

Homework Assignment 3

Due Tuesday, February 28

The answers to the homework assignment should be your own individual work. Please hand in a hard copy of your solutions in class on the due date.

1. (25 points, 5 points each) Consider the following function and relation constants:

- **isResearcher**(x): is a unary relation stating that person x is a researcher
- **isReviewer**(x, y): is a binary relation stating that person x is a reviewer of paper y
- **isAuthorOf**(x, y): is a binary relation stating that person x is an author of paper y
- **areCoauthors**(x, y): is a binary relation stating that persons x and y are coauthors
- **advisor**(x): is a unary function denoting person x 's PhD advisor

Give a translation from the following English sentences to the first-order language defined by the function and relation constants above.

- (a) Two people are coauthors if and only if they have written at least one paper together
 - (b) Every researcher is coauthors with at least one other researcher
 - (c) Some researchers write all of their papers with at least one other researcher
 - (d) If any person x coauthors a paper with another person y , then x can never review y 's papers
 - (e) Every researcher has written at least one paper with their PhD advisor
2. (25 points, 5 points each) Given binary relation constants p and q and a unary function constant f , consider the following structure defined by the universe of discourse $U = \{\star, \circ\}$ and the following interpretation I :

$$\begin{aligned} I(p) &= \{\langle \circ, \star \rangle, \langle \star, \circ \rangle\} \\ I(q) &= \{\langle \circ, \circ \rangle, \langle \circ, \star \rangle\} \\ I(f) &= \{\star \mapsto \circ, \circ \mapsto \circ\} \end{aligned}$$

Under this structure and variable assignment $\sigma : [x \mapsto \star, y \mapsto \circ, z \mapsto \circ]$, state whether the following formulas evaluate to true or false. No partial credit will be given for wrong answers, and no explanation is necessary.

- (a) $p(x, y) \rightarrow (\forall z. \exists w. q(z, f(w)))$
- (b) $p(y, x) \rightarrow (\exists z. \forall w. q(z, w))$
- (c) $\forall x. \forall y. (p(f(x), y) \rightarrow (\exists z. (q(y, z) \leftrightarrow p(f(z), x))))$
- (d) $\forall x. \exists y. ((p(x, y) \wedge q(y, x)) \rightarrow p(y, x))$
- (e) $q(z, f(z)) \rightarrow \forall x. \neg \exists y. (p(x, y) \wedge q(y, x))$

3. (40 points, 10 points each) For each of the following sentences, identify whether it is valid, satisfiable, or unsatisfiable. If the formula is valid or unsatisfiable, prove it using the semantic argument method. If the formula is satisfiable but not valid, provide (i) a structure that satisfies the formula, and (ii) a structure that falsifies the formula.

(a) $(\exists x.p(x) \vee q(x)) \leftrightarrow \exists x.p(x) \vee \exists x.q(x)$

(b) $(\forall x, y. (p(x, y) \rightarrow p(y, x))) \rightarrow \forall z.p(z, z)$

(c) $\exists x, y. (p(x, y) \rightarrow (p(y, x) \rightarrow \forall z.p(z, z)))$

(d) $(\forall x.(\neg p(x) \vee \neg q(x))) \leftrightarrow (\neg \forall x.(p(x) \wedge q(x)))$

(e) $\exists y.\forall x.p(y, x) \wedge \exists x.\forall y.\neg p(y, x)$