

CS243 Homework Assignment 6

Due Thursday, Nov 27 2012

Max points: 70

Please hand in a hard copy of your solutions before class on the due date. The answers to the homework assignment should be typeset using LaTeX and must be your own individual work. You may discuss problems with other students in the class; however, your write-up must mention the names of these individuals.

1. (4 points) In a given state, license plates consist of either 4 upper-case letters followed by 2 digits $[0 - 9]$ or 2 digits $[0 - 9]$ followed by 4 upper case letters. Furthermore, no license plate can start with a 0. How many possible license plates are there? Explain your reasoning.
2. (5 points) How many bitstrings of length 10 contain 4 consecutive ones? Justify your answer.
3. (5 points) What is the coefficient of x^3y^4 in the expansion of $(4x+5y)^7$?
4. (5 points) At least how many students must a class have to guarantee that there are at least 20 students of the same gender? Explain your reasoning.
5. (7 points) Suppose a coin is flipped 9 times. How many possible outcomes contain more heads than tails? Explain your reasoning.
6. (4 points) Suppose that you bought 3 apple, 4 orange, and 2 fig trees. In how many different ways can these trees be planted in a row?
7. (10 points) How many ways are there to place 9 women and 6 men to stand in a line so that no two men stand next to each other? (Hint: First position women and then consider possible positions for the men.)

8. (10 points) Suppose a small library has five book shelves labeled A–E. In how many different ways can 30 books be placed on these shelves if the books are ...
- (a) (5 points) all different?
 - (b) (5 points) all identical?
9. (10 points) A bakery sells 4 different kinds of croissants: plain, chocolate, vanilla, and almond. Assuming order of selection does not matter, how many ways are there to select ...
- (a) (4 points) 12 croissants?
 - (b) (6 points) 20 croissants with at least two of each type of croissant?
10. (5 points) 25 students in a class took a test containing 40 questions. Given that the maximum number of mistakes made by any student in the class is 10, prove that there are at least 3 students in the class who made the same number of mistakes.
11. (5 points) How many ways are there to distribute a deck of 52 cards to 4 players? Explain your reasoning.