CS311H: Discrete Mathematics

Propositional Logic

Instructor: Işıl Dillig

Course Staff

- **Instructor:** Prof. Işıl Dillig
- **TA:** Jessica Hoffmann
- **Proctor:** Cody Freitag
- **Class meets every Tuesday, Thursday 2:00 pm - 3:30 pm**
- **Course webpage:** http://www.cs.utexas.edu/~isil/cs311h/
- **Contains contact info, office hours, slides from lectures, homework assignments etc.**

About this Course

- Give mathematical background you need for computer science
- **Topics:** Logic, proof techniques, number theory, combinatorics, graph theory, basic complexity theory . . .
- These will come up again and again in higher-level CS courses
  - Master CS311H material if you want to do well in future courses!

Textbook

- **Textbook (optional):** Discrete Mathematics and Its Applications by Kenneth Rosen
- **Textbook not a substitute for lectures:**
  - Class presentation may not follow book
  - Skip many chapters and cover extra material

Piazza

- **Piazza page:** https://piazza.com/utexas/fall2015/cs311h/home
- If you have any questions about material or homework, please post questions on Piazza
- You are encouraged to answer each other’s questions
- Homework #0: Make sure you can access Piazza page!

Discussion Sections and Office Hours

- **Discussion sections on Mondays and Wednesdays 2-3 pm**
- Assigned to one of two sections, but can attend both
- Discussion section will answer questions, solve new problems, and go over previous homework
- Isil’s office hours: Thursdays before class (1-2 pm)
- Cody’s office hours: Mon, Wed 3-4 pm
- Jess’s office hours: Tuesday 4-6pm
### Requirements

- Weekly written homework assignments
- Two midterm exams: in-class, closed-book
  - Allowed to bring 3 pages of hand-prepared notes
- Scheduled for October 6, November 12
- Final exam on December 15
- No make-up exams given unless you have serious, documented medical emergency

### Grading

- Final exam: 40% of final grade
- Each midterm: 20% of final grade
- Homework: 20% of final grade
- Final grades may be curved, but lower bounds guaranteed (e.g., get at least A- if grade is 90% or higher)

### Homework Policy

- Homework due at the beginning of class on due date
  - No credit unless turned in by 2 PM on due date
  - Late submissions not allowed, but lowest homework score dropped when calculating grades
- We will only give hard copies of homework solutions in class

### Honor Code

- Homework write-up must be your own
- May not copy answers from on-line resources or other students
- If you discuss with others, write-up must mention their names
- May not share homework solutions on Piazza
- Honor code taken very seriously at UT
  - May be expelled for violating honor code!
  - Please read departmental guidelines (link from course webpage)

### Class Participation

- Everyone expected to attend lectures
  - Key to learning material!
- Ask questions!
  - No question is a stupid question!
  - Other students also benefit from your questions
- Make class fun by participating!
  - Might win chocolate if you answer questions :)

### Let's get started!
Logic

- Logic: study of valid reasoning; fundamental to CS
- Allows us to represent knowledge in precise, mathematical way
- Allows us to make valid inferences using a set of precise rules
- Many applications in CS: AI, programming languages, databases, computer architecture, automated testing and program analysis, . . .

Propositional Logic

- Simplest logic is propositional logic
- Building blocks of propositional logic are propositions
- A proposition is a statement that is either true or false
- Examples:
  - "CS311 is a course in discrete mathematics": True
  - "Austin is located in California": False
  - "Pay attention": Not a proposition
  - "x+1 = 2": Not a proposition

Propositional Variables, Truth Value

- Truth value of a proposition identifies whether a proposition is true (written T) or false (written F)
- What is truth value of "Today is Friday"? F
- Variables that represent propositions are called propositional variables
- Truth value of a propositional variable is either T or F.
- Denote propositional variables using lower-case letters, such as p, p₁, p₂, q, r, s, . . .

Conjunction

- Conjunction of two propositions p and q, written p ∧ q, is the proposition "p and q"
- p ∧ q is T if both p is true and q is true, and F otherwise.
- What is the conjunction and the truth value of p ∧ q for . . .
  - p = "It is fall semester", q = "Today is Thursday"?
  - p = "It is Thursday", q = "It is morning"?
Disjunction

- Disjunction of two propositions \( p \) and \( q \), written \( p \lor q \), is the proposition \( "p or q" \).
- \( p \land q \) is \( T \) if either \( p \) is true or \( q \) is true, and \( F \) otherwise.
- What is the disjunction and the truth value of \( p \lor q \) for...
  - \( p = \text{"It is spring semester"}, q = \text{"Today is Thursday"} \)?
  - \( p = \text{"It is Friday"}, q = \text{"It is morning"} \)?

Propositional Formulas and Truth Tables

- Truth table for propositional formula \( F \) shows truth value of \( F \) for every possible value of its constituent atomic propositions.

Examples

Construct truth tables for the following formulas:

1. \((p \lor q) \land \neg p\)
2. \((p \land q) \lor (\neg p \land \neg q)\)
3. \((p \lor q \lor \neg r) \land r\)

More Logical Connectives

- \(\land, \lor, \neg\) most common boolean connectives, but there are other boolean connectives as well.
- Other connectives: exclusive or \( \oplus \), implication \( \rightarrow \), biconditional \( \leftrightarrow \)
- Exclusive or: \( p \oplus q \) is true when exactly one of \( p \) and \( q \) is true, and false otherwise

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \oplus q )</th>
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<tbody>
<tr>
<td>T</td>
<td>T</td>
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Implication (Conditional)

- An implication (or conditional) \( p \rightarrow q \) is read "if \( p \) then \( q \)" or "\( p \) implies \( q \)"
- It is false if \( p \) is true and \( q \) is false, and true otherwise
- Observe: If \( p \) is false, \( p \rightarrow q \) is true, regardless of \( q \)'s value.
- Exercise: Draw truth table for \( p \rightarrow q \)
- In an implication \( p \rightarrow q \), \( p \) is called antecedent and \( q \) is called consequent.
Converting English into Logic

Let \( p = "I major in CS" \) and \( q = "I will find a good job". How do we translate following English sentences into logical formulas?

- "If I major in CS, then I will find a good job":
  \( p \rightarrow q \)
- "I will not find a good job unless I major in CS":
  \( 
eg q \rightarrow 
eg p \)
- "It is sufficient for me to major in CS to find a good job":
  \( p \rightarrow q \)
- "It is necessary for me to major in CS to find a good job":
  \( q \rightarrow p \)

More English - Logic Conversions

Let \( p = "I major in CS" \), \( q = "I will find a good job" \), and \( r = "I can program" \). How do we translate following English sentences into logical formulas?

- "I will not find a good job unless I major in CS or I can program":
  \( 
eg q \rightarrow (p \lor r) \)
- "I will not find a good job unless I major in CS and I can program":
  \( 
eg q \rightarrow (p \land r) \)
- "A prerequisite for finding a good job is that I can program":
  \( r \rightarrow q \)
- "If I major in CS, then I will be able to program and I can find a good job":
  \( p \rightarrow (r \land q) \)

Converse of a Implication

- The converse of an implication \( p \rightarrow q \) is \( q \rightarrow p \).
- What is the converse of "If I am a CS major, then I can program"?
- What is the converse of "If I get an A in CS311, then I am smart"?
- **Note:** It is possible for a implication to be true, but its converse to be false, e.g., \( F \rightarrow T \) is true, but converse false

Inverse of an Implication

- The inverse of an implication \( p \rightarrow q \) is \( \neg p \rightarrow \neg q \).
- What is the inverse of "If I am a CS major, then I can program"?
- What is the inverse of "If I get an A in CS311, then I am smart"?
- **Note:** It is possible for a implication to be true, but its inverse to be false. \( F \rightarrow T \) is true, but inverse is false

Contrapositive of Implication

- The contrapositive of an implication \( p \rightarrow q \) is \( \neg q \rightarrow \neg p \).
- What is the contrapositive of "If I am a CS major, then I can program"?
- What is the contrapositive of "If I get an A in CS311, then I am smart"?
- **Question:** Is it possible for an implication to be true, but its contrapositive to be false?

Conditional and its Contrapositive

**A conditional** \( p \rightarrow q \) and its **contrapositive** \( \neg q \rightarrow \neg p \) always have the same truth value.

- **Proof:** We consider all four possible cases:
  - \( p = T, q = T \): Both \( T \rightarrow T \) and \( T \rightarrow T \) are true
  - \( p = T, q = F \): Both \( T \rightarrow F \) and \( T \rightarrow F \) are false
  - \( p = F, q = T \): Both \( F \rightarrow T \) and \( F \rightarrow T \) are true
  - \( p = F, q = F \): Both \( F \rightarrow F \) and \( F \rightarrow F \) are true
Consider a conditional $p \rightarrow q$

Is it possible that its converse is true, but inverse is false?

Conditional is of the form $p \rightarrow q$

Converse: $q \rightarrow p$

Inverse: $\neg p \rightarrow \neg q$

Contrapositive: $\neg q \rightarrow \neg p$

Conditional and contrapositive have same truth value

Inverse and converse always have same truth value

A biconditional $p \leftrightarrow q$ is the proposition “$p$ if and only if $q$”.

The biconditional $p \leftrightarrow q$ is true if $p$ and $q$ have same truth value, and false otherwise.

Exercise: Construct a truth table for $p \leftrightarrow q$

Question: How can we express $p \leftrightarrow q$ using the other boolean connectives?

Given a formula $p \land q \lor r$, do we parse this as $(p \land q) \lor r$ or $p \land (q \lor r)$?

Without settling on a convention, formulas without explicit parantheses are ambiguous.

To avoid ambiguity, we will specify precedence for logical connectives.

Negation ($\neg$) has higher precedence than all other connectives.

Question: Does $\neg p \land q$ mean (i) $\neg(p \land q)$ or (ii) $(\neg p) \land q$?

Conjunction ($\land$) has next highest precedence.

Question: Does $p \land q \lor q$ mean (i) $(p \land q) \lor q$ or (ii) $p \land (q \lor q)$?

Disjunction ($\lor$) has third highest precedence.

Next highest is precedence is $\rightarrow$, and lowest precedence is $\leftrightarrow$

Which is the correct interpretation of the formula

$p \lor q \land r \leftrightarrow q \rightarrow \neg r$

(A) $(p \lor (q \land r)) \leftrightarrow q \rightarrow (\neg r)$

(B) $(p \lor q) \land r \leftrightarrow q \rightarrow (\neg r)$

(C) $(p \lor (q \land r)) \leftrightarrow (q \rightarrow (\neg r))$

(D) $(p \lor ((q \land r) \leftrightarrow q)) \rightarrow (\neg r)$
Summary

- Formulas in propositional logic are formed using propositional variables and boolean connectives.
- Connectives: negation $\neg$, conjunction $\land$, disjunction $\lor$, conditional $\rightarrow$, biconditional $\leftrightarrow$.
- Truth table shows truth value of formula under all possible assignments to variables.