CS311H Problem Set 2

Due Tuesday, September 17

1. (20 points, 4 points each) Consider domain $D = \{\star, \circ\}$, binary predicate p, and unary predicate q with the following interpretation I:

   - $q(\star) = \text{false}$, $q(\circ) = \text{true}$
   - $p(\star, \star) = \text{true}$, $p(\star, \circ) = \text{false}$, $p(\circ, \star) = \text{false}$, $p(\circ, \circ) = \text{true}$

   For each of the first-order logic formulas below, state their truth value under domain D and interpretation I given above.

   (a) $\forall x.\exists y.p(x, y)$
   (b) $\exists x.\forall y.p(x, y)$
   (c) $\forall x.(q(x) \rightarrow (\exists y.p(x, y)))$
   (d) $\forall x.\forall y.(p(x, y) \rightarrow q(x))$
   (e) $\exists x.(q(x) \rightarrow (\forall y.p(x, y)))$

2. (20 points, 4 points each) Consider the following predicates:

   - girl(x), which represents x is a girl
   - guy(x), which represents x is a guy
   - likes(x, y), which represents x likes y
   - goodlooking(x), which represents x is goodlooking

   Translate the following English sentences into first-order logic:

   (a) Every guy likes a girl.
   (b) Some guys like all girls.
   (c) Every girl likes all goodlooking guys.
   (d) There are some girls who don't like any guys.

(e) Every goodlooking girl is liked by some guy.

3. (10 points) Prove that the following formula is contingent:

$$\forall x.\forall y.(p(x,y) \to p(y,x))$$

4. (10 points) Prove that the following formulas F_1 and F_2 are equivalent:

$$\begin{array}{ll} F_1: & \neg(\exists x.(p(x) \wedge (\exists y.(q(y) \wedge \neg r(x,y))))) \\ F_2: & \forall x.(p(x) \to (\forall y.(q(y) \to r(x,y)))) \end{array}$$

Clearly label each equivalence used in your proof.

5. (15 points) Consider the following hypotheses:

$$\begin{array}{ll} \text{H1}: & \exists x.(p(x) \wedge q(x)) \\ \text{H2}: & \forall x.(q(x) \to r(x)) \end{array}$$

Use rules of inference to prove that the following conclusion follows from these hypotheses:

$$\text{C}: \ \exists x.(p(x) \wedge r(x))$$

Clearly label the inference rules used at every step of your proof.

6. (15 points) Consider the following hypotheses:

$$\begin{array}{ll} \text{H1}: & \forall x.(\neg C(x) \to \neg A(x)) \\ \text{H2}: & \forall x.(A(x) \to \forall y.B(y)) \\ \text{H3}: & \exists x.A(x) \end{array}$$

Use rules of inference to prove that the following conclusion follows from these hypotheses:

$$\text{C}: \ \exists x.(B(x) \wedge C(x))$$

Clearly label the inference rules used at every step of your proof.