

CS311H Problem Set 6

Due Thursday, October 24

1. (4 points) In a given state, license plates consist of either 4 upper-case letters followed by 2 digits $[0 - 9]$ or 2 digits $[0 - 9]$ followed by 4 upper case letters. Furthermore, no license plate can start with a 0. How many possible license plates are there? Explain your reasoning.
2. (5 points) What is the coefficient of x^3y^4 in the expansion of $(4x+5y)^7$?
3. (5 points) At least how many students must a class have to guarantee that there are at least 20 students of the same gender? Explain your reasoning.
4. (7 points) Suppose a coin is flipped 9 times. How many possible outcomes contain more heads than tails? Explain your reasoning.
5. (4 points) Suppose that you bought 3 apple, 4 orange, and 2 fig trees. In how many different ways can these trees be planted in a row?
6. (10 points) How many ways are there to place 9 women and 6 men to stand in a line so that no two men stand next to each other? (Hint: First position women and then consider possible positions for the men.)
7. (10 points) Suppose a small library has five book shelves labeled A–E. In how many different ways can 30 books be placed on these shelves if the books are ...
 - (a) (5 points) all different?
 - (b) (5 points) all identical?
8. (10 points) A bakery sells 4 different kinds of croissants: plain, chocolate, vanilla, and almond. Assuming order of selection does not matter, how many ways are there to select ...
 - (a) (4 points) 12 croissants?

- (b) (6 points) 20 croissants with at least two of each type of croissant?
9. (5 points) 25 students in a class took a test containing 40 questions. Given that the maximum number of mistakes made by any student in the class is 10, prove that there are at least 3 students in the class who made the same number of mistakes.
10. (5 points) How many ways are there to distribute a deck of 52 cards to 4 players? Explain your reasoning.