1. (10 points) For each description given below, either give a simple (undi-rected) graph $G$ with the stated property or prove that no such simple graph can exist.

   (a) $G$ contains 4 vertices and 12 edges
   (b) $G$ contains 4 vertices with degrees 1, 2, 2, 3
   (c) $G$ contains 8 vertices with degrees 0, 1, 2, 3, 4, 5, 6, 7

2. (10 points) Prove or disprove the following claim about a simple undi-rected graph $G$ with at least two vertices: “It is possible that all vertices in $G$ have different degrees.”

3. (10 points) Prove that, if $G$ is a bipartite graph with $n$ vertices and $e$ edges, then $e \leq n^2/4$.

4. (10 points) Let $K'_n$ be a graph that is obtained by removing an arbitrary edge from $K_n$. What is the chromatic number of $K'_n$? Prove your answer.

5. (10 points) A $k$-regular graph is a simple undirected graph where each vertex has degree $k$. Is it possible to construct a $k$-regular graph for all $k \geq 1$? If so, prove your answer; otherwise give a counterexample.

6. (10 points) Consider a simple graph $G = (V, E)$ such that for any vertex $v \in V$, $\deg(v) \geq 2$. Is it possible that $G$ does not contain a cycle? If so, give an example of such a graph. If not, prove that such a graph $G$ must contain a cycle.