Slide Credit:
http://www.siggraph.org/education/materials/HyperGraph/aliasing/alias0.htm

CS354 Computer Graphics
Sampling and Aliasing

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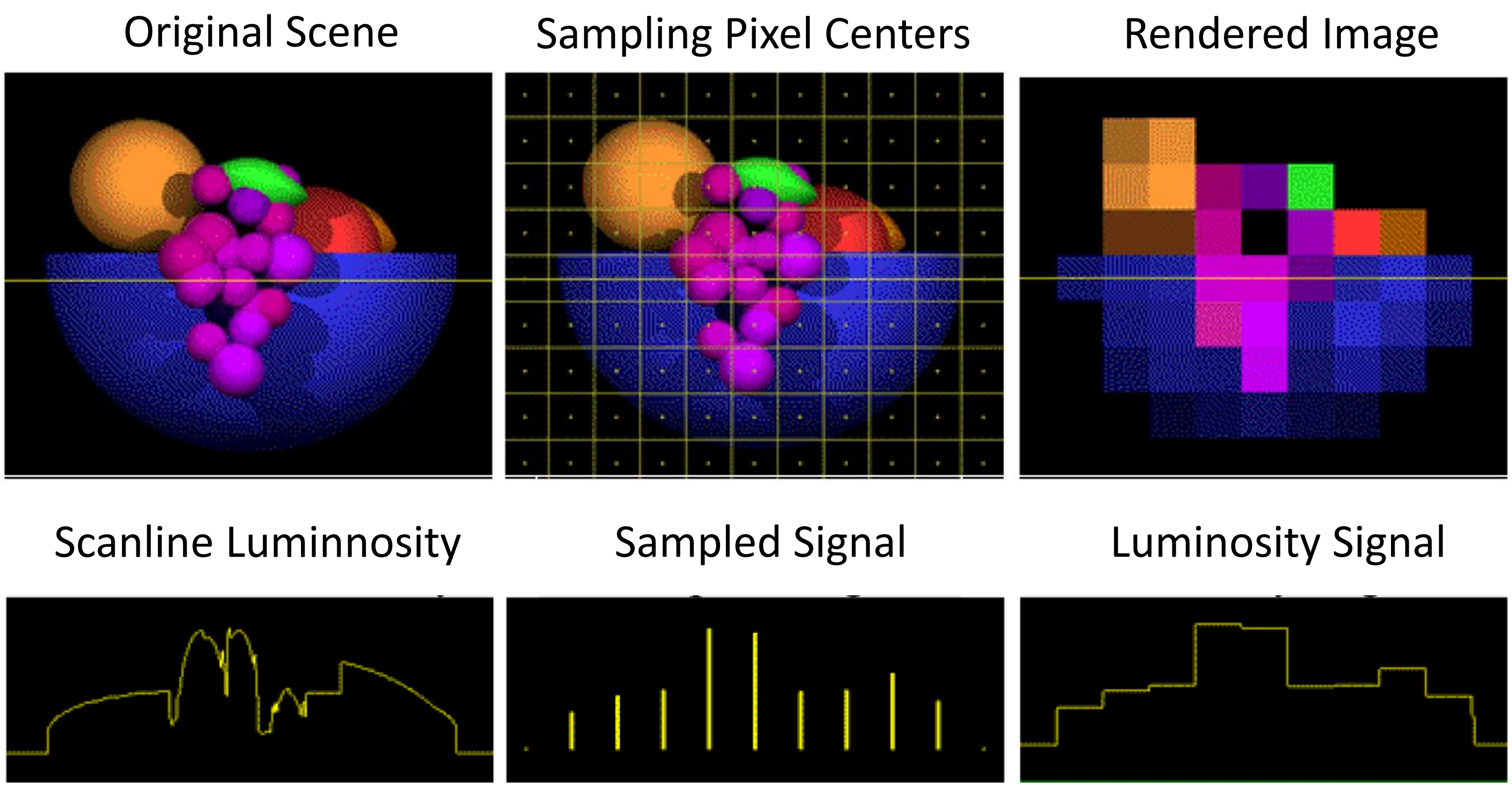
Sampling and Aliasing

- Display is discrete and world is continuous
  (at least at the level we perceive)
- Sampling: Convert continuous into discrete
- Reconstruction: Converting from discrete to continuous
- Aliasing: Artifacts arising from sampling and consequent loss of information
- Anti-aliasing: Attempts to overcome aliasing

Sampling

- Occurs when the sampling inherent in rendering does not contain enough information for an accurate image



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Effects Caused by Aliasing

Aliasing

- The errors caused by aliasing are called artefacts
-
- Common aliasing artefacts include
  - Jagged profiles
  - Disappearing or improperly rendered fine detail
  - Disintegrating textures

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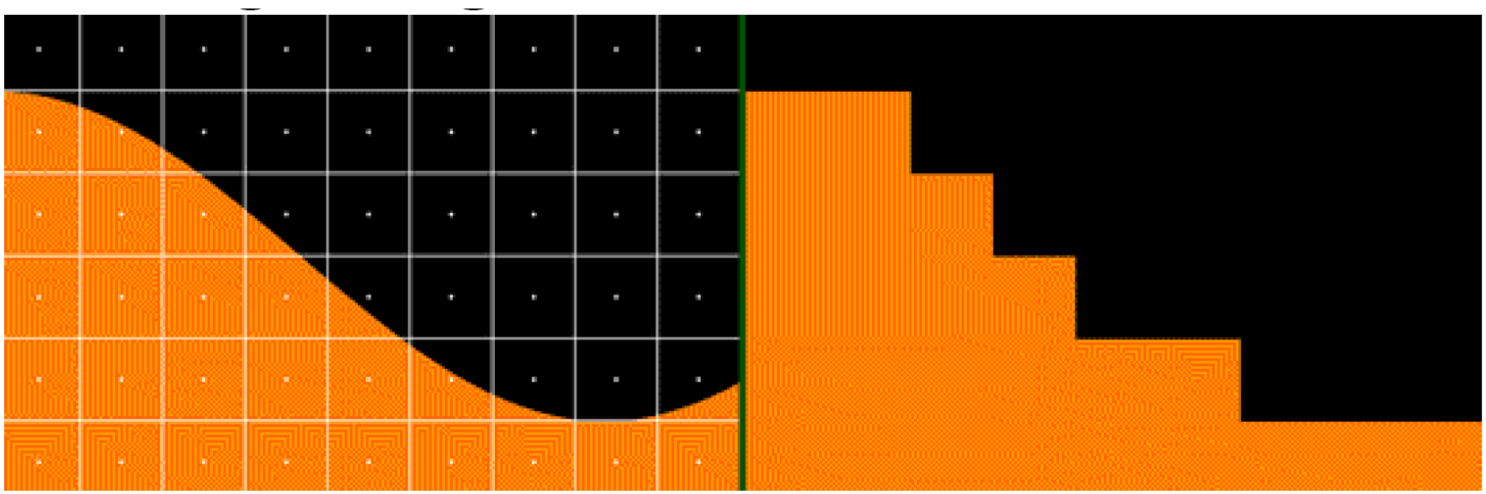
Jagged Profiles

- The picture on the left shows the sampling grid superimposed on the original scene. The picture on the right is the rendered image. A jagged profile is quite evident in the rendered image
- Especially noticeable when there is high contrast between the interior/exterior of the silhouette



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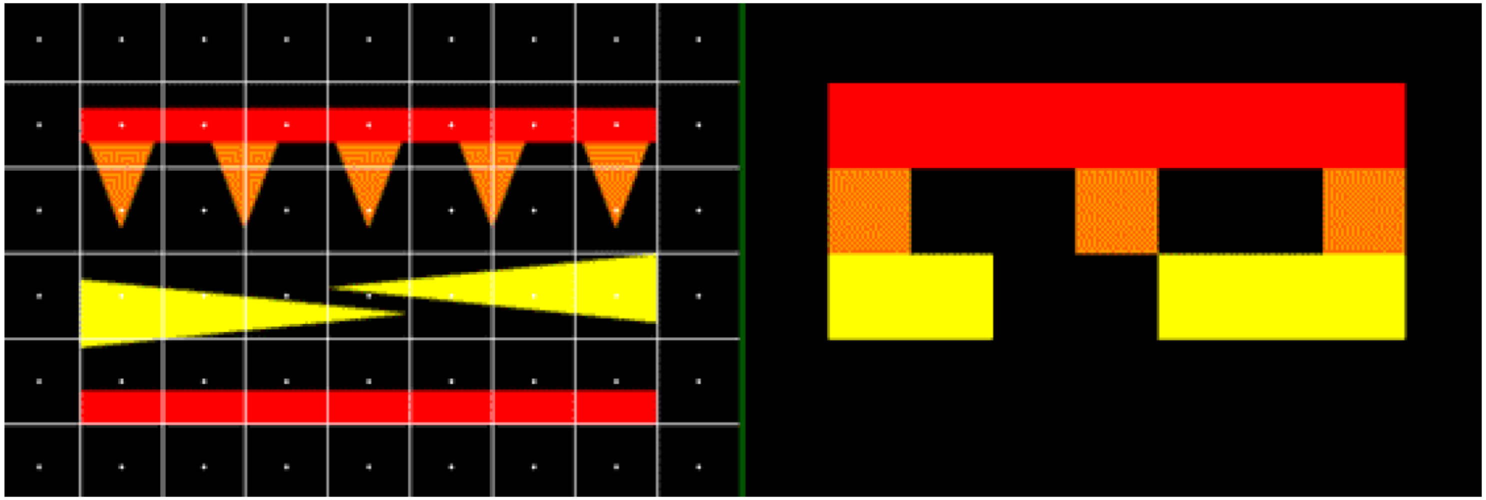
Improperly Rendered Detail

- When "rounding" off the value of each pixel with respect to the value of pixel-centers, one of the two red rectangles disappears entirely, and the other doubles in width. Two of the orange triangles disappear. Although the two yellow triangles are identical in size, one is larger than the other in the rendered image.



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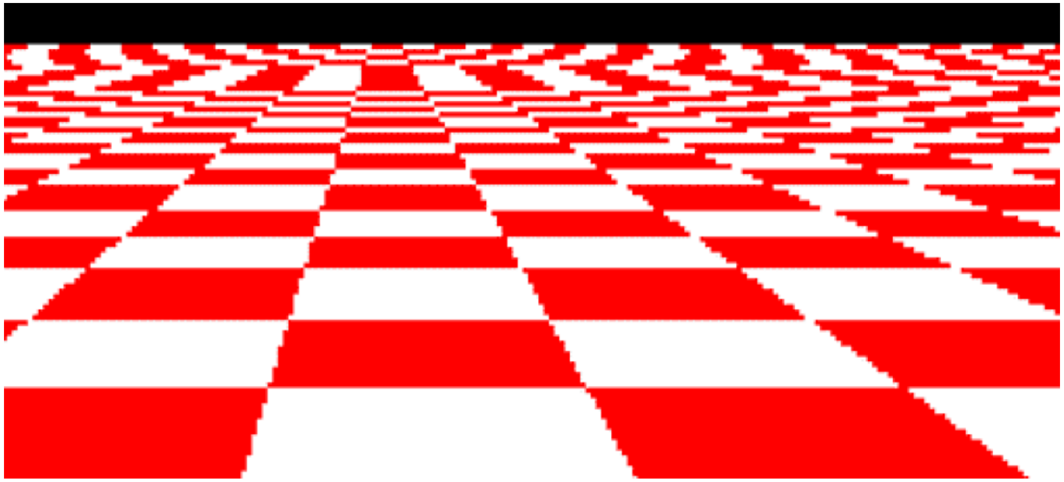
Disintegrating Texture

- The checkers on a plane should become smaller
  - Think about how you see a regular pattern in real life
- Aliasing effects
  - Bigger and/or irregular
- Increasing resolution does not address this issue
  - moves the artefact closer to the horizon



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Antialiasing

Antialiasing

- Antialiasing methods were developed to combat the effects of aliasing

- The two major categories of antialiasing techniques are
  - Prefiltering
  - Postfiltering

- Prefiltering and postfiltering are motivated from sampling theory

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Prefiltering

- Prefiltering methods treat a pixel as an area, and compute pixel color based on the overlap of the scene's objects with a pixel's area

- It is considered as a convolution operator to remove high-frequency signals
  - The Nyquist Theorem states that in order to adequately reproduce a signal it should be periodically sampled at a rate that is 2X the highest frequency you wish to record.

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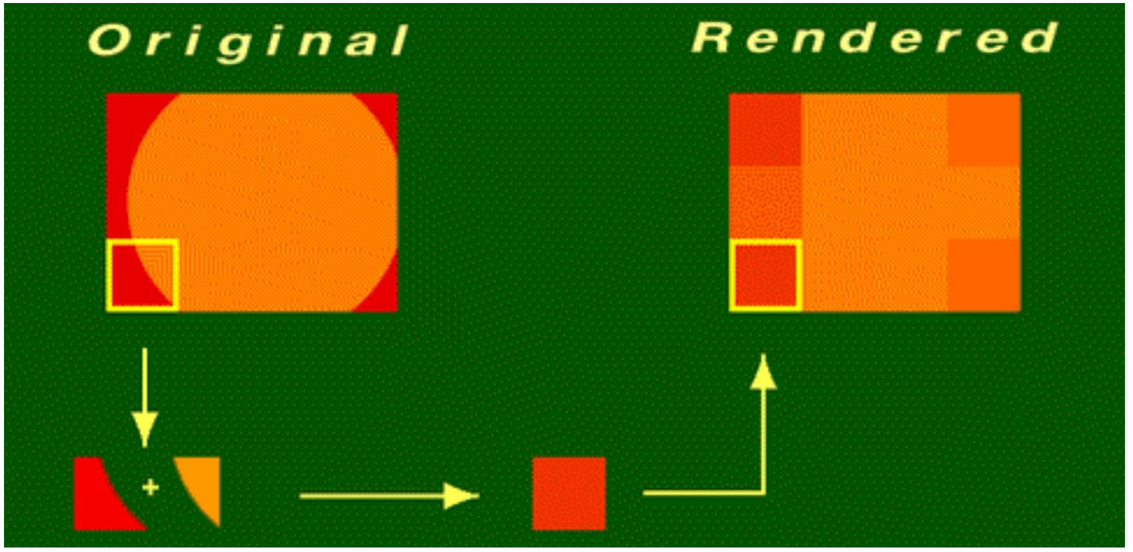
Basis for Prefiltering Algorithms



- All of the pixels inside the circle are 100 percent orange. All the pixels on the boundary of the circle have some area that is red and some area that is orange. Forty percent of the highlighted pixel is orange and 60 percent of its area is red.

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Prefiltering Demonstration

- Both phrases were rendered at a resolution of 512x512. Even at this resolution, the jaggies are apparent in the phrase that hasn't been antialiased

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Closeup

- Without antialiasing, the jaggies are harshly evident

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Closeup of Prefiltered image

- Along the character's border, the colors are a mixture of the foreground and background colors

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Postfiltering

- For each displayed pixel, a postfiltering method takes several samples from the scene and computes an average of the samples to determine the pixel's color
- The two steps in the postfiltering process are:
  - Sample the scene at n times the display resolution. For example, suppose the display resolution is 512x512. Sampling at three times the width and three times the height of the display resolution would yield 1536x1536 samples
  - The color of each pixel in the rendered image will be an average of several samples. For example, if sampling were performed at three times the width and three times the height of the display resolution, then a pixel's color would be an average of nine samples. A filter provides the weights used to compute the average

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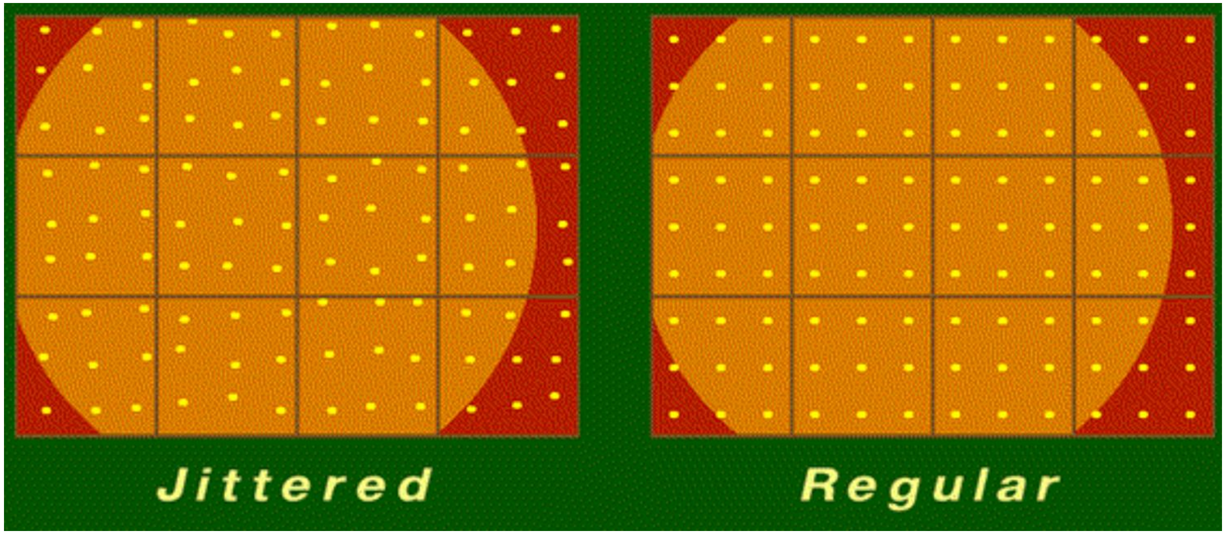
Sampling in the Postfiltering Method



- In the right figure, the samples are regularly spaced. In the left figure, the positions of samples are displaced by a random amount (small relative to the size of the pixel). This method of perturbing the sample positions is known as "jittering." Jittering adds noise to the rendered image. The advantage of jittering is that the human eye tolerates noise more easily than it tolerates aliasing artefacts

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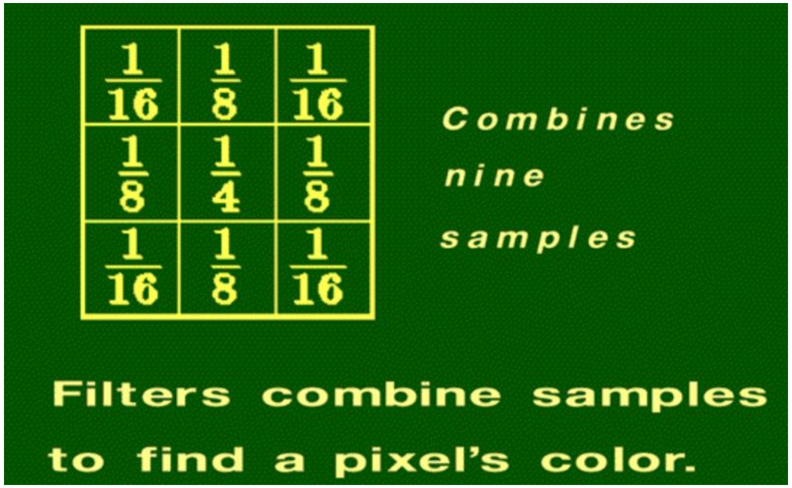
Filters



- Filters combine samples to compute a pixel's color. The weighted filter shown on the slide combines nine samples taken from inside a pixel's boundary. Each sample is multiplied by its corresponding weight and the products are summed to produce a weighted average, which is used as the pixel color. There are weighted and unweighted averages

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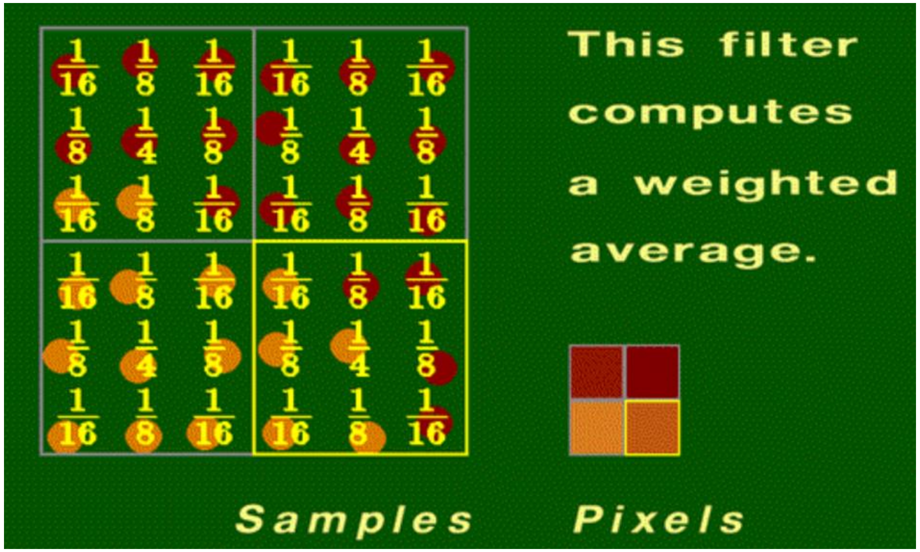
Using a filter to compute a pixel's color



- On the left is a group of samples that will be combined to produce pixel colors. Some of the samples are orange, and some are red. The superimposed numbers are the weights from the filter shown in the last slide. The highlighted group of samples on the left are combined using the filter to produce the color of the highlighted pixel on the right

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Example Images

No antialiasing

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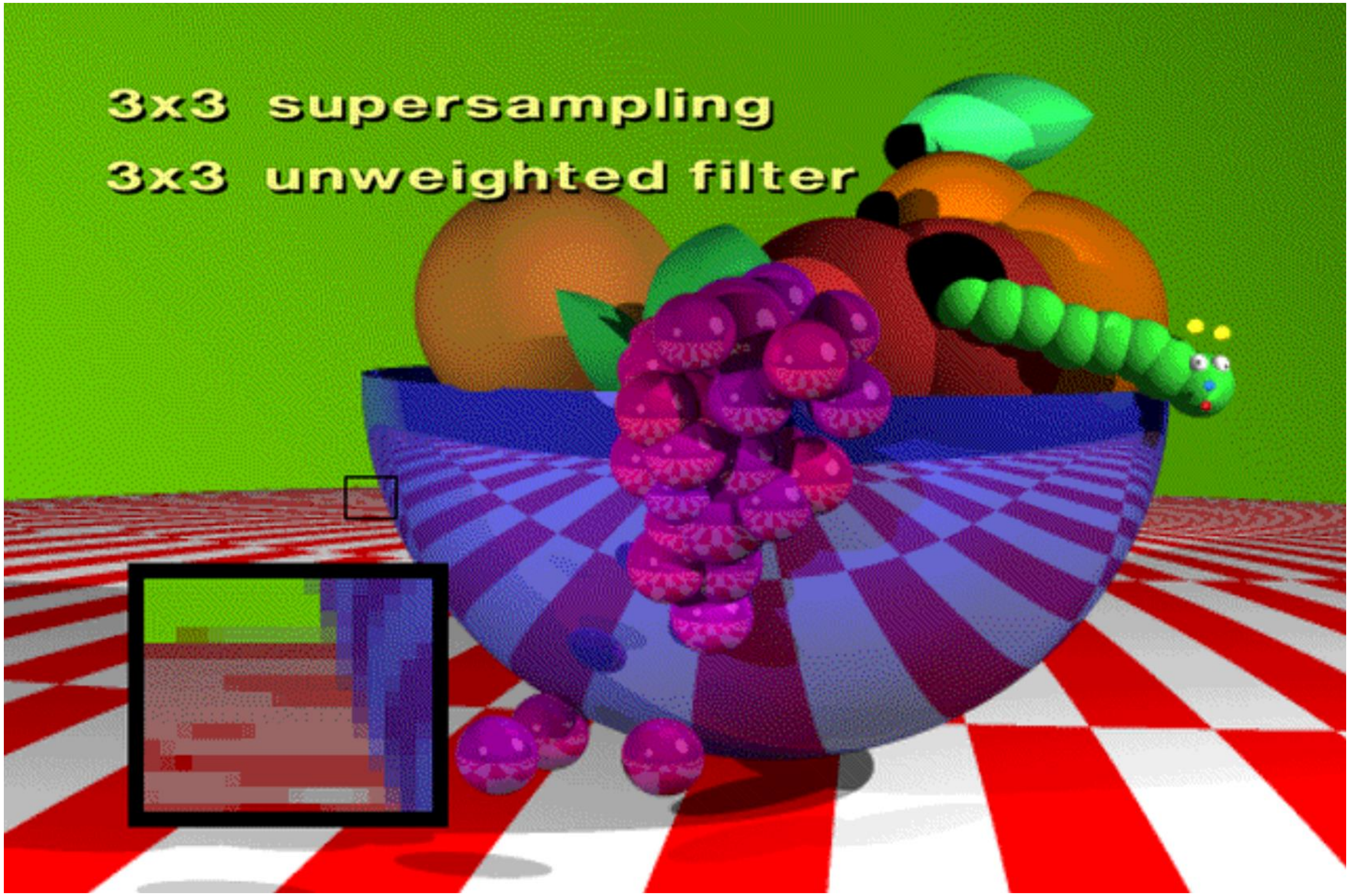
3x3 supersampling, 3x3 unweighted filter



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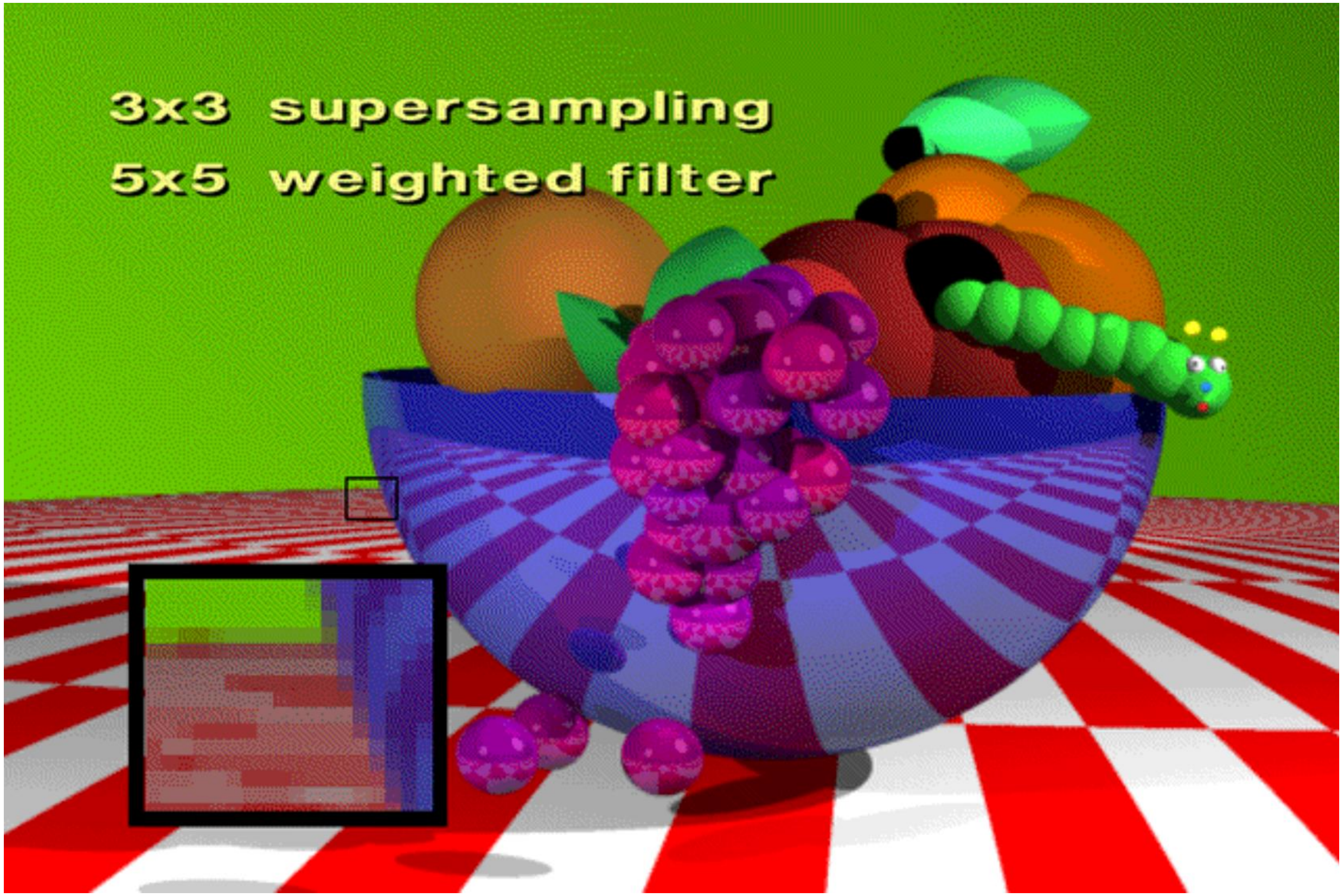
3x3 supersampling, 5x5 weighted filter



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3x3 supersampling, jittered samples, 3x3 weighted filter

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Aliasing - Point Sampling Theory

Keypoints of Point Sampling Theory

- Point sampling in the spatial domain corresponds to the point by point multiplication of a sampling impulse function times the analog signal to be sampled
- The Fourier Transform of an impulse function with spacing T in the spatial domain results in an impulse function with spacing 1/T in the frequency domain
- The multiplication of two functions in one domain corresponds to the convolution of the Fourier Transforms of the two functions in the other domain
- The convolution of an impulse function with another function is the second function instantiated at the impulse function positions

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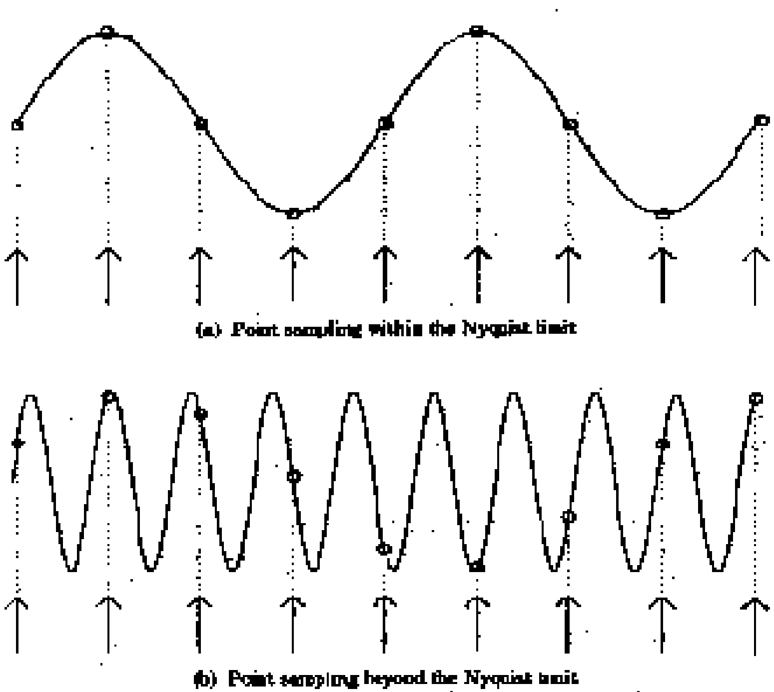
Example in the time domain



- The top signal is sampled at the Nyquist limit and so is not aliased but the bottom signal is sampled at a rate below the Nyquist limit and is aliased

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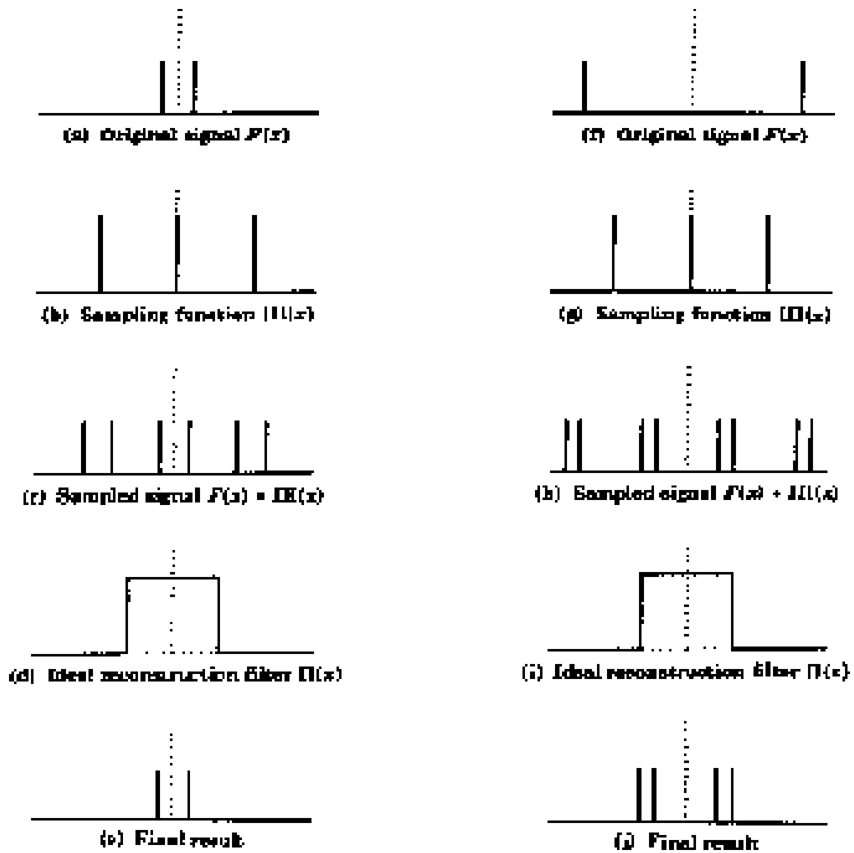
Example in the frequency domain



The left side corresponds to the top in the above image and the right side corresponds to the lower signal.

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Questions?