

CS311H Homework Assignment 4

Due Thursday, October 3

Please hand in a hard copy of your solutions before class on the due date. You may discuss problems with other students in the class; however, your write-up must mention the names of these individuals.

1. (20 points) Prove or disprove each of the following claims:
 - (a) The function f defined by $f(x) = 2x + 5$ is a bijection from \mathbb{R} to \mathbb{R} .
 - (b) The function f defined by $f(x) = 2x + 5$ is a bijection from \mathbb{Z} to \mathbb{Z} .

2. (15 points) Let x be a real number. Prove the following:

$$\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor$$

3. (15 points) Let f be a function from B to C , g be a function from A to B , and suppose that $f \circ g$ is injective.
 - (a) Does g have to be injective? Prove your answer or give a counterexample.
 - (b) Does f have to be injective? Prove your answer or give a counterexample.
4. (15 points) State whether each of the statements below are true or false. Briefly justify your answer.
 - (a) Let A be a countable set and B be a countably infinite set. Then, $A \cup B$ is always countable.
 - (b) The set $\mathbb{R} - \mathbb{N}$ is countably infinite.
 - (c) Let A and B be two uncountably infinite sets. Then $A \cap B$ is uncountable.