Homework Assignment 3
Due Tuesday, February 21 2019

1. (40 points, 10 points each) For each of the following sentences, identify whether it is valid, satisfiable, or unsatisfiable. If the formula is valid or unsatisfiable, prove it using the semantic argument method. If the formula is satisfiable but not valid, provide (i) a structure that satisfies the formula, and (ii) a structure that falsifies the formula.

   (a) $(\exists x.p(x) \vee q(x)) \leftrightarrow \exists x.p(x) \vee \exists x.q(x)$

   (b) $(\forall x, y.\ (p(x, y) \rightarrow p(y, x))) \rightarrow \forall z.p(z, z)$

   (c) $\exists x, y.\ (p(x, y) \rightarrow (p(y, x) \rightarrow \forall z.p(z, z)))$

   (d) $\exists y.\forall x.p(y, x) \wedge \exists x.\forall y.\neg p(y, x)$

2. (10 points) For each pair of expressions below, state whether they are unifiable, and if so, give a most general unifier.

   (a) $p(a, f(y), y)$ and $p(a, x, f(x))$

   (b) $p(f(x, a), f(f(b, a)))$ and $p(z, f(z))$

   (c) $p(f(x, y), f(y, z))$ and $p(z, f(w, f(y, w)))$

3. (10 points) Convert the following sentence to clausal form:

$$\exists x.\forall y.\exists z.\forall w.((p(x, y) \wedge \neg q(z, w)) \rightarrow \exists x.r(x, w))$$

4. (10 points) Give an example of a sentence in first-order logic where the resulting formula after applying skolemization is not equivalent to the original formula. Show the formula in Skolem normal form and explain why it is not equivalent to the original formula.

5. (10 points) Give a resolution refutation of the set of clauses shown below. For each new derived clause, clearly label the pair of clauses from which it was derived and indicate most general unifiers.

$$\begin{aligned}
C1 &: \{\neg p(x_1, x_2), q(x_1, x_2, f(x_1, x_2))\} \\
C2 &: \{\neg r(x_3, x_4), q(a, x_3, x_4)\} \\
C3 &: \{r(x_5, x_6), \neg q(a, x_5, x_6)\} \\
C4 &: \{p(x_7, g(x_7)), q(x_7, g(x_7), x_8)\} \\
C5 &: \{\neg r(x_9, x_{10}), \neg q(x_9, x_{11}, x_{12})\}
\end{aligned}$$

6. (20 points) Consider the following formula:

$$\begin{array}{lll}
 & \forall x.(course(x) \wedge easy(x)) \rightarrow (\exists y.student(y) \wedge happy(y)) & (1) \\
\wedge & \forall x.\forall y.(course(x) \wedge hasFinal(x) \wedge student(y) \rightarrow \neg happy(y)) & (2) \\
\wedge & \neg(\forall x.(course(x) \wedge hasFinal(x) \rightarrow \neg easy(x))) & (3)
\end{array}$$

   (a) (5 points) For each line marked (1), (2), (3) above, explain the meaning of the corresponding formula in English.

   (b) (15 points) Prove the unsatisfiability of this formula by giving a resolution refutation.