

## CS311H Problem Set 3

Due Thursday, September 26

1. (30 points, 6 points each) Prove or disprove each of the following statements below. If you believe the statement is true, prove the statement using one of the proof techniques we discussed in class and clearly state the proof strategy you use. If you believe the statement is false, disprove the claim by proving a counterexample.
  - (a) The product of an odd and even integer is even.
  - (b) If  $r$  is a rational number and  $p$  is an irrational number, then  $r + p$  is irrational.
  - (c) If  $r$  is a rational number and  $p$  is an irrational number, then  $rp$  is irrational.
  - (d) If  $7x + 9$  is even, then  $x$  is odd.
  - (e) For all real numbers  $a, b, c$ ,  $\max(a, \max(b, c)) = \max(\max(a, b), c)$
2. (5 points) What is the Cartesian product of the sets  $\{\emptyset\}$  and  $\{a, b\}$ ?
3. (5 points) Prove or disprove: There exists a set  $S$  whose powerset is equal to  $\{S\}$
4. (15 points) Prove that  $A \cup (A \cap B) = A$ .
5. (10 points) Explain what is wrong with each of the following “proof”s that  $-1 = 1$ .
  - (a) **Proof 1:** Assume  $-1 = 1$ . By squaring both sides, we get  $1 = 1$ , which is true. Hence, we have  $-1 = 1$
  - (b) **Proof 2:** First, we know  $1 = \sqrt{1}$ . Second,  $\sqrt{1} = \sqrt{-1 \cdot -1} = \sqrt{-1}\sqrt{-1}$ . Hence,  $1 = \sqrt{-1}\sqrt{-1} = \sqrt{-1}^2$ . Squaring both sides, we have  $1 = -1$ .