Structure-aware computation of predicate abstraction

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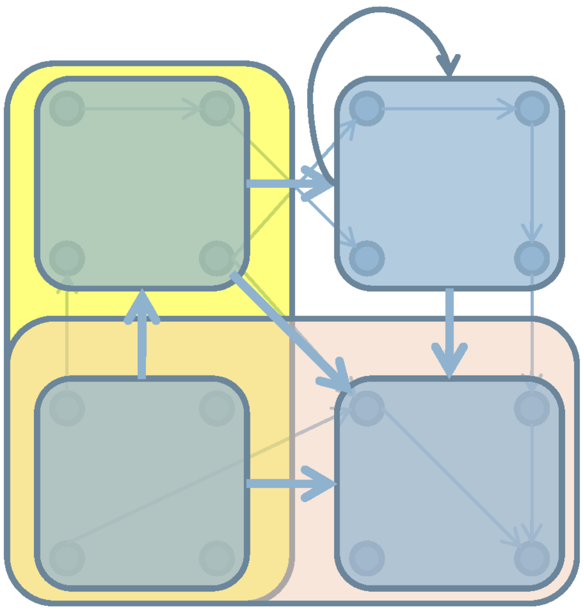
Predicate abstraction

- Concrete program C over states S
- Predicates Ψ_i induce partition over S
- Each partition is a state of the abstract program
- Transitions in abstract space
  - from as to as' iff c-transition from cs to cs', with cs in as, and cs' in as'



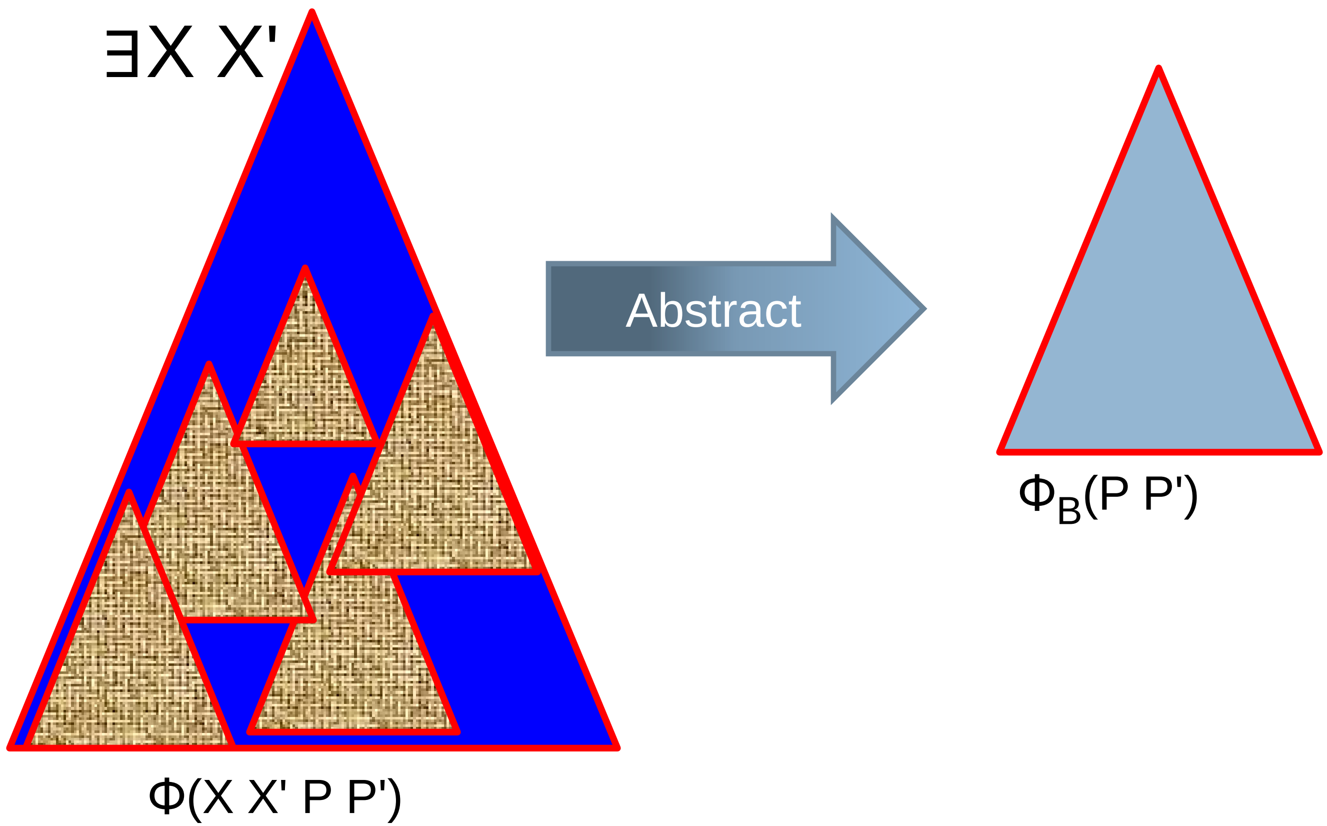
Predicate abstraction: symbolic view

- Concrete state as assignment to X variables
  - booleans, bit vectors, reals, integers, …
- Concrete program as SMT formula CR(X, X')
- Abstract state as assignment to boolean variables P_i
- Predicates as SMT formulae $\Psi_i(X)$
- Abstraction function Abstr(X X' P P') as $\bigwedge_i P_i \leftrightarrow \Psi_i(X)$

- Computing predicate abstraction:
  - Obtain a boolean representation for AR(P,P')
  - Amenable to symbolic model checking

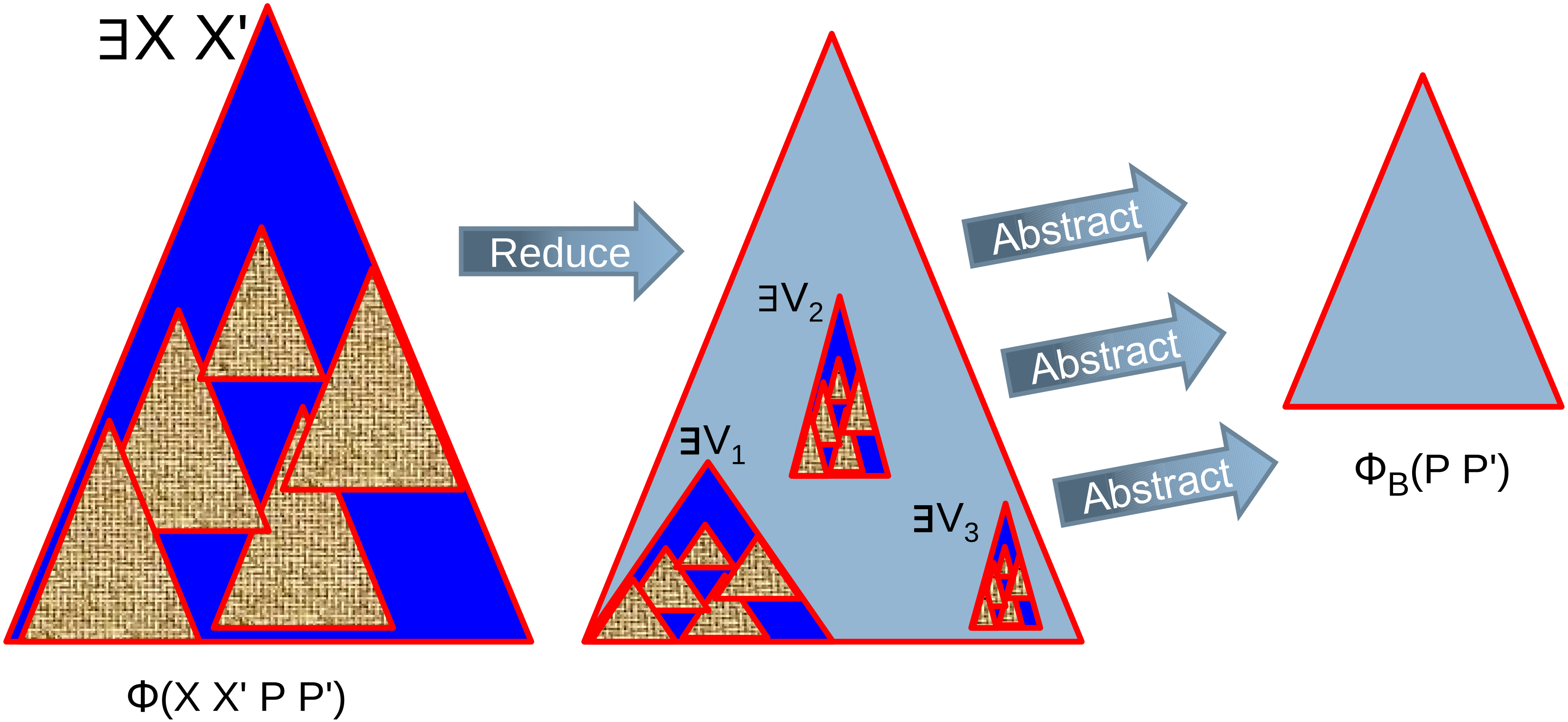
- $AR(P,P') = \exists\, X\, X'.(CR(X, X') \wedge \bigwedge_i P_i \leftrightarrow \Psi_i(X) \wedge \bigwedge_i P_i' \leftrightarrow \Psi_i(X')\,)$

From Q-SMT to Boolean



- Predicate Abstraction
  - at the core of many verification approaches
  - often a bottleneck

Avoid Monolithic Computation



Structure-aware predicate abstraction

- New procedure for predicate abstraction
- Exploits the available problem structure
- At the high level
  - structure of system being abstracted
  - modules, scope of variables, nature of transitions
- At the low level
  - structure of quantified formula
  - reduce scope of quantification

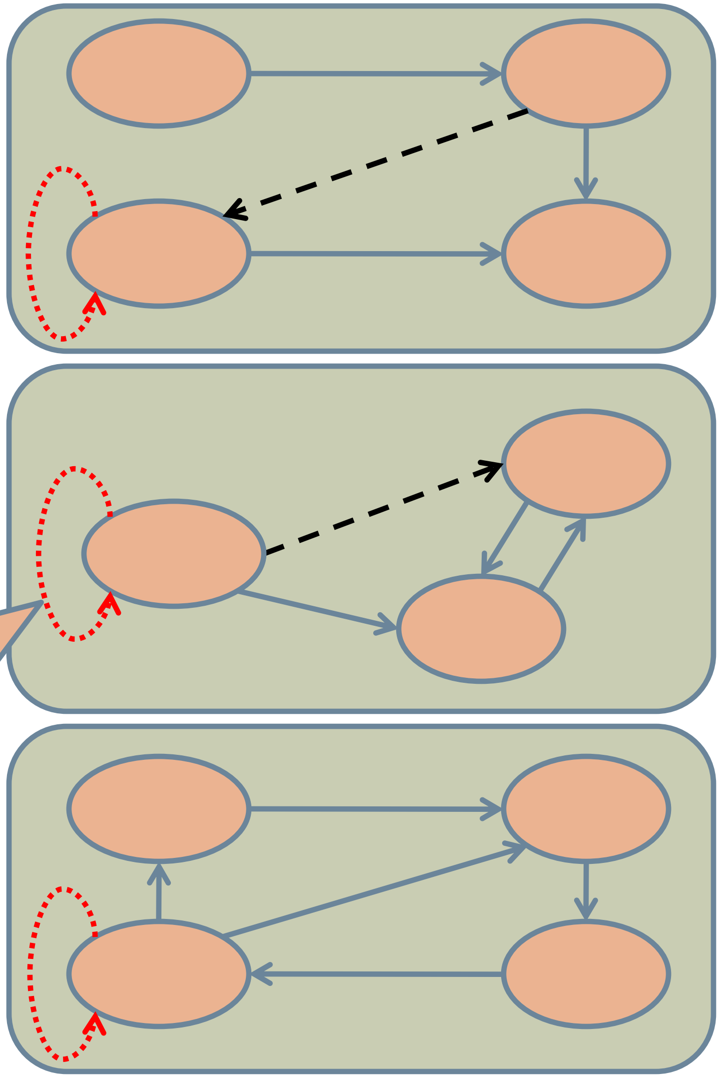
High level framework

- System structured in several components
- Asynchronously composed via interleaving
- Transitions:
  - local transitions
  - synchronizing transitions
  - timed transitions

Invariants: x in [10, 20]
SMT: $10 \le x$ & $x \le 20$

Flow condition: der(x) in [1.1, 1.3]
SMT: $x + 1.1 \cdot \delta \le x'$ & $x' \le x + 1.3 \cdot \delta$

Global: the same δ for all components!



Predicate abstraction procedure

- Ingredients
  - disjunctively partitioning the concrete program
  - inlining
  - clustering
  - blocking and restricting models
  - value sampling

Disjunctive Partitioning

- $AR(P\ P') = \boxed{\exists\ X\ X'.(CR(X\ X') \wedge Abstr(X\ X'\ P\ P'))}$
- Present CR as disjunction of
  - $\vee_m \vee_l E_t(X\ X')$ local transitions
  - $\vee_{m,m'} E_\sigma(X\ X')$ synchronizing transitions
  - $\vee_l E_\delta(X\ X')$ timed transitions
- Distribute Abstr(X X' P P') over disjuncts
- Push Quantification inside disjunction
- $AR(P\ P') =$

$$(\vee_m \vee_l \boxed{\exists\ X\ X'.(E_t(X\ X') \wedge Abstr(X\ X'\ P\ P'))}) \vee$$
$$(\vee_{m,m'} \boxed{\exists\ X\ X'.(E_\sigma(X\ X') \wedge Abstr(X\ X'\ P\ P'))}) \vee$$
$$\vee_l \boxed{\exists\ X\ X'.(E_\delta(X\ X') \wedge Abstr(X\ X'\ P\ P'))}$$

Abstracting one transition

- During transitions, several components may not change
- In local transitions
  - only active process is modified
  - loc' = loc, x' = x, …
- synchronizing transitions
  - similarly, only active processes change
- timed transitions
  - discrete locations do not change
- Lots of potential for inlining

Rules for inlining

- $\exists X.(\beta \wedge (u{=}\alpha))$ rewrites to $\exists X.(\beta[u / \alpha])$
  - where u in X, and not in α
- $\exists X.(\beta \wedge (q \leftrightarrow \alpha))$ rewrites to $(q \leftrightarrow \alpha) \wedge \exists X.(\beta[q / \alpha])$
  - where α propositional, and q not in α
- $\exists X.(\beta \wedge (\gamma \leftrightarrow \alpha))$ rewrites to $\exists X.(\beta[\gamma / \alpha]) \wedge (\gamma \leftrightarrow \alpha))$
  - where α propositional but γ has vars in X

Practical Limitations

- Variable in one component may be referred to in flow conditions of other components
  - this indirectly influences its behaviour.

- Predicates can introduce correlations that are not directly present in the original system
  - e.g. $(x + y < 10)$ connects x and y

Clustering

- $\exists X.(\Phi_1(X_1\ P) \wedge \Phi_2(X_2\ P) \wedge \ldots \wedge \Phi_n(X_n\ P)\ )$
- Each variable in X occurs in at most one of the clusters X_i
- Each cluster can be dealt with independently
- Trade one big quantification for many (hopefully smaller) quantifications

$$(\exists X_1.\Phi_1(X_1\ P)) \wedge (\exists X_2.\Phi_2(X_2\ P)) \wedge \ldots \wedge (\exists X_n.\Phi_n(X_n\ P))$$

Blocking and Restricting Models

- When computing $\Phi_B(P) \vee \exists X.\Phi(X\ P)$
- Replace $\exists X.\Phi(X\ P)$ with $\exists X.(\neg\Phi_B(P) \wedge \Phi(X\ P))$
- Rationale
  - boolean reasoning cheaper than SMT reasoning
  - models in Φ_B have already been visited
  - force exploration to other models within $\neg\Phi_B$

- When computing
  - $\Phi_{B0}(P) \wedge \exists X_1.\Phi_1(X_1\ P) \wedge \exists X_2.\Phi_2(X_2\ P) \wedge \ldots \wedge \exists X_n.\ \Phi_n(X_n\ P)$
- We can use previously computed conjuncts to prune quantification
  - $\exists X_1.(\ \Phi_1(X_1\ P) \wedge \neg\Phi_{B0}(P))$
  - $\exists X_2.(\ \Phi_2(X_2\ P) \wedge \neg\Phi_{B01}(P))$
  - $\exists X_3.(\ \Phi_3(X_3\ P) \wedge \neg\Phi_{B012}(P))$
- Restrict to models still worth exploration

Variable Sampling

- "Quasi clustering": a single w prevents clustering
  - $\exists\, X.(\Phi_1(w\ X_1\ P) \wedge \Phi_2(w\ X_2\ P) \wedge \ldots \wedge \Phi_n(w\ X_n\ P))$
- Pick one value c for w, replace, and cluster
  - $\exists\, X\backslash w.(\Phi_{1,w/c}(X_1\ P) \wedge \Phi_{2,w/c}(X_2\ P) \wedge \ldots \wedge \Phi_{n,w/c}(X_n\ P)$
- Result: underapproximation $\Phi_{w/c}(P)$
  - computed one cofactor with respect to w = c
  - we have to cover the case w≠c
  - $\exists\, X.(w \neq c \wedge \Phi_1(w\ X_1\ P) \wedge \Phi_2(w\ X_2\ P) \wedge \ldots \wedge \Phi_n(w\ X_n\ P))$
- The process can be iterated
  - need to block already covered models
  - need to find a suitable sequence of instantiations

Sampling-driven quantification

```
SamplingAllSMT(Phi, X, W) {
res := False;
(sat, mu) := SMTSolve(Phi);
while sat do
  c := PickValue(mu, W);
  new := AllSMT(not res and Phi[W / c]);
  res := res or new;
  (sat, mu) := SMTSolve(Phi and not res);
end while
return res;
}
```

Implementation

- Extended NuSMV
  - empowered with SMT functionalities
  - types: reals, integers, bit-vectors, …
- MathSAT SMT solver used as backend
- High level simplifications
  - network of automata
  - python script to generate disjunctive partitioned representation
- Low level simplifications as rewriter over quantified formulae
- Abstraction based on AllSMT version of MathSAT

Experimental Set up

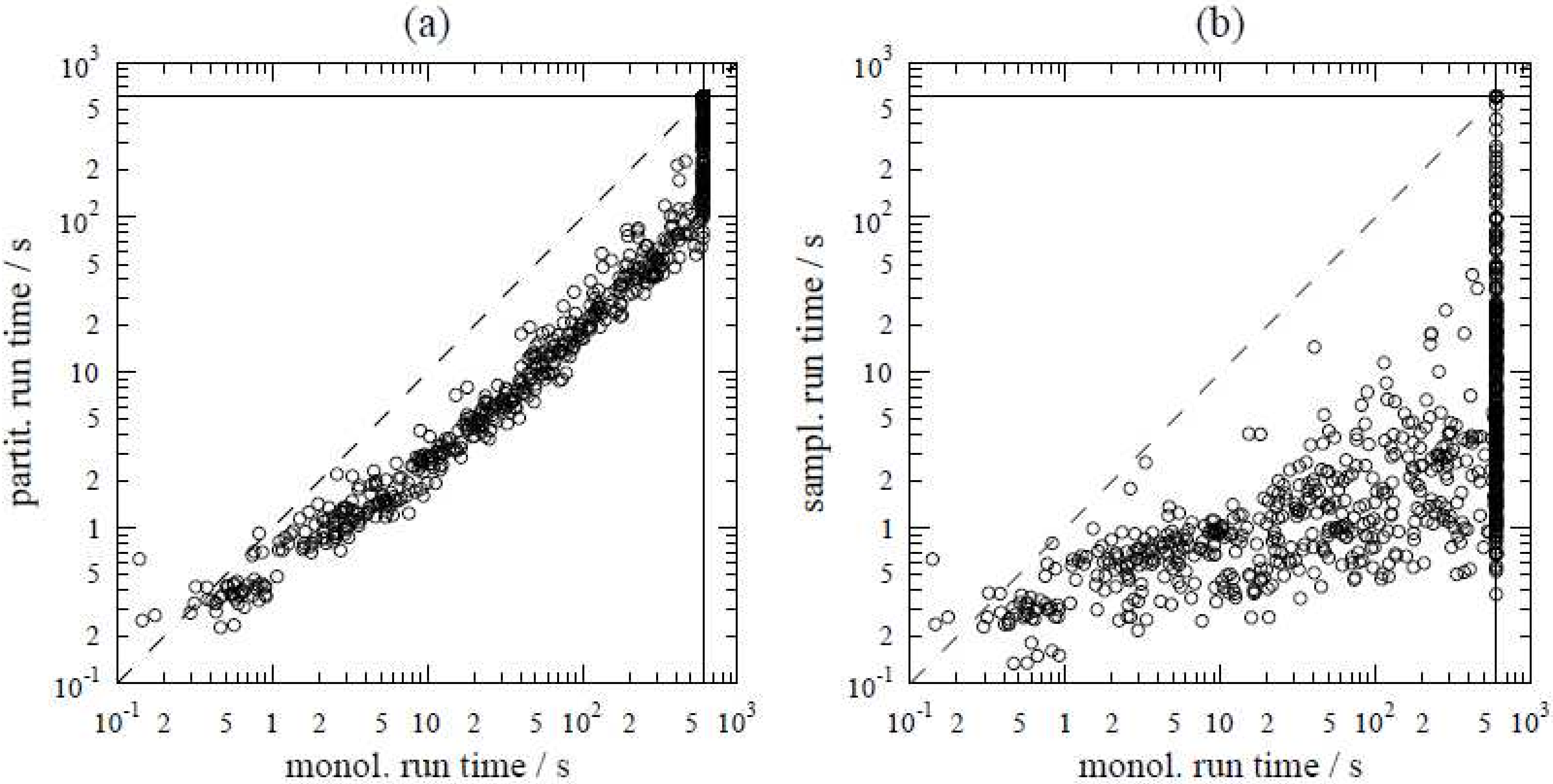
- Two classes of problems
  - from HyTech distribution
  - randomly generated networks of automata

- Compared Algorithms
  - mono
  - + partitioning
  - + clustering
  - + v-sampling

Results on Hytech models

Model	$\lvert\vec{P}\rvert$	$\lvert\vec{V}\rvert$	$\lvert$disj$\rvert$	computation time (s) monol.	partit.	clust.	sampl.	sampling clu	sam
active	34	5	27	54.626	18.847	2.410	0.937	5	1
active-trace	34	7	27	51.781	22.171	2.473	0.952	5	1
audio	30	6	15	13.826	4.547	0.448	0.442	2	2
audio-timing	29	7	15	10.910	3.915	0.947	0.690	2	6
billiard-timed	25	3	5	0.910	0.732	0.732	1.044	2	13
dist-controller	8	7	12	0.320	0.232	0.195	0.147	5	1
grc-ver	24	5	11	33.068	19.599	10.421	0.455	4	8
new-grc	22	5	11	38.649	17.840	7.395	0.383	4	7
railroad	16	3	8	0.170	0.140	0.131	0.112	2	5
reactor-clock	19	4	5	0.181	0.133	0.069	0.050	2	2
reactor-rect	17	4	5	0.132	0.112	0.051	0.045	2	2

Results on Random LHA's



Related Work

- Imprecise techniques
  - Cartesian Abstraction
- Boolean Quantification
  - BDD-based
  - SAT-based
- Monolithic SMT-based predicate abstraction
  - AllSMT [CAV06]
  - BDD + SMT [FMCAD07]

- Software model checking: BLAST, SATABS
  - Partitioning transition by transition in CFG
  - Forward image computations by inlining unmodified variables

- Avoid abstraction computation
  - Directly compute abstract violations [FM09]
  - No need for AllSMT functionality

Conclusions

- A structure-aware procedure for the exact computation of predicate abstraction
- Exploit high level structure
  - transition partitioning
  - variable scope
- Exploit low level structure
  - formula quantification, clustering
  - value sampling
- Significant speed-ups

Future Work

- Comprehensive comparison with other methods
  - Experiment with BDD-based abstraction
- Measure impact on CEGAR loop
- Application to post-image computation
  - Reachability in abstract space
- Full incrementality