CS311H Homework Assignment 5

Due Monday, October 23

1. (10 points) Use the extended Euclidian algorithm to find $\gcd(215, 35)$, and find integers s, t such that $\gcd(215, 35) = s \cdot 215 + t \cdot 35$. Show every step of the algorithm.

2. (3 points) Determine if the linear congruence $12x \equiv 15 \pmod 8$ has any solutions. Explain your reasoning.

3. (7 points) Find the set of all solutions to the linear congruence $3x \equiv 4 \pmod 7$. Show all your work.

4. (10 points) For any integer $n \geq 1$, prove that there always exist n consecutive composite numbers. (Hint: You might want to use the factorial function.)

5. (10 points) Prove by induction that the sum of the first n positive odd integers is n^2. Explicitly state whether you are using regular or strong induction.

6. (10 points) Use induction to prove that the following equality holds for every positive integer n:

$$\sum_{i=1}^{n} i \cdot 2^i = (n-1) \cdot 2^{n+1} + 2$$

   State whether you are using regular or strong induction.

7. (5 points) Explain what is wrong with the following "proof":

   Claim: $\forall n \geq 0.\ 3n = 0$
   Proof: By strong induction on n. For the base case, we have $n = 0$. Since $3 \cdot 0 = 0$, the claim holds for the base case.
   For the inductive step, we prove the property for $k+1$, i.e., $3 \cdot (k+1) =$

0. Observe that $k+1$ can be written as $i+j$ for some natural numbers i, j where $i \leq k$ and $j \leq k$. Now, by the inductive hypothesis, we have $3i = 0$ and $3j = 0$. Hence, $3 \cdot (k+1) = 3i + 3j = 0 + 0 = 0$. Hence the property holds.

8. (10 points) Prove by induction that $3^n < n!$ for all integers n greater than 6. State whether you are using regular or strong induction.

9. (15 points) Prove by induction that every integer $n > 17$ can be written in the form $n = 4a + 7b$ where $a, b \geq 0$. State whether you are using regular or strong induction.