Boolean Synthesis via Decomposition

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**Boolean Synthesis**

**Our goal:** To decompose the specification into formulas that are easier to synthesize.

**Specification of a system encoded as a Boolean formula**

\[
(x_0 \oplus y_0 = 0) \\
\land \\
(x_1 \oplus y_1 \oplus (x_0 \land y_0) = 0)
\]

**Boolean function implementing system behavior**

\[
y_0 := x_0 \\
y_1 := x_1 \oplus x_0
\]

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Decomposition using Factored Formulas

\[ F(\vec{x}, y_1, y_2, y_3, y_4) = F_1(\vec{x}, y_2, y_4) \land F_2(\vec{x}, y_1, y_2, y_3) \land F_3(\vec{x}, y_3) \]

- Easy to perform decomposition.
- Has been shown to significantly improve synthesis algorithms.

However: Dependences between factors prevent us from taking full advantage of the decomposition.
Sequential Decomposition

**Given:** A Boolean formula $F(\vec{x}, \vec{y})$ between input variables $\vec{x}$ and output variables $\vec{y}$.

**Return:** Two Boolean formulas $F_1(\vec{x}, \vec{z})$ and $F_2(\vec{z}, \vec{y})$ that can be composed back into $F$. 
Sequential Decomposition

\[ F \]

\[ F_1 \quad F_2 \]

\[ g_1 \quad g_2 \]
Sequential Decomposition

\[ \vec{x} \xrightarrow{g_1} \vec{z} \xrightarrow{g_2} \vec{y} \]