CS311H Problem Set

Due Nov 21, 2019

1. (10 points) Prove or disprove the following claim about a simple undirected graph G with at least two vertices: "It is possible that all vertices in G have different degrees."

2. (10 points) Prove that, if G is a bipartite graph with n vertices and e edges, then $e \leq n^2/4$.

3. (10 points) Let K_n' be a graph that is obtained by removing an arbitrary edge from K_n. What is the chromatic number of K_n'? Prove your answer.

4. (10 points) A k-regular graph is a simple undirected graph where each vertex has degree k. Is it possible to construct a k-regular graph for all $k \geq 1$? If so, prove your answer; otherwise give a counterexample.

5. (10 points) Consider a simple graph $G = (V, E)$ such that for any vertex $v \in V$, $\deg(v) \geq 2$. Is it possible that G does not contain a cycle? If so, give an example of such a graph. If not, prove that such a graph G must contain a cycle.

6. (10 points) Let G be a planar graph with e edges such that $e \geq 3$ and suppose G has at least 2 regions. Let r denote the sum of the degrees of all the regions of R, i.e.,

$$r = \sum_{r \in \text{region}(G)} \deg(R)$$

   What is the relationship between r and e? Prove your answer.

7. (10 points) Consider a graph T' that is obtained by adding an arbitrary edge (but no vertices) to some tree T. Can T' be a tree? If yes, give an example of such a T and T'. If not, prove that T' can never be a tree.

8. (10 points) Consider a tree T with maximum degree $m \geq 2$. Prove that T has at least m leaves.