CS243 Homework Assignment 7

Due Thursday, Dec 6 2012
Max points: 90

Please hand in a hard copy of your solutions before class on the due date. The answers to the homework assignment should be typeset using LaTeX and must be your own individual work. You may discuss problems with other students in the class; however, your write-up must mention the names of these individuals.

1. (8 points) Bob, Kate, and Jack enter a contest in which there are 100 participants in total. Given that a different participant is selected for first, second, and third prize, what is the probability that...
   (a) (4 points) Bob, Kate, and Jack each receive a prize?
   (b) (4 points) Bob, Kate, and Jack recieve first, second, and third prizes respectively?
2. (7 points) What is the probability of rolling at least one six when a die is rolled three times?
3. (10 points) Suppose that A and B are independent events. Prove that $\overline{A}$ and $\overline{B}$ are also independent events.
4. (8 points) What is the conditional probability that exactly 3 heads appear when a fair coin is flipped 5 times, given that the first flip came up tails?
5. (7 points) A neighbor agrees to water your plant while you are on vacation, and you are 90% that the neighbor will remember to water the plant. Without water, the plant has an 80% chance of dying, but with water, it only has a 15% chance of dying. What is the probability that the plant will be alive when you return from your vacation?
6. (10 points) A laptop was stolen from a university dorm called Stern Hall at night, and a witness identified the thief as being male. The police tested the reliability of the witness under the same circumstances that existed on the night of the theft and concluded that the witness

identified a person's gender correctly 80% of the time. Given that the thief is a resident of Stern Hall and given that 75% of students in Stern Hall are female, what is the probability that the thief is indeed male, as the witness claims?

7. (10 points) In a lottery, 3 balls are randomly selected without replacement from an urn containing 11 balls, labeled $0 - 10$. 3 of the balls in the urn are green, 3 are red, and 5 are black. Suppose that you win \$1 for each green ball selected, and you lose \$1 for each red ball selected. If X is a random variable representing the total winnings/losses from the lottery, compute the following probabilities:

   (a) (5 points) $p(X = 0)$

   (b) (5 points) $p(X = 2)$

8. (10 points) A group of 120 high school students are driven in 3 buses to an art museum. There are 36, 40, and 44 students respectively in each of the three buses. In the museum, a student is randomly chosen as the winner of a gift from the museum shop. If X is a random variable representing the number of students in the bus of the winner, what is the expected value of X?

9. (10 points) Consider a gambling game in which you win 8 dollars with probability 10%, win 4 dollars with probability 20%, lose 2 dollars with probability 40% and lose 4 dollars with probability 30%. Let X be a random variable representing wins/losses in this game (e.g., if s is an outcome in which you lose 2 dollars, then $X(s) = -2$).

   (a) (2 points) Compute $E(X)$

   (b) (3 points) Compute $V(X)$ using the definition of variance.

   (c) (3 points) Compute $V(X)$ using $V(X) = E(X^2) - E(X)^2$

   (d) (2 points) Compute $\sigma(X)$, the standard deviation of X.

10. (10 points) Prove that $V(cX) = c^2V(X)$. (Hint: Use Theorem 3 from the book which states $E(aX + b) = aE(X) + b$. This is called the *linearity of expectation*.)