CS389L: Problem Set 6

1. Solve the following linear program using Simplex:

$$\begin{array}{ll} \text{Maximize :} & x_1 + 3x_2 \\ \text{Subject to :} & \\ & \begin{array}{rcl} -x_1 + x_2 & \leq & -1 \\ -2x_1 - 2x_2 & \leq & -6 \\ -x_1 + 4x_2 & \leq & 2 \\ x_1, x_2 & \geq & 0 \end{array} \end{array}$$

   Show the initial slack form representation and the auxiliary linear program needed to obtain a feasible basic solution. Also, show each step of the Simplex algorithm after performing a pivot operation, and use Bland's rule for pivot selection.

2. Consider the following inequality system over integers:

$$\begin{array}{rcl} 4y & \leq & 2x \\ 2y & \leq & -x + 3 \\ 4y & \geq & 1 \end{array}$$

   Use the Omega test to determine whether this system is satisfiable or not.

3. Consider an arbitrary CNF formula ϕ in propositional logic.

   (a) Is it possible to convert ϕ to a conjunctive $T_{\mathbb{Q}}$ formula ψ such that ϕ and ψ are equisatisfiable? If so, describe a technique for doing this; if not, explain why this is not possible. Note that ψ is not allowed to contain disjunctions.

   (b) Is it possible to convert χ to a conjunctive $T_{\mathbb{Z}}$ formula ψ such that ϕ and χ are equisatisfiable? If so, describe a technique for doing this; if not, explain why this is not possible. Note that ψ is not allowed to contain disjunctions.

   (c) If you answered "yes" to either part (a) or (b), demonstrate how your conversion works on the following formula:

$$(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge \neg p$$

   How can you use the translated formula to determine if the original formula is satisfiable?