1. (10 points) For each description given below, either give a simple (undirected) graph $G$ with the stated property or prove that no such simple graph can exist.
   
   (a) $G$ contains 4 vertices and 12 edges
   (b) $G$ contains 4 vertices with degrees 1, 2, 2, 3
   (c) $G$ contains 8 vertices with degrees 0, 1, 2, 3, 4, 5, 6, 7

2. (10 points) Prove or disprove the following claim about a simple undirected graph $G$ with at least two vertices: “It is possible that all vertices in $G$ have different degrees.”

3. (10 points) Prove that, if $G$ is a bipartite graph with $n$ vertices and $e$ edges, then $e \leq n^2/4$.

4. (10 points) Let $K_n'$ be a graph that is obtained by removing an arbitrary edge from $K_n$. What is the chromatic number of $K_n'$? Prove your answer.

5. (10 points) A $k$-regular graph is a simple undirected graph where each vertex has degree $k$. Is it possible to construct a $k$-regular graph for all $k \geq 1$? If so, prove your answer; otherwise give a counterexample.

6. (10 points) Let $G$ be a planar graph with $e$ edges such that $e \geq 3$ and suppose $G$ has at least 2 regions. Let $r$ denote the sum of the degrees of all the regions of $R$, i.e.,

   $$r = \sum_{R \in \text{region}(G)} \deg(R)$$

   What is the relationship between $r$ and $e$? Prove your answer.
7. (20 points) For each of the properties listed below, draw a connected multi-graph with at least three vertices and at least three edges that has these properties or explain why no such graph can exist. For each graph you draw, also explain why it has these properties.

(a) A bipartite graph with an odd number of vertices and contains an Euler circuit
(b) A bipartite graph with an odd number of vertices that also contains a Hamilton circuit
(c) A graph with three vertices that does not contain an Euler circuit, but contains an Euler path and a Hamilton circuit
(d) A bipartite graph that has a Hamilton circuit, but no Euler circuit