CS311H Homework Assignment 1

Due Tuesday, September 10

1. (8 points) Construct a truth table for the following formula:

$$((p \land \neg q) \to (r \lor p))$$

2. (8 points, 2 points each) Let s be the proposition "I will go swimming", h the proposition "It's hot", and r the proposition "It's raining". Express the following sentences in propositional logic:

   (a) "I will go swimming provided that it's hot and not raining"

   (b) "I will go swimming unless it is raining"

   (c) "I will go swimming only if it is hot"

   (d) "For me to go swimming, it is necessary that it's not raining"

3. (4 points, 1 point each) Consider the proposition "If an animal is a rabbit, then it is also a mammal."

   (a) State in English the contrapositive of this proposition

   (b) State in English the converse of this proposition

   (c) State in English the inverse of this proposition

   (d) Of the three propositions above in (a)-(c), identify which ones are true and which ones are false

4. (12 points, 4 points each) For each of the formulas below, state whether they are valid, unsatisfiable, or contingent, and prove your answer.

   (a) $(\neg p \lor q) \to q$

   (b) $(((p \to q) \to p) \to p)$

   (c) $\neg((\neg(p \land q) \to (p \to \neg q)))$

5. (8 points) Prove that $\neg((p \vee q) \rightarrow \neg q)$ and q are equivalent by using the logical equivalences we showed in class. You should clearly label the equivalence you use (e.g., De Morgan's law, absorption law etc.)

6. (10 points) Consider the following argument:

   - George and Mary are not both innocent.
   - If George is not lying, Mary must be innocent.
   - Therefore, if George is innocent, then he is lying.

   Let g be the proposition "George is innocent", m be the proposition "Mary is innocent", and let l be the proposition "George is lying".

   (a) (4 points) Write a propositional formula F involving variables g, m, l such that the above argument is valid if and only if F is valid.

   (b) (6 points) Is the above argument valid? If so, prove its validity by proving the validity of F. If not, give an interpretation under which F evaluates to false.