CS311H Homework Assignment 9

Due Nov 22, 2016

Please hand in a hard copy of your solutions before class on the due date. The answers to the homework assignment should be your own individual work. You may discuss problems with other students in the class; however, your write-up must mention the names of these individuals.

1. (10 points) Let $G = (V, E)$ be a simple, undirected graph such that $\forall v \in V.\ \text{degree}(v) \geq 2$. Is the statement "$G$ must contain a cycle" true or false? If true, prove your answer. Otherwise, give a counterexample.

2. (10 points) Consider a graph T' that is obtained by adding an arbitrary edge to some tree T. Can T' be a tree? If yes, give an example of such a T and T'. If not, prove that T' can never be a tree.

3. (10 points) Consider a tree T with maximum degree $m \geq 2$. Prove that T has at least m leaves.

4. (10 points) Let G be a planar graph with e edges such that $e \geq 3$ and suppose G has at least 2 regions. Recall the **Region Handshaking Theorem** we discussed in class, which states:

$$\sum_{R \in \text{region}(G)} \deg(R) = 2e$$

   Prove the region handshaking theorem using induction.

5. (20 points) For each of the properties listed below, draw a *connected* multi-graph with at least three vertices and at least three edges that has these properties or explain why no such graph can exist. For each graph you draw, also explain why it has these properties.

   (a) A bipartite graph with an odd number of vertices and contains an Euler circuit

(b) A bipartite graph with an odd number of vertices that also contains a Hamilton circuit

(c) A graph with three vertices that does not contain an Euler circuit, but contains an Euler path and a Hamilton circuit

(d) A bipartite graph that has a Hamilton circuit, but no Euler circuit