CS243 Homework Assignment 4

Due Tuesday, October 18
Max points: 75

Please hand in a hard copy of your solutions before class on the due date. The answers to the homework assignment should be typeset using LaTeX and must be your own individual work. You may discuss problems with other students in the class; however, your write-up must mention the names of these individuals.

1. **Summations:** (10 points) Derive a closed form expression for the following summation:

$$\sum_{i=2}^{n}(2 \cdot 3^i + 4i)$$

   Show how you derive this closed form expression.

2. **Countability:** (9 points, 3 points each) State whether each of the statements below are true or false. Briefly justify your answer.

   (a) Let A be a countable set and B be a countably infinite set. Then, $A \cup B$ is always countable.

   (b) The set $\mathbb{R} - \mathbb{N}$ is countably infinite.

   (c) Let A and B be two uncountably infinite sets. Then $A \cap B$ is uncountable.

3. **Divisibility:** (22 points, 11 points each)

   (a) Let a, b, c, m be integers such that $m \geq 2$ and $c > 0$. Prove that if $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{cm}$.

   (b) Let a, b, m be integers such that $m \geq 2$. Prove that if $a \equiv b \pmod{m}$, then $\gcd(a, m) = \gcd(b, m)$.

4. **Euclidian Algorithm:** (10 points) Use the extended Euclidian algorithm to find $\gcd(215, 35)$, and find integers s, t such that $\gcd(215, 35) = s \cdot 215 + t \cdot 35$. Show every step of the algorithm.

5. **Linear Congruences:** (12 points)

   (a) (4 points) Determine if the linear congruence $12x \equiv 15 \pmod 8$ has any solutions. Explain your reasoning.

   (b) (8 points) Find the set of all solutions to the linear congruence $3x \equiv 4 \pmod 7$. Show all your work.

6. **Chinese Remainder Theorem:** (12 points) Use the Chinese Remainder Theorem to find the set of all solutions to the following system of linear congruences:

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 1 \pmod 2$$

   You must show all your work to get full credit.