Networks of Elastic Circuits

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with

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Latency Insensitive Design

- Challenge in nanoscale technology: Implement a given functionality in a way that tolerates the latency changes of components and wires connecting them.

- Pioneering work: Carloni, McMillan, Sangiovanni-Vincentelli (CAV 1999)

- Intel project **SELF (Synchronous Elastic Flow)**: Kishinevsky, Cortadella, Grundmann (TAU2005, DAC 2006)

- This presentation: Theoretical foundation for SELF
Elastic Circuits

• Ordinary (non-elastic) adder

\[
\begin{array}{c}
\cdots & 3 & 5 & 2 & 1 \\
\cdots & 4 & 1 & 0 & 2 \\
\end{array}
\rightarrow
\begin{array}{c}
+ \\
\end{array}
\rightarrow
\begin{array}{c}
\cdots & 7 & 6 & 2 & 3 \\
\end{array}
\]

• Elastic adder

\[
\begin{array}{c}
\cdots & 3 & 5 & 2 & 1 \\
\cdots & 4 & 1 & 0 & 2 \\
\end{array}
\rightarrow
\begin{array}{c}
+^e \\
\end{array}
\rightarrow
\begin{array}{c}
\cdots & 7 & 6 & 2 & 3 \\
\end{array}
\]

• Elasticization of a wire: Var. Latency Empty Elastic Buffer

\[
\begin{array}{c}
\cdots & 7 & 6 & 2 & 3 \\
\end{array}
\rightarrow
\begin{array}{c}
\text{Id}^e \\
\end{array}
\rightarrow
\begin{array}{c}
\cdots & 7 & 6 & 2 & 3 \\
\end{array}
\]
SELF Approach to Elasticization

- Wires of $A$ become channels—triples of wires—in $A^e$.
  - $X$ vs. $\langle X, \text{valid}_X, \text{stop}_X \rangle$

  ![Diagram showing states of SELF channels: transfer, idle, retry](image)

- States of SELF channels:
  - Transfer
  - Idle
  - Retry
Questions

✶ Given a circuit $A$, how to construct its elasticization(s) $A^e$?

- SELF does it

✶ If $N$ is an ordinary network and we elasticize its components and connect channels accordingly, will we get an elasticization of $N$?

✶ If we insert an empty elastic buffer into a channel of an elastic network, will the resulting network be “equivalent” to the given one?
More Basic Questions

✶ What is precisely the “equivalence” of an ordinary and an elastic circuit?

✶ What is an elastic circuit?

✶ What is a circuit?
Ordinary Circuits and Networks
Systems

• Set of wires $W$
  - Example: for the system Adder, $W = \{\text{in1, in2, out}\}$

• Set of $W$-behaviors $[W]$: $W$-indexed records of streams
  - Example: $\sigma = \langle \sigma.\text{in1}, \sigma.\text{in2}, \sigma.\text{out} \rangle$
    $\sigma.\text{in1} = \langle 2, 2, 2, \ldots \rangle$  $\sigma.\text{in2} = \langle 1, 2, 3, \ldots \rangle$  $\sigma.\text{out} = \langle 3, 4, 5, \ldots \rangle$

• A $W$-system is a set of $W$-behaviors
  - Example: $\text{Adder} = \{\sigma \mid \sigma.\text{out} = \sigma.\text{in1} \oplus \sigma.\text{in2}\}$
    $\langle 3, 4, 5, \ldots \rangle = \langle 2, 2, 2, \ldots \rangle \oplus \langle 1, 2, 3, \ldots \rangle$
  - Example: $\text{Conn} = \{\sigma \mid \sigma.\text{out} = \sigma.\text{in}\}$
System Operations: Hiding, Composition, Networks

- \( \text{hide}_V(S) = \{ \sigma_{W-V} \mid \sigma \in S \} \subseteq [W - V] \)
- \( S_1 \uplus S_2 = \{ \sigma \mid \sigma_{W_1} \in S_1 \land \sigma_{W_2} \in S_2 \} \subseteq [W_1 \cup W_2] \)
- Networks of systems:

\[
\langle S_1, \ldots, S_m \mid u_1 = v_2, \ldots, u_n = v_n \rangle = \text{hide}_{\{u_1, \ldots, u_n, v_1, \ldots, v_n\}}(S_1 \uplus \cdots \uplus S_m \uplus \text{Conn}(u_1, v_1) \uplus \cdots \uplus \text{Conn}(u_n, v_n))
\]

\[
= \langle A, B, C, D \mid 2 = 5, 4 = 7, 10 = 11, 8 = 9, 12 = 1 \rangle
\]
Measuring Distance Between Streams (Behaviors)

- **Definition** \( a \sim_n b \) iff \( \text{prefix}(n, a) = \text{prefix}(n, b) \)

- **Definition** \( \sigma \sim_n \tau \) iff \( (\forall w \in W) \sigma.w \sim_n \tau.w \)

- **Example:**

\[
\begin{align*}
\sigma.\text{in1} &= \langle 2, 2, 2, \ldots \rangle & \sigma.\text{in2} &= \langle 1, 2, 3, \ldots \rangle & \sigma.\text{out} &= \langle 3, 4, 5, \ldots \rangle \\
\tau.\text{in1} &= \langle 2, 2, 2, \ldots \rangle & \tau.\text{in2} &= \langle 1, 2, 5, \ldots \rangle & \tau.\text{out} &= \langle 3, 4, 7, \ldots \rangle
\end{align*}
\]

\[
\therefore \sigma \sim_2 \tau & \quad \therefore \sigma \not\sim_3 \tau
\]
Machines (Circuits Abstractly)

**Definition** An \((I, O)\)-machine is an \((I \cup O)\)-system given by a function \(F: [I] \rightarrow [O]\) satisfying the causality property

\[
(\forall \sigma, \sigma' \in [I])(\forall k \geq 0) \quad \sigma \sim_k \sigma' \implies F(\sigma) \sim_k F(\sigma')
\]

Outputs at the first \(k\) cycles are determined by inputs at the first \(k\) cycles.
Feedback: When is it a machine?

**Definition** An input-output pair \((u, v)\) is **sequential** if

\[
(\forall \sigma, \sigma' \in [I] \quad \forall k \geq 0) \quad \sigma.u \sim_{k-1} \sigma'.u \quad \land \quad (\forall x \neq u) \quad \sigma.x \sim_k \sigma'.x 
\] 

\[\Rightarrow F(\sigma).v \sim_k F(\sigma').v\]
Combinational Loop Theorem

**Definition** \( \Gamma(\mathcal{N}) \): Vertices are wires of \( \mathcal{N} \); directed edges drawn for non-sequential wire pairs.

**Theorem** If \( \Gamma(\mathcal{N}) \) is acyclic, then \( \mathcal{N} \) is a machine.
Elastic Circuits and Networks
[\(I, O\)]-Elastic Machine

★ Input-output structure

- inputs: \(I \cup \{\text{valid}_X \mid X \in I\} \cup \{\text{stop}_Y \mid Y \in O\}\)
- outputs: \(O \cup \{\text{valid}_Y \mid Y \in O\} \cup \{\text{stop}_X \mid X \in I\}\)

★ Persistence

- \(\mathcal{S} \models G (\text{valid}_Y \land \text{stop}_Y \Rightarrow (\text{valid}_Y)^+)\) for every \(Y \in O\)
[\textit{I, O}]-Elastic Machine (ctd)

\begin{itemize}
  \item Transfer and token count
  \begin{center}
  \begin{tabular}{l|cccccccccccccc}
  cycle & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \ldots \\
  \hline
  \text{valid} & * & A & B & B & B & C & * & * & D & D & \ldots \\
  \text{stop} & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & \ldots \\
  \text{tct} & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & \ldots \\
  \end{tabular}
  \end{center}
\end{itemize}

\begin{itemize}
  \item Transfer behavior $\omega^T$ (data from transfer cycles)
  \item $\omega^T.X = (A, B, C, D, \ldots)$
  \item Components $\omega^T.X$ of $\omega^T$ are perhaps finite sequences
\end{itemize}
[I, O]-Elastic Machine (ctd)

*Liveness*

\[ (\forall Y \in O) \quad S \models G (\text{min}_tct_O \geq tct_Y \land \text{min}_tct_I > tct_Y \Rightarrow F \text{valid}_Y) \]

\[ (\forall X \in I) \quad S \models G (\text{min}_tct_{I \cup O} \geq tct_X \Rightarrow F \neg \text{stop}_X) \]

- Serve only the hungriest channels:

- Liveness guarantees that all transfer behaviors \( \omega^T.Z \) are infinite (in an “elastic environment”)

\[ \therefore \quad \text{The transfer system } S^T = \{ \omega^T | \omega \in S \sqcup \text{Env}_{I,O} \} \]
[\(I, O\)]-Elastic Machine (ctd)

- **Determinism**

\[
(\forall \omega_1, \omega_2 \in S) \quad \omega_1^T.I = \omega_2^T.I \implies \omega_1^T.O = \omega_2^T.O
\]

**Definition** \(S\) is an \([I, O]\)-elastic machine if it has the input-output structure as described, and satisfies the persistence, liveness, and determinism conditions.

**Theorem** If \(S\) is an \([I, O]\)-elastic machine, then \(S^T\) is an \((I, O)\)-machine.

- \(S\) is an elasticization of \(M\) when \(M = S^T\)
Elastic Networks

Suppose $S_1, \ldots, S_m$ are elastic machines.

$$\mathcal{N} = \langle \langle S_1, \ldots, S_m \rangle \mid X_1 = Y_1, \ldots, X_n = Y_n \rangle$$

$$\triangleq$$

$$\langle S_1, \ldots, S_m \mid X_i = Y_i, \text{valid}_X = \text{valid}_Y, \text{stop}_X = \text{stop}_Y \ (1 \leq i \leq n) \rangle$$

- Is $\mathcal{N}$ an elastic machine?
- Do we have $\mathcal{N}^T = \langle S_1^T, \ldots, S_m^T \mid X_1 = Y_1, \ldots, X_n = Y_n \rangle$?
Elastic Feedback

\[ \mathcal{F} = \langle S | P = Q \rangle = \langle S | P = Q, \text{valid}_P = \text{valid}_Q, \text{stop}_P = \text{stop}_Q \rangle \]

**Definition** An i/o channel pair \((P, Q)\) **sequential** for \(S\) if

\[ S \models G (\text{min}_t \text{ct}_{I \cup O} \geq \text{tct}_Q \land \text{min}_t \text{ct}_{I-\{P\}} > \text{tct}_Q \Rightarrow F \text{valid}_Q) \]

and the graph \(\Gamma(\mathcal{F})\) is acyclic.
Elastic Network Theorem

- $\mathcal{N} = \langle S_1, \ldots, S_m \mid X_1 = Y_1, \ldots, X_n = Y_n \rangle$
- $\delta_i$ : a sequentiality interface for $S_i$

\[ \delta_i(Z) = \text{set of input wires "jointly sequential" wrt } Z \]

**Definition** $\Delta(\mathcal{N})$ : Vertices are channels of $\mathcal{N}$ (i.e., $X_j$ and $Y_j$ are identified); a directed edge drawn for each pair $(P, Q) \in I_i \times O_i$ such that $P \notin \delta_i(Q)$.

- $\mathcal{N}' = \langle S_1^T, \ldots, S_m^T \mid X_1 = Y_1, \ldots, X_n = Y_n \rangle$

**Theorem** If $\Delta(\mathcal{N})$ is acyclic, then $\mathcal{N}$ is an elastic machine, $\mathcal{N}'$ is a machine, and $\mathcal{N}^T = \mathcal{N}'$. 

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Inserting Empty Buffers

**Theorem** Suppose $\mathcal{N}_1$ and $\mathcal{N}_2$ are elastic networks obtainable from each other by insertion and deletion of empty elastic buffers. If $\Delta(\mathcal{N}_1)$ is acyclic, then

- $\Delta(\mathcal{N}_2)$ is acyclic
- $\mathcal{N}_1^\top = \mathcal{N}_2^\top$
What’s Coming Next?

- Prove that SELF creates elastic circuits
- Weaken the definition of elasticity to include all existing “elastic” designs
- Extend theory to more complex SELF protocols
Background: Patient Systems
(Carloni, McMillan, Sangiovanni-Vincentelli)

- Behavior: for each wire, a stream in which each element is either
  a value or □ (“bubble”)

- Example:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>A</th>
<th>B</th>
<th>B</th>
<th>C</th>
<th>*</th>
<th>*</th>
<th>D</th>
<th>D</th>
<th>...</th>
</tr>
</thead>
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<tr>
<td>elastic</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>valid_X</td>
<td></td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>stop_X</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>patient</td>
<td>□</td>
<td>A</td>
<td>□</td>
<td>□</td>
<td>B</td>
<td>□</td>
<td>C</td>
<td>□</td>
<td>□</td>
<td>D</td>
</tr>
</tbody>
</table>

- Precise definition when a collection of such behaviors is a patient process

- Compositionality Theorem for patient processes; construction of a patient process latency equivalent to a given circuit

- “Elastic” and “patient” are difficult to compare