Data-Driven Shape Correspondence

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December 5th 2016
Maps between objects
Maps for propagation and interpolation

Propagation:

Interpolation:
Aggregating information

Segmentation and Skeleton Extraction
Matching in other domains

Protein docking

Brain matching

http://step.polymtl.ca/~rv101/images/research-brains.png
Li Keqiang will initiate the annual dialogue mechanism between premiers of China and Canada during this visit, and hold the first annual dialogue with Premier Trudeau of Canada.
Matching is ubiquitous

- Introduction to machine learning (Coursera)
  - 40K code submissions for linear SVM

Credit: Jonathan Huang
Matching is ubiquitous

- Grading using maps between AST trees
  - Graders mark a very small portion of them
  - Propagate labels to other programs
Matching is Hard!
Classification

1000 classes

[He et al. 16]

Segmentation

[Faster R-CNN: 4-step training]

Step 4: Fine tune FC layers of Fast R-CNN using same shared convolutional layers as in 3.

[Ren et al. 16]

Recognition

Correspondence

[Zheng et al. 16]

[Liu et al. 08]
Fundamental Challenge: Lack of Training Data
Hard to label dense correspondences
State-of-the-art pair-wise methods

SIFTFlow [Liu et al. 08]

DSP [Kim et al. 13]

4-points voting [Aigor et al. 08]

Blended intrinsic maps [Kim et al. 11]

HubAlign [Hashemifar et al. 14]
Outline

Map synchronization

Supervising neural networks
Ambiguities in assembling pieces
Resolving ambiguities by looking at additional pieces
Resolving ambiguities by looking at additional pieces
Matching through intermediate objects

--- map propagation

Blended intrinsic maps
[Kim et al. 11]
Pair-wise maps usually contain sufficient information.

Network of approximately correct blended intrinsic maps
Map synchronization problem

Identify correct maps among a (sparse) network of maps
A natural constraint on maps is that they should be consistent along cycles.

Q. Huang, G. Zhang, L. Gao, S. Hu, A. Bustcher, and L. Guibas. An Optimization Approach for Extracting and Encoding Consistent Maps in a Shape Collection, SIGGRAPHAsia’ 12
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Literature on utilizing the cycle-consistency constraint

- **Spanning tree optimization** [Huber et al. 01, Huang et al. 06, Cho et al. 08, Crandel et al. 11, Huang et al. 12]

- **Sampling inconsistent cycles** [Zach et al. 10, Nyugen et al. 11, Zhou et al. 15]
Compressive sensing view of map synchronization

Cycle-consistency

Input maps

Compressible

Noisy observations
Matrix representation of maps

\[ X_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \]

\[ T_{12} = \begin{bmatrix} R_{12} & t_{12} \\ 0 & 1 \end{bmatrix} \]
Map synchronization as constrained matrix recovery

\[ X = \begin{bmatrix}
    I_m & X_{12} & \cdots & X_{1n} \\
    X_{21} & I_m & \cdots & \vdots \\
    \vdots & \vdots & \ddots & X_{n-1,n} \\
    X_{n1} & \cdots & X_{n,n-1} & I_m \\
\end{bmatrix} \]

\[ X_{ij} = X_{j1}X_{i1}^T \]

\[ \begin{bmatrix}
    I_m \\
    \vdots \\
    X_{n1} \\
\end{bmatrix} \begin{bmatrix}
    I_m & \cdots & X_{n1}^T \\
\end{bmatrix} \]
Map synchronization as constrained matrix recovery

Parameter-free
Theoretical guarantees

Noisy measurements of matrix blocks

Q. H and L. Guibas, *Consistent Shape Maps via Semidefinite Programming*, Sym. on Geometry Processing’13
Q. H, F. Wang, L. Guibas, Functional Map Networks for Analyzing and Exploring Large Shape Collections, SIGGRAPH’ 14
S. Shen, Q.H., N. Srebro, S. Sunghavi, Normalized Spectral Map Synchronization, NIPS’ 16
Permutation synchronization

Objective function:

$$\text{minimize} \sum_{(i,j) \in G} \| X_{ij}^{\text{input}} - X_{ij} \|_1$$

Constraints:

$$X \succeq 0$$

$$X_{ii} = I_m, \quad 1 \leq i \leq n$$

$$X_{ij}1 = 1, \quad X_{ij}^T1 = 1, \quad 1 \leq i < j \leq n$$

$$0 \leq X \leq 1$$
Deterministic guarantee

- **Theorem:** Given noisy input maps, permutation synchronization recovers the underlying maps if

\[ \text{#incorrect corre. of each point} < \frac{\lambda_2(G)}{4} \]
Optimality when the object graph $G$ is a clique

- 25% incorrect correspondences
- Worst-case scenario
  - Two clusters of objects of equal size
  - Wrong correspondences between objects of different clusters only (50%)
Justification of maximizing $\lambda_2(G)$ for map graph construction

Imageweb [Heath et al 10]

Fuzzy correspondences on shapes [Kim et al 12]
Variants

Partial maps [CGH’14]
Spectral Sync. [SHSS’16]

Near-optimal!

Rotation Sync.
[Wang and Singer’14,...]

Near-optimal?
Experimental Results
Constrained matrix recovery achieves state-of-the-art performance
Constrained matrix recovery achieves state-of-the-art performance
Outline

Supervising neural networks
Matrix recovery perspective
Map synchronization versus learning pair-wise matching

<table>
<thead>
<tr>
<th></th>
<th>Pair-wise (RANSAC)</th>
<th>Joint Matching (from RANSAC)</th>
<th>Pairwise (Learning) Leordeanu et al. 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>64.1%</td>
<td>97.4%</td>
<td>94.6%</td>
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<tr>
<td>Joint Matching</td>
<td></td>
<td></td>
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<tr>
<td>(from Learning)</td>
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</tr>
<tr>
<td></td>
<td>100%</td>
<td>95.1%</td>
<td>100%</td>
</tr>
<tr>
<td>Joint Matching</td>
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<tr>
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</table>
Unsupervised Learning of Relative Camera Poses Using Neural Networks
Alternating minimization converges

<table>
<thead>
<tr>
<th>Network</th>
<th>Iter. 0</th>
<th>Iter. 1</th>
<th>Iter. 2</th>
<th>Iter. 3</th>
<th>Iter. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SydneyHouse [Chu et al.16]</td>
<td>125K Training, 30K Testing</td>
<td>6.13</td>
<td>5.02</td>
<td>4.95</td>
<td>4.89</td>
</tr>
</tbody>
</table>

SydneyHouse

PoseNet
Predicted maps become more consistent
Cycle-consistency perspective

[Zhou-Krähenbühl-Abruy-Huang-Efros, CVPR’ 16]
Connecting real images through synthetic images

\[ \tilde{f}_{s_1, s_2} = f_{s_1, r_1} \circ f_{r_1, r_2} \circ f_{r_2, s_2} \]
Flow architecture

FlowNet [Fischer et al 15]
Multi-lingual Translation [Cho et al.]
Looking Ahead
Maps as functions between sets

- **Surjection**: $f : X \to Y$ with $X = D_f$
- **Injection**: $g : X \to Y$ with $X = D_g$
- **Bijection**: $h : X \to Y$ with $X = D_h$
How to define sets on objects?
Sets (or representations) and maps should be optimized together.

- **Pixel-wise correspondences**
- **Segment correspondences**
Questions?