CS311H Homework Assignment 8

Due Nov 13, 2016

1. (10 points) For each description given below, either give a simple (undirected) graph G with the stated property or prove that no such simple graph can exist.

   (a) G contains 4 vertices and 12 edges

   (b) G contains 4 vertices with degrees 1, 2, 2, 3

   (c) G contains 8 vertices with degrees 0, 1, 2, 3, 4, 5, 6, 7

2. (10 points) Prove or disprove the following claim about a simple undirected graph G with at least two vertices: "It is possible that all vertices in G have different degrees."

3. (10 points) Prove that, if G is a bipartite graph with n vertices and e edges, then $e \leq n^2/4$.

4. (10 points) Let K'_n be a graph that is obtained by removing an arbitrary edge from K_n. What is the chromatic number of K'_n? Prove your answer.

5. (10 points) A k-regular graph is a simple undirected graph where each vertex has degree k. Is it possible to construct a k-regular graph for all $k \geq 1$? If so, prove your answer; otherwise give a counterexample.

6. (10 points) Consider a simple graph $G = (V, E)$ such that for any vertex $v \in V$, $\deg(v) \geq 2$. Is it possible that G does not contain a cycle? If so, give an example of such a graph. If not, prove that such a graph G must contain a cycle.