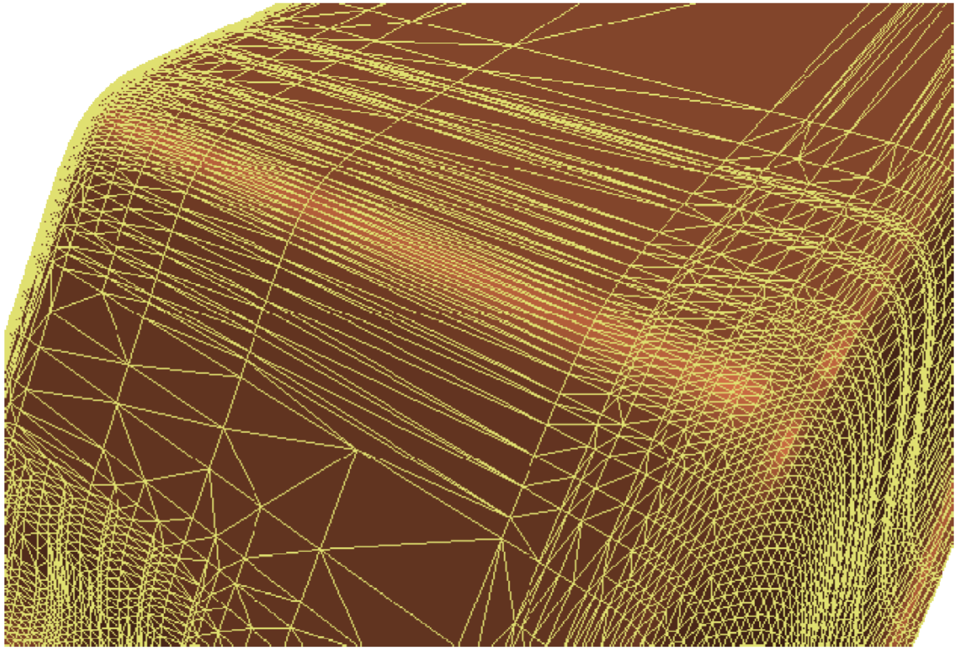

Fairness Criteria

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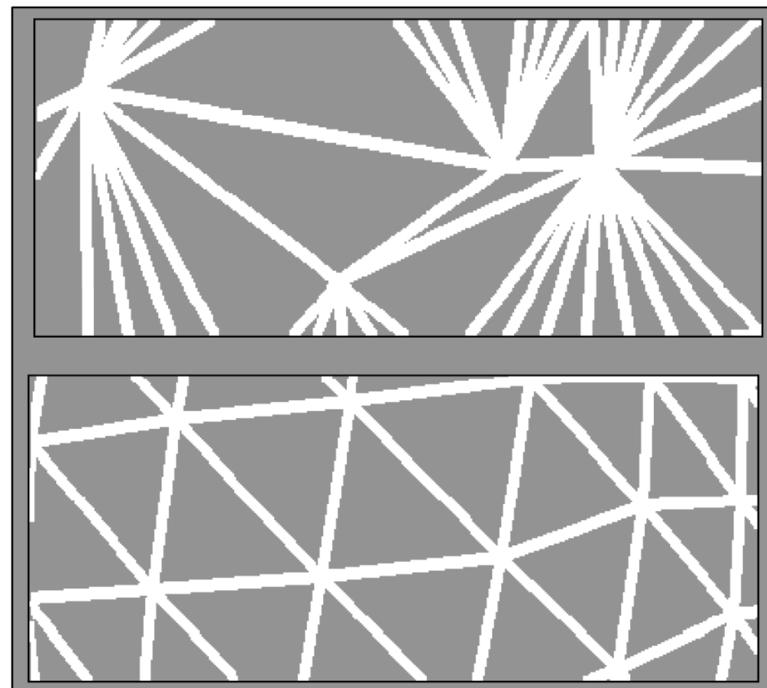


Comparison

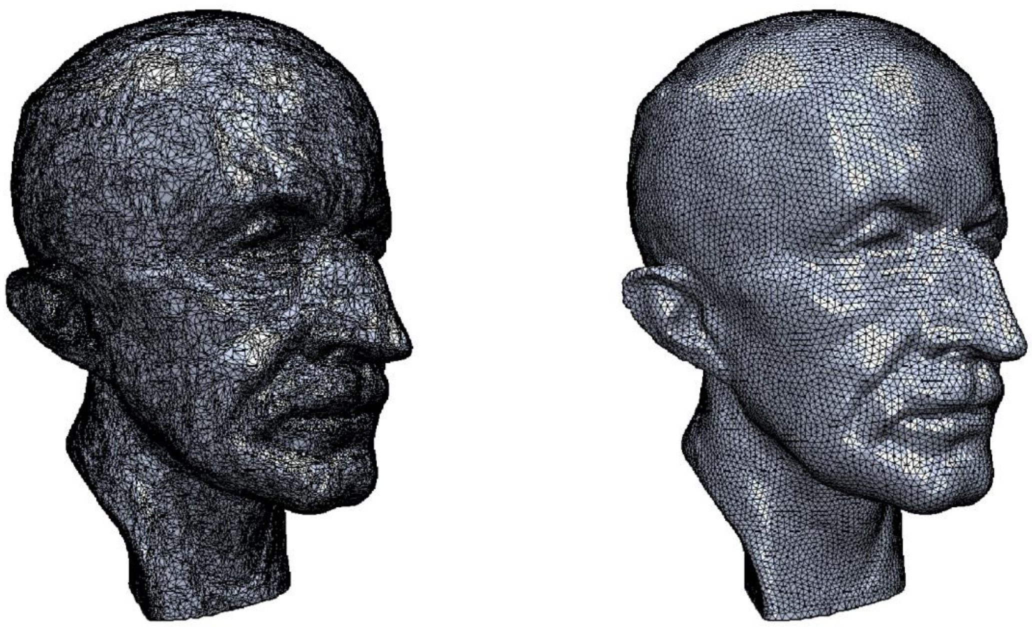
- Vertex clustering
  - fast, but difficult to control simplified mesh
  - Topology changes, non-manifold meshes
  - Global error bound, but often not close to optimum

- Incremental decimation with quadratic error metrics
  - good trade-off between mesh quality and speed
  - explicit control over mesh topology
  - restricting normal deviation improves mesh quality

Remeshing

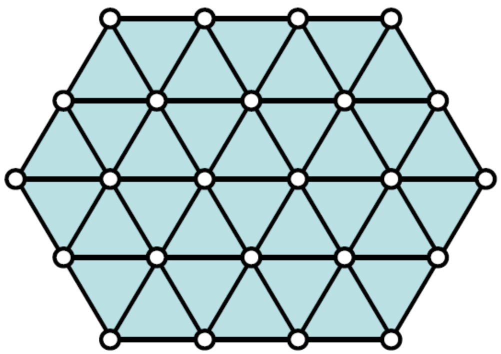
Remeshing

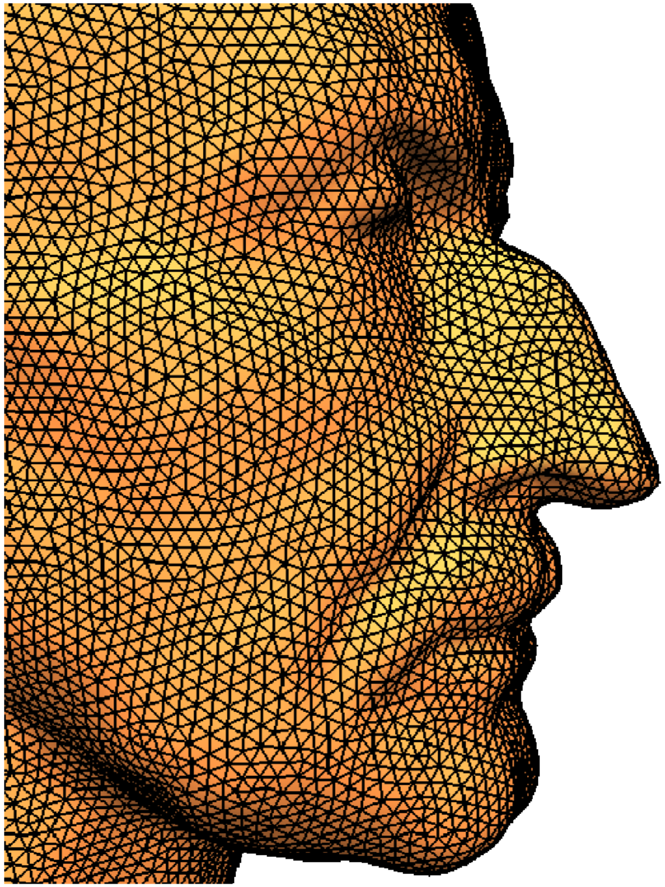
Given a 3D mesh, find a “better” discrete representation of the underlying surface



What is a good mesh?

- Equal edge lengths
- Equilateral triangles
- Valence close to 6





What is a good mesh?

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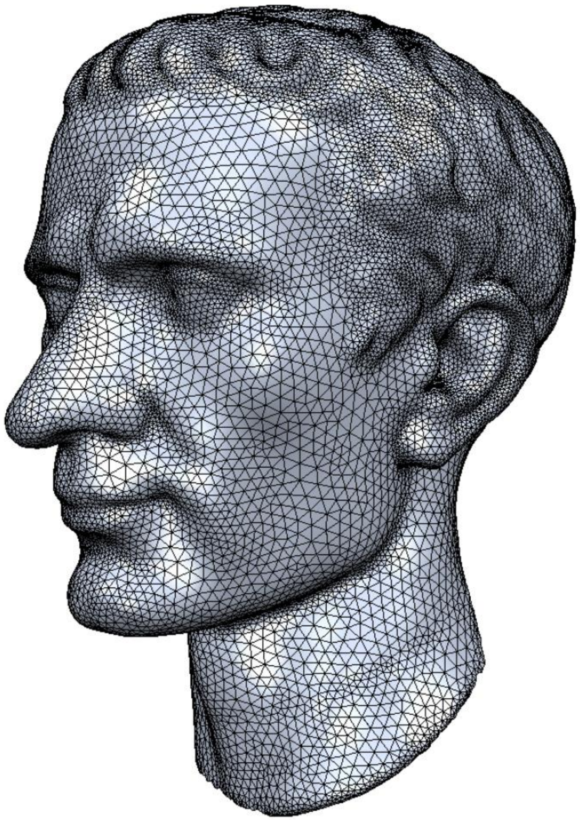
- Uniform vs. adaptive sampling



What is a good mesh?

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- Equilateral triangles
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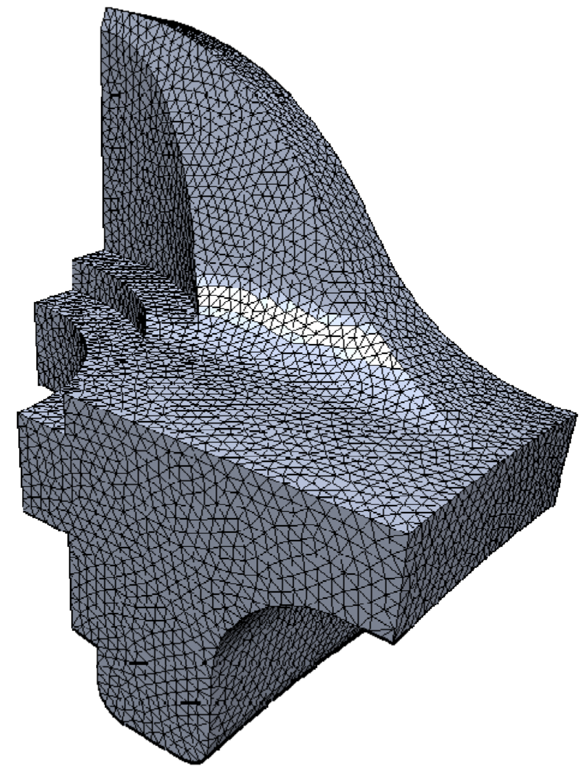
- Uniform vs. adaptive sampling
- Feature preservation



What is a good mesh?

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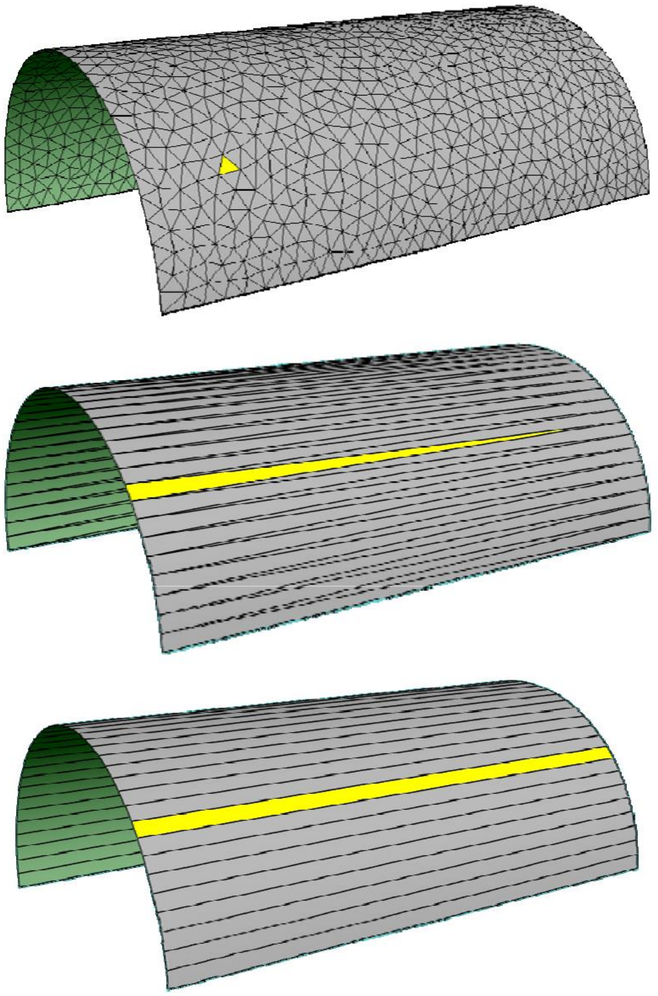
- Uniform vs. adaptive sampling
- Feature preservation
- Alignment to curvature lines
- Isotropic vs. anisotropic



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- Triangles vs. quadranges