# CS243 Homework Assignment 3

Due Tuesday, September 25

Please hand in a hard copy of your solutions before class on the due date. The answers to the homework assignment should be typeset using LaTeX and must be your own individual work. You may discuss problems with other students in the class; however, your write-up must mention the names of these individuals.

1. (40 points, 8 points each) Prove or disprove each of the following statements below. If you believe the statement is true, prove the statement using one of the proof techniques we discussed in class and clearly state the proof strategy you use. If you believe the statement if false, disprove the claim by proving a counterexample.

   - The product of an odd and even integer is even.
   - If $r$ is a rational number and $p$ is an irrational number, then $r+p$ is irrational.
   - If $r$ is a rational number and $p$ is an irrational number, then $rp$ is irrational.
   - If $7x + 9$ is even, then $x$ is odd.
   - For all real numbers $a, b, c$, $\max(a, \max(b, c)) = \max(\max(a, b), c)$

2. (5 points) What is the Cartesian product of the sets $\{\emptyset\}$ and $\{a, b\}$

3. (5 points) Prove or disprove: There exists a set $S$ whose powerset is equal to $\{S\}$

4. (15 points) Prove that $A \cup (A \cap B) = A$. Your proof should **not** use any of the set identities from Table 1, p. 124 of the Rosen book.

5. (15 points) Prove or disprove: The function $f$ defined by $f(x) = 2x+5$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

6. (5 points) Prove or disprove: The function $f$ defined by $f(x) = 2x + 5$ is a bijection from $\mathbb{Z}$ to $\mathbb{Z}$.

7. (15 points) Let $x$ be a real number. Prove the following:

$$\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor$$