Homework Assignment 4
Due Thursday, March 1 2018 at 3:30pm

1. (10 points) For each pair of expressions below, state whether they are unifiable, and if so, give a most general unifier.

   (a) $p(a, f(y), y)$ and $p(a, x, f(x))$

   (b) $p(f(x, a), f(f(b, a)))$ and $p(z, f(z))$

   (c) $p(f(x, y), f(y, z))$ and $p(z, f(w, f(y, w)))$

2. (10 points) In this question, we will explore a pathological case for Robinson's unification algorithm. Consider the following two terms e, e':

$$e = f(x_0, x_1, \ldots, x_{n-1}, x_0)$$
$$e' = f(g(x_1, x_1), g(x_2, x_2), \ldots, g(x_n, x_n), x_0)$$

   (a) Let σ be the MGU of e, e' such that σ' maps each x_i to a term t_i. What is the size of t_i as a function of n, i?

   (b) Using your answer from part (a), explain why Robinson's algorithm exhibits its worst-case exponential behavior on this example.

3. (10 points) Convert the following sentence to clausal form:

$$\exists x.\forall y.\exists z.\forall w.((p(x, y) \land \neg q(z, w)) \rightarrow \exists x.r(x, w))$$

4. (10 points) Give an example of a sentence in first-order logic where the resulting formula after applying skolemization is not equivalent to the original formula. Show the formula in Skolem normal form and explain why it is not equivalent to the original formula.

5. (10 points) Give a resolution refutation of the set of clauses shown below. For each new derived clause, clearly label the pair of clauses from which it was derived and indicate most general unifiers.

$$C1 : \{\neg p(x_1, x_2), q(x_1, x_2, f(x_1, x_2))\}$$
$$C2 : \{\neg r(x_3, x_4), q(a, x_3, x_4)\}$$
$$C3 : \{r(x_5, x_6), \neg q(a, x_5, x_6)\}$$
$$C4 : \{p(x_7, g(x_7)), q(x_7, g(x_7), x_8)\}$$
$$C5 : \{\neg r(x_9, x_{10}), \neg q(x_9, x_{11}, x_{12})\}$$

6. (15 points) Consider the following formula:

$$\begin{array}{ll} & \forall x.(course(x) \land easy(x)) \rightarrow (\exists y.student(y) \land happy(y)) \quad (1) \\ \land & \forall x.\forall y.(course(x) \land hasFinal(x) \land student(y) \rightarrow \neg happy(y)) \quad (2) \\ \land & \neg(\forall x.(course(x) \land hasFinal(x) \rightarrow \neg easy(x))) \quad (3) \end{array}$$

   Prove the unsatisfiability of this formula by giving a resolution refutation.