Course Staff

- **Instructor:** Prof. Ihsil Dillig
- **TAs:** Ben Mariano (PhD student), Maruth Goyal (2nd year Turing scholar)
- Class meets every Tuesday, Thursday 2-3:15 pm
- Course webpage: [http://www.cs.utexas.edu/~isil/cs311h/](http://www.cs.utexas.edu/~isil/cs311h/)
- Contains contact info, office hours, slides from lectures, homework assignments etc.

About this Course

- Give mathematical background you need for computer science
- **Topics:** Logic, proof techniques, number theory, combinatorics, graph theory, basic complexity theory . . .
- These will come up again and again in higher-level CS courses
  - Master CS311H material if you want to do well in future courses!

Textbook

- **Textbook (optional):** Discrete Mathematics and Its Applications by Kenneth Rosen
- Textbook not a substitute for lectures:
  - Class presentation may not follow book
  - Skip many chapters and cover extra material

Piazza

- **Piazza page:** [https://piazza.com/utexas/fall2019/cs311h/home](https://piazza.com/utexas/fall2019/cs311h/home)
- **Homework #0:** Make sure you can access Piazza page!
- Please post class-related questions on Piazza instead of emailing instructor TA’s
  - If something is not clear to you, it won’t be to others either
  - You’ll get answers a lot quicker
- Please use common sense when posting questions on Piazza
  - Hints/ideas ok, but cannot post full solutions!!

Discussion Sections and Office Hours

- **Discussion sections on Fridays 1-2 pm and 2-3 pm**
- Discussion section will answer questions, solve new problems, and go over previous homework
- Isil’s office hours: Tuesday, Thursday: after class to 4 pm in GDC 5.726
- Ben’s office hours: Mondays: 9-11 am in GDC 5.710B
- Maruth’s office hours: Wednesdays, 2-4 pm in GDC 5.710B
Requirements

- Weekly written homework assignments
- Three exams: in-class, closed-book
  - Allowed to bring 3 pages of hand-prepared notes
- Scheduled for October 8, November 7, December 5
- No final exam :-)
- No make-up exams given unless you have serious, documented medical emergency

Grading

- Each midterm: 25% of final grade
- Homework: 25% of final grade
- Final grades may be curved, but lower bounds guaranteed (e.g., get at least A- if grade is 90% or higher)

Homework Policy

- Homework due at the beginning of class on due date
- Late submissions not allowed, but lowest homework score dropped when calculating grades
- We will only give hard copies of homework solutions in class
- Homework solutions must be typeset in LaTeX – see instructions on first homework!

Honor Code

- Homework write-up must be your own
- May not copy answers from on-line resources or other students
- If you discuss with others, write-up must mention their names
- Honor code taken very seriously at UT
  - May be expelled for violating honor code!
  - Please read departmental guidelines (link from course webpage)

Class Participation

- Everyone expected to attend lectures
  - Please do not use cell phone, laptop, or tablets during lecture!
- Ask questions!
  - No question is a stupid question!
  - Other students also benefit from your questions
- Make class fun by participating!
  - Might win chocolate if you answer questions :)

Let’s get started!
Logic

▶ Logic: study of valid reasoning; fundamental to CS
▶ Allows us to represent knowledge in precise, mathematical way
▶ Allows us to make valid inferences using a set of precise rules
▶ Many applications in CS: AI, programming languages, databases, computer architecture, automated testing and program analysis, . . .

Propositional Logic

▶ Simplest logic is propositional logic
▶ Building blocks of propositional logic are propositions
▶ A proposition is a statement that is either true or false
▶ Examples:
    ▶ "CS311 is a course in discrete mathematics": True
    ▶ "Austin is located in California": False
    ▶ "Pay attention": Not a proposition
    ▶ "x+1 =2": Not a proposition

Propositional Variables, Truth Value

▶ Truth value of a proposition identifies whether a proposition is true (written T) or false (written F)
▶ What is truth value of "Today is Friday"? F
▶ Variables that represent propositions are called propositional variables
▶ Denote propositional variables using lower-case letters, such as p, p1, p2, q, r, s, . . .

Compound Propositions

▶ More complex propositions formed using logical connectives (also called boolean connectives)
▶ Three basic logical connectives:
    1. ∧: conjunction (read "and")
    2. ∨: disjunction (read "or")
    3. ¬: negation (read "not")
▶ Propositions formed using these logical connectives called compound propositions; otherwise atomic propositions
▶ A propositional formula is either an atomic or compound proposition

Negation

▶ Negation of a proposition p, written ¬p, represents the statement "It is not the case that p".
▶ If p is T, ¬p is F and vice versa.
▶ In simple English, what is ¬p if p stands for . . .
    ▶ "Austin is located in California"?
    ▶ "Less than 80 students are enrolled in CS311"?

Conjunction

▶ Conjunction of two propositions p and q, written p ∧ q, is the proposition "p and q"
▶ p ∧ q is T if both p is true and q is true, and F otherwise.
▶ What is the conjunction and the truth value of p ∧ q for . . .
    ▶ p = "It is fall semester", q = "Today is Wednesday"?
    ▶ p = "It is Tuesday", q = "It is the afternoon"?
**Disjunction**

- Disjunction of two propositions \( p \) and \( q \), written \( p \lor q \), is the proposition "\( p \) or \( q \)"
- \( p \lor q \) is \( T \) if either \( p \) is true or \( q \) is true, and \( F \) otherwise.
- What is the disjunction and the truth value of \( p \lor q \) for...
  - \( p = \) "It is spring semester", \( q = \) "Today is Wednesday"?
  - \( p = \) "It is Friday", \( q = \) "It is morning"?

**Propositional Formulas and Truth Tables**

- Truth table for propositional formula \( F \) shows truth value of \( F \) for every possible value of its constituent atomic propositions
- Example: Truth table for \( \neg p \)
  - \( p \) \( \neg p \)
  - \( T \) \( F \)
  - \( F \) \( T \)

- Example: Truth table for \( p \lor q \)
  - \( p \) \( q \) \( p \lor q \)
  - \( T \) \( T \) \( T \)
  - \( T \) \( F \) \( T \)
  - \( F \) \( T \) \( T \)
  - \( F \) \( F \) \( F \)

**Examples**

Construct truth tables for the following formulas:

1. \((p \lor q) \land \neg p\)
2. \((p \land q) \lor (\neg p \land \neg q)\)
3. \((p \lor q \lor \neg r) \land r\)

**More Logical Connectives**

- \( \land, \lor, \neg \) most common boolean connectives, but there are other boolean connectives as well
- Other connectives: exclusive or \( \oplus \), implication \( \rightarrow \), biconditional \( \leftrightarrow \)
- Exclusive or: \( p \oplus q \) is true when exactly one of \( p \) and \( q \) is true, and false otherwise

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \oplus q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( T )</td>
<td>( F )</td>
</tr>
<tr>
<td>( T )</td>
<td>( F )</td>
<td>( T )</td>
</tr>
<tr>
<td>( F )</td>
<td>( T )</td>
<td>( T )</td>
</tr>
<tr>
<td>( F )</td>
<td>( F )</td>
<td>( F )</td>
</tr>
</tbody>
</table>

- Truth table:

**Implication (Conditional)**

- An implication (or conditional) \( p \rightarrow q \) is read "if \( p \) then \( q \)" or "\( p \) implies \( q \)"
- It is false if \( p \) is true and \( q \) is false, and true otherwise
- Intuition: Consider the sentence "If I’m late, I’ll pay you $100". When am I lying?
- Exercise: Draw truth table for \( p \rightarrow q \)
- In an implication \( p \rightarrow q \), \( p \) is called **antecedent** and \( q \) is called **consequent**

**Converting English into Logic**

Let \( p = \) "I major in CS" and \( q = \) "I will find a good job". How do we translate following English sentences into logical formulas?

- "If I major in CS, then I will find a good job":
- "I will not find a good job unless I major in CS":
- "It is sufficient for me to major in CS to find a good job":
- "It is necessary for me to major in CS to find a good job":

![Image of a table with truth values for logical connectives]
More English - Logic Conversions

Let \( p = " \text{I major in CS}" \), \( q = " \text{I will find a good job}" \), \( r = " \text{I can program}" \). How do we translate following English sentences into logical formulas?

- "I will not find a good job unless I major in CS or I can program":
- "I will not find a good job unless I major in CS and I can program":
- "A prerequisite for finding a good job is that I can program":
- "If I major in CS, then I will be able to program and I can find a good job":

Converse of a Implication

- The converse of an implication \( p \rightarrow q \) is \( q \rightarrow p \).
- What is the converse of "If I am a CS major, then I can program"?
- What is the converse of "If I get an A in CS311, then I am smart"?
- Question: Do an implication and its converse always have the same truth value?

Inverse of an Implication

- The inverse of an implication \( p \rightarrow q \) is \( \neg p \rightarrow \neg q \).
- What is the inverse of "If I am a CS major, then I can program"?
- What is the inverse of "If I get an A in CS311, then I am smart"?
- Question: Do an implication and its inverse always have the same truth value?

Contrapositive of Implication

- The contrapositive of an implication \( p \rightarrow q \) is \( \neg q \rightarrow \neg p \).
- What is the contrapositive of "If I am a CS major, then I can program"?
- What is the contrapositive of "If I get an A in CS311, then I am smart"?
- Question: Is it possible for an implication to be true, but its contrapositive to be false?

Conditional and its Contrapositive

A conditional \( p \rightarrow q \) and its contrapositive \( \neg q \rightarrow \neg p \) always have the same truth value.

- Prove it!

Question

- Given \( p \rightarrow q \), is it possible that its converse is true, but inverse is false?
### Summary

- Conditional is of the form $p \rightarrow q$
- Converse: $q \rightarrow p$
- Inverse: $\neg p \rightarrow \neg q$
- Contrapositive: $\neg q \rightarrow \neg p$
- Conditional and contrapositive have same truth value
- Inverse and converse always have same truth value

---

### Biconditionals

- A biconditional $p \leftrightarrow q$ is the proposition "$p$ if and only if $q$".
- The biconditional $p \leftrightarrow q$ is true if $p$ and $q$ have same truth value, and false otherwise.
- Exercise: Construct a truth table for $p \leftrightarrow q$
- Question: How can we express $p \leftrightarrow q$ using the other boolean connectives?

---

### Operator Precedence

- Given a formula $p \land q \lor r$, do we parse this as $(p \land q) \lor r$ or $p \land (q \lor r)$?
- Without settling on a convention, formulas without explicit paranthesization are ambiguous.
- To avoid ambiguity, we will specify precedence for logical connectives.

---

### Operator Precedence Example

- Which is the correct interpretation of the formula
  \[ p \lor q \land r \leftrightarrow q \rightarrow \neg r \]

- (A) $(p \lor (q \land r)) \leftrightarrow q \rightarrow (\neg r)$
- (B) $((p \lor q) \land r) \leftrightarrow q \rightarrow (\neg r)$
- (C) $(p \lor (q \land r)) \leftrightarrow (q \rightarrow (\neg r))$
- (D) $(p \lor ((q \land r) \leftrightarrow q)) \rightarrow (\neg r)$

---

### Summary

- Formulas in propositional logic are formed using propositional variables and boolean connectives
- Connectives: negation $\neg$, conjunction $\land$, disjunction $\lor$, conditional $\rightarrow$, biconditional $\leftrightarrow$
- Truth table shows truth value of formula under all possible assignments to variables