CS311H Homework Assignment 7

Due Nov 5, 2014

Please hand in a hard copy of your solutions before class on the due date. The answers to the homework assignment should be your own individual work. You may discuss problems with other students in the class; however, your write-up must mention the names of these individuals.

1. (10 points) For each description given below, either give a simple (undirected) graph $G$ with the stated property or prove that no such simple graph can exist.
   
   (a) $G$ contains 4 vertices and 12 edges
   
   (b) $G$ contains 4 vertices with degrees 1, 2, 2, 3
   
   (c) $G$ contains 8 vertices with degrees 0, 1, 2, 3, 4, 5, 6, 7

2. (10 points) Prove or disprove the following claim about a simple undirected graph $G$ with at least two vertices: “It is possible that all vertices in $G$ have different degrees.”

3. (10 points) Prove that, if $G$ is a bipartite graph with $n$ vertices and $e$ edges, then $e \leq n^2/4$.

4. (10 points) Let $K'_n$ be a graph that is obtained by removing an arbitrary edge from $K_n$. What is the chromatic number of $K'_n$? Prove your answer.

5. (10 points) A $k$-regular graph is a simple undirected graph where each vertex has degree $k$. Is it possible to construct a $k$-regular graph for all $k \geq 1$? If so, prove your answer; otherwise give a counterexample.

6. (10 points) Consider a simple graph $G = (V, E)$ such that for any vertex $v \in V$, $\deg(v) \geq 2$. Is it possible that $G$ does not contain a cycle? If so, give an example of such a graph. If not, prove that such a graph $G$ must contain a cycle.