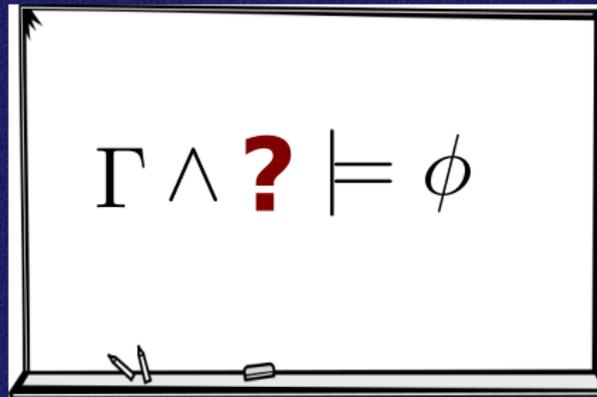


Abductive Inference and its Applications in Program Analysis, Verification, and Synthesis



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What is Abduction?

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- i.e., given invalid formula $\Gamma \Rightarrow \phi$, find a “simple” formula ψ such that $\Gamma \wedge \psi \Rightarrow \phi$ is valid and ψ does not contradict Γ .

Simple Example



- Premises: “If it rains, then it is wet and cloudy”, “If it is wet, then it is slippery”:
 $(R \Rightarrow W \wedge C) \wedge (W \Rightarrow S)$

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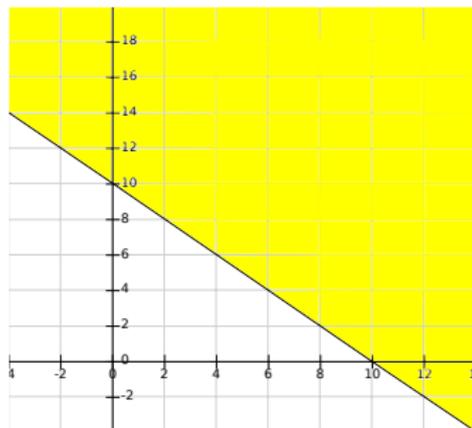


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- Possible solution: R , i.e., “It is rainy”

Arithmetic Example

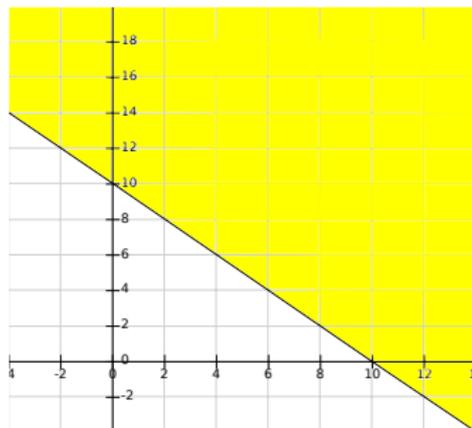
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Arithmetic Example



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Arithmetic Example



- Suppose we know $x \geq -2$
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- Abductive explanation: $y > 12$

Outline of Talk



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- ① Properties of desired solutions
- ② Algorithm for performing abduction in LRA/LIA

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Outline of Talk



- 1 Properties of desired solutions
- 2 Algorithm for performing abduction in LRA/LIA
- 3 Loop invariant generation using abduction
- 4 Compositional verification using abduction
- 5 Use of abduction in program synthesis
- 6 Conclusion and future directions

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- So, what kind of solutions do want to compute?

Which Abductive Explanations Are Good?

Guiding Principle:
Occam's Razor



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Which Abductive Explanations Are Good?

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- **Generality:** If explanation A is logically weaker than explanation B , always prefer A
- **Succinctness:** Minimize number of variables



Want to compute logically weakest solutions with fewest variables



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- First talk about how to compute solutions with fewest variables



Want to compute logically weakest solutions with fewest variables

- First talk about how to compute solutions with fewest variables
- Then talk about how to obtain most general solution containing these variables

Minimum Satisfying Assignments

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Minimum satisfying assignment (MSA):



- ✓ assigns values to a subset of variables in formula
- ✓ sufficient to make formula true
- ✓ Among all other partial satisfying assignments, contains fewest variables

Example

- Consider the following formula in linear integer arithmetic:

$$x + y + w > 0 \vee x + y + z + w < 5$$

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- **Note:** Algorithm for computing MSAs given in our CAV'12 paper, "Minimum Satisfying Assignments for SMT"

Why Are MSAs Useful for Abduction?

- Given facts Γ and conclusion ϕ , **MSA** σ of $\Gamma \Rightarrow \phi$ consistent with Γ is a solution to abduction problem:

$$\sigma \models \Gamma \Rightarrow \phi \quad \text{hence} \quad \sigma \wedge \Gamma \models \phi$$

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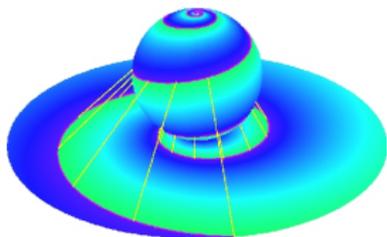
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- Furthermore, it uses a fewest number of variables
- But it is not the most general solution

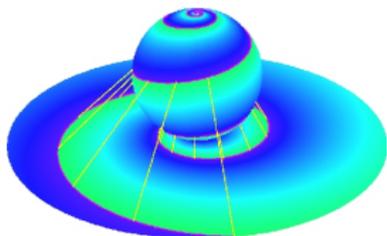
Finding Most General Solutions



Key idea:
Quantifier elimination

- To find most general solution containing variables in the MSA, **universally quantify** all other variables \overline{V} and apply quantifier elimination to $\forall \overline{V}. \Gamma \Rightarrow \phi$

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- This yields most general solution with fewest variables

Abduction Algorithm

- `abduce` yields formula ψ such that

$$\Gamma \wedge \psi \models \phi$$

and ψ is consistent with Γ and θ

```
abduce( $\Gamma, \phi, \theta$ ) {
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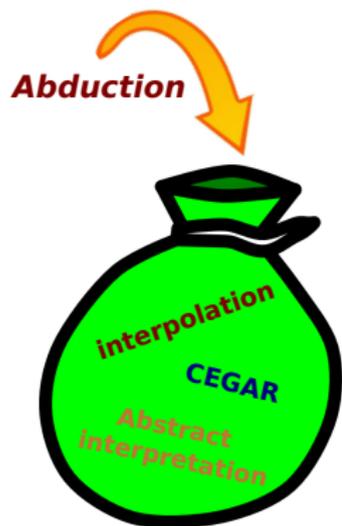
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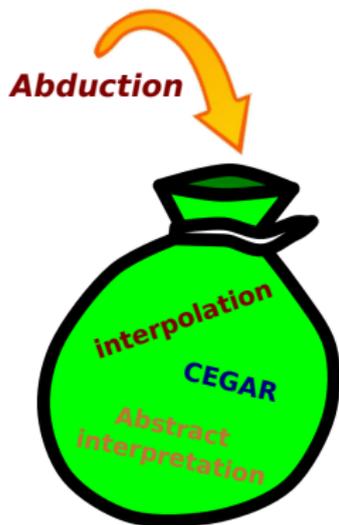
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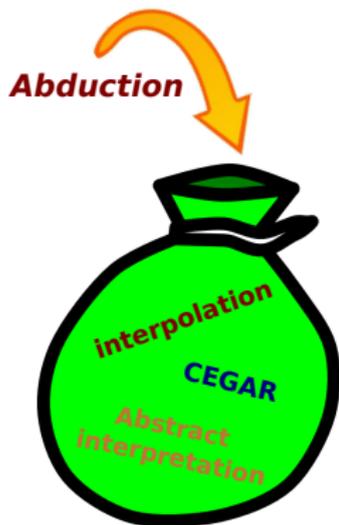
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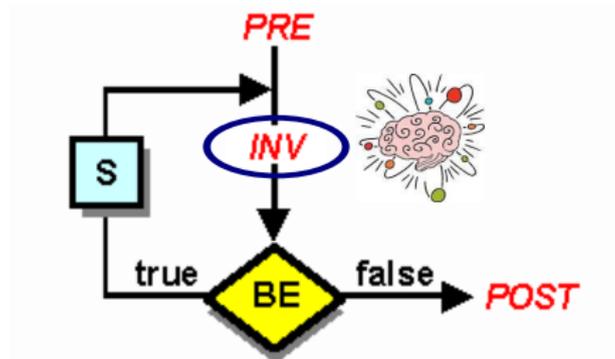


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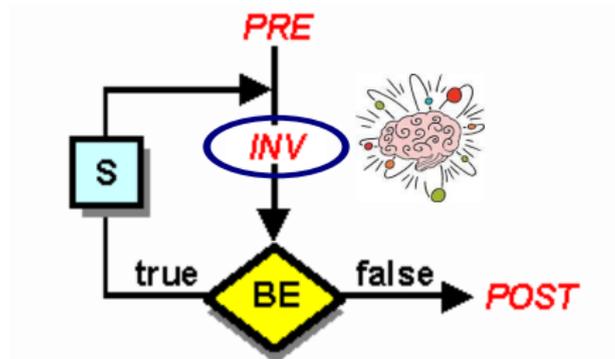


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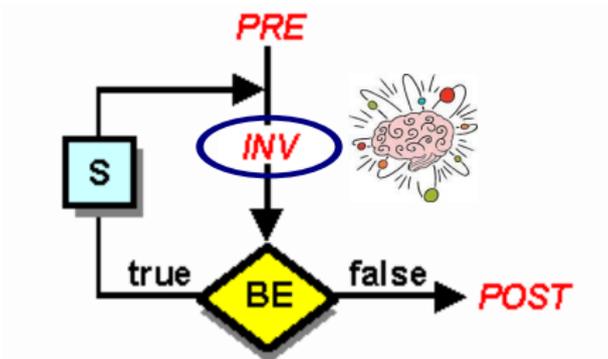


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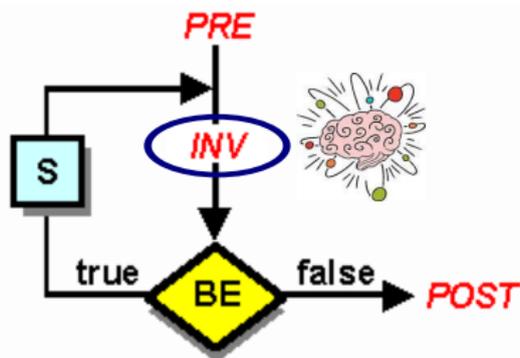
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- Most challenging aspect of program verification: **loop invariant generation**
- Inductive loop invariant **Inv** is implied by **Pre** and preserved in each iteration assuming only **Inv**
- But **Inv** is only useful if it is sufficient to prove **Post**



Key idea 1: Given loop L and postcondition $Post$, use **abduction** to speculate candidate invariants

High Level Idea



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- Use abduction to speculate an invariant I that implies post-condition Q :

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- Now, check if $I \wedge I'$ is inductive, if not, keep strengthening using (i) proof goes through, or (ii) get contradiction

Simple Example

- Start by solving abduction problem:

$$i > n \wedge ?? \Rightarrow i \geq 1$$

```
{i=1, j = 0, n<5}
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```
while(i <= n) {
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  j := j+i;
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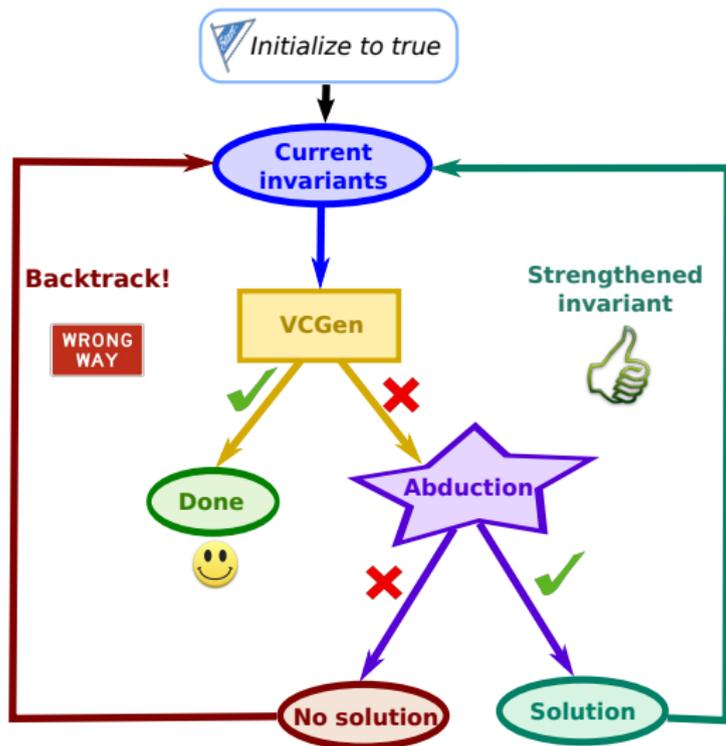
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- This is inductive, so algorithm terminates

Overall Algorithm



How to Perform Backtracking

- **Recall:** Abduction procedure takes Γ , ϕ , and set θ



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abduce(Γ, ϕ, θ)

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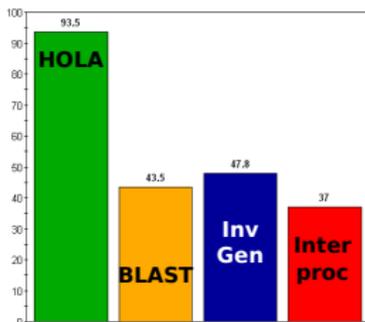
Algorithm lazily generates abductive explanations

Some Experimental Results

- Evaluated this technique on 46 loop invariant benchmarks

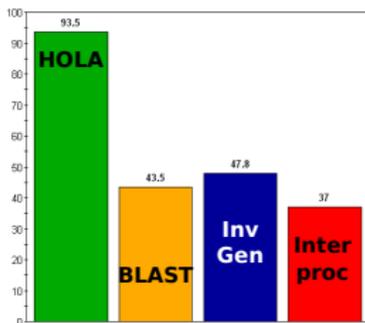
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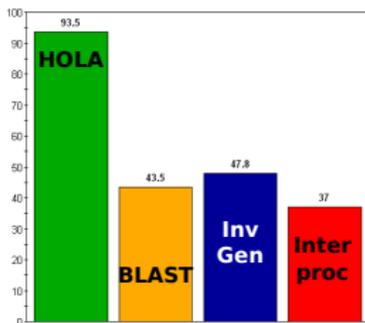
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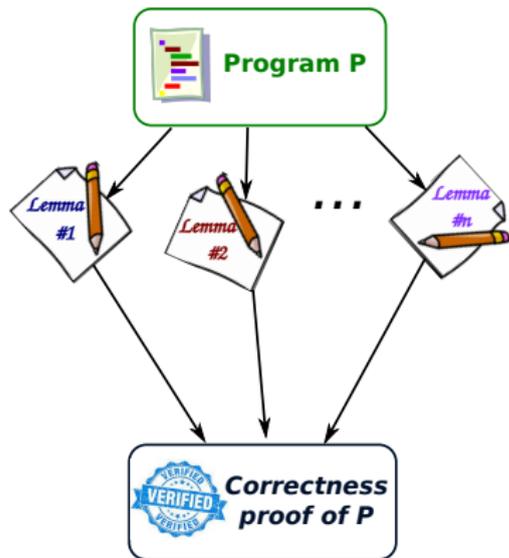
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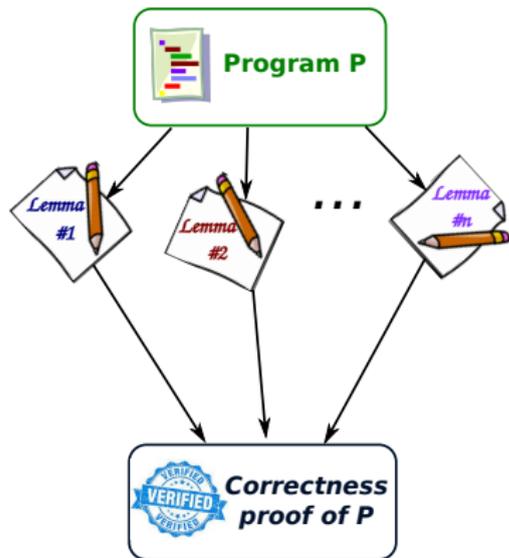


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Application #2: Compositional Verification

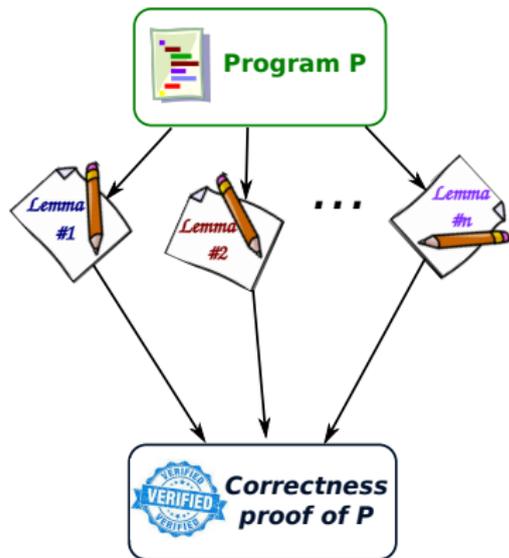


Application #2: Compositional Verification



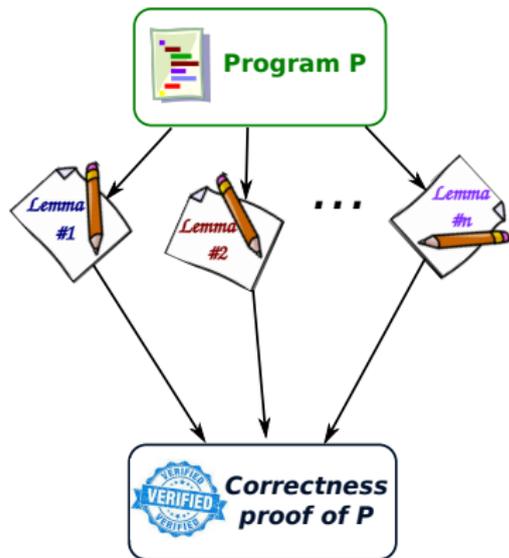
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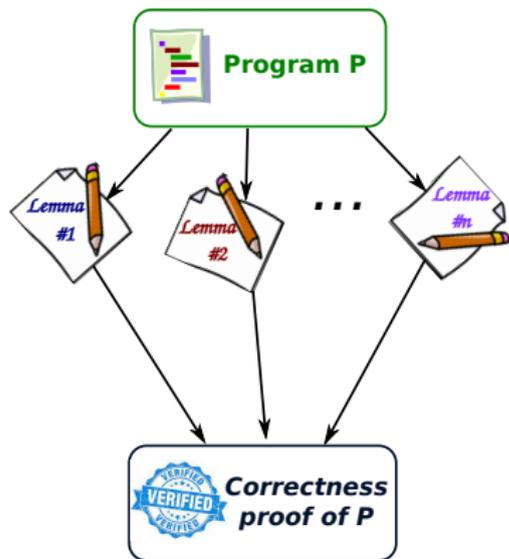
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- Compositional approaches decompose proof into lemmas
- Two key advantages:
 - 1 **Scalability:** Each lemma concerns small syntactic part \Rightarrow reason about program fragments in isolation

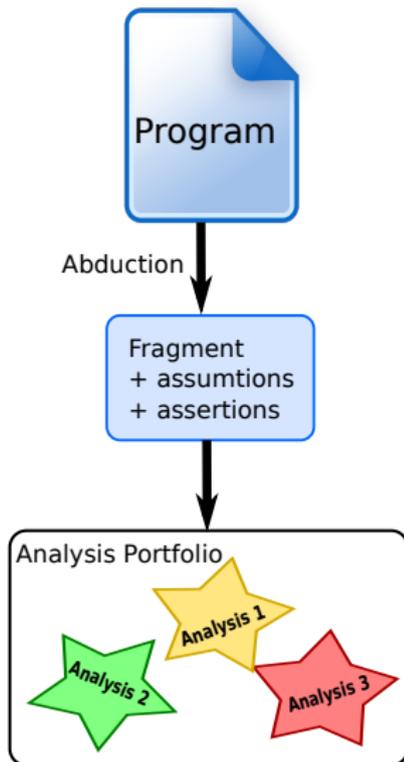
Application #2: Compositional Verification



- Compositional approaches decompose proof into lemmas
- Two key advantages:
 - 1 **Scalability:** Each lemma concerns small syntactic part \Rightarrow reason about program fragments in isolation
 - 2 **Abstraction:** Each lemma can be proven using a different abstraction \Rightarrow combine strengths of different techniques

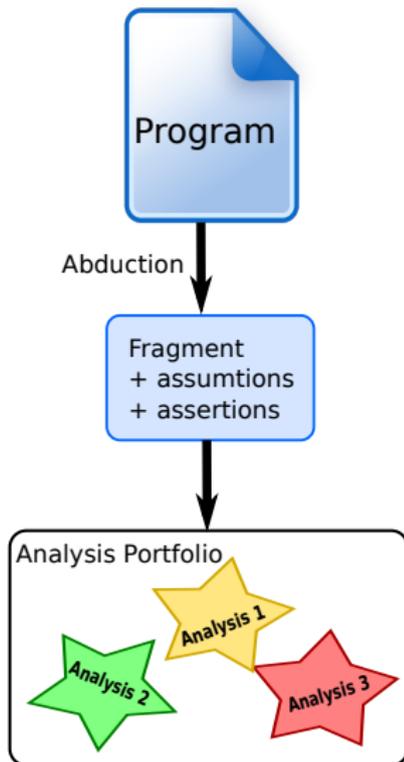
Overview of Compositional Verification Approach

- **Key idea:** Use **abduction** to decompose proof into auxiliary lemmas



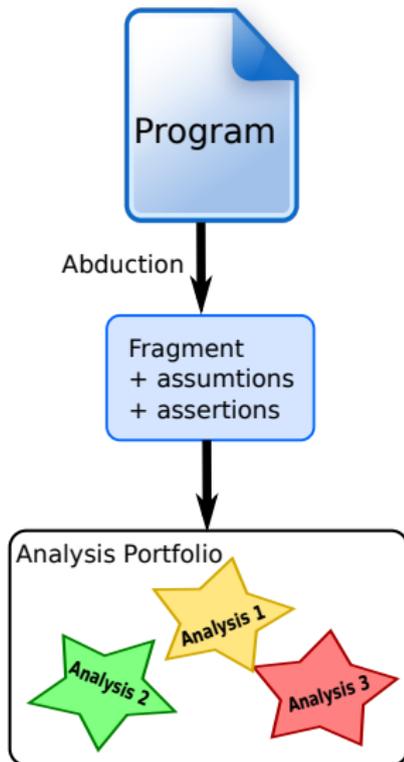
Overview of Compositional Verification Approach

- **Key idea:** Use **abduction** to decompose proof into auxiliary lemmas
 - Lemmas are snippets annotated with assertions and assumptions



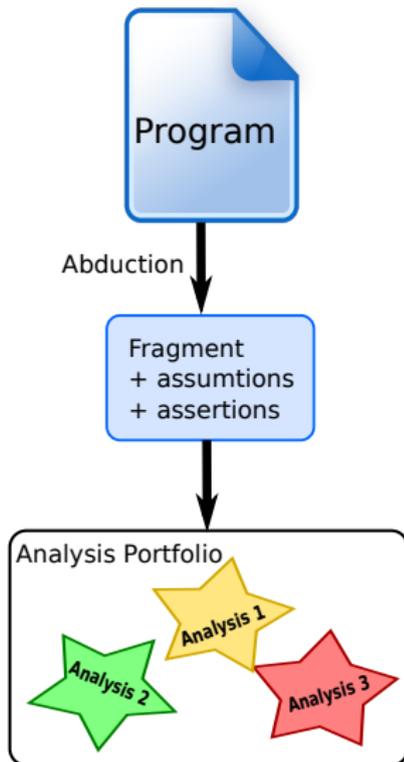
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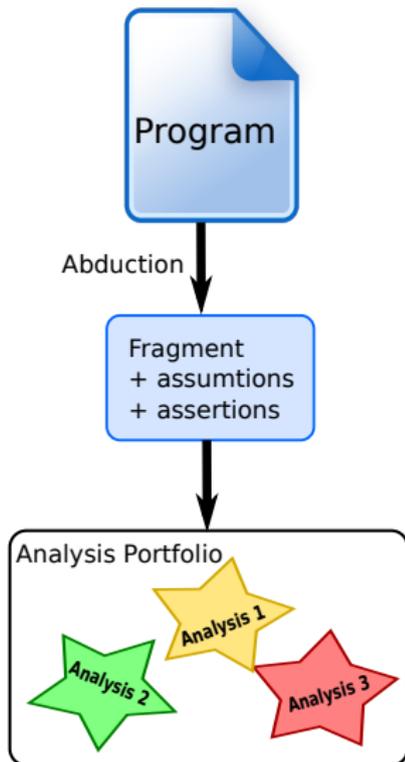
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Overview of Compositional Verification Approach

- **Key idea:** Use **abduction** to decompose proof into auxiliary lemmas
 - Lemmas are snippets annotated with assertions and assumptions
- Lemmas are discharged using portfolio of client analyses
- Combine lemmas into overall proof using **circular compositional reasoning**
 - Each lemma can assume correctness of all other lemmas



Example

- Consider this code snippet

```
int i=1, j=0;
while(*) {j++; i+=3;}
int z = i-j;
int x=0, y=0, w=0;
while(*) {
    assert(x==y);

    z+=x+y+w;
    y++;
    x+=z%2;
    w+=2;
}
```

Example

- Consider this code snippet
- Want to reason about two fragments in isolation

```
int i=1, j=0; Fragment 1  
while(*) {j++; i+=3;}  
int z = i-j;
```

```
int x=0, y=0, w=0;  
while(*) {  
    assert(x==y);  
  
    z+=x+y+w;  
    y++;  
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```

Fragment 2

Example

- Consider this code snippet
- Want to reason about two fragments in isolation
- Focus on fragment containing assertion

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int i=1, j=0;  
while(*) {j++; i+=3;}  
int z = i-j;
```

```
int x=0, y=0, w=0;  
while(*) {  
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Example

- Consider this code snippet
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- Cannot verify it yet because need precondition “z is odd”

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Example

- Consider this code snippet
- Want to reason about two fragments in isolation
- Focus on fragment containing assertion
- Cannot verify it yet because need precondition “z is odd”
- Want to automatically infer such missing assumptions!

```
int i=1, j=0;
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int x=0, y=0, w=0;
while(*) {
    assert(x==y);

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```

Example cont.

Program Decoration and VC Generation

```
int i=1, j=0;
while(*) {j++; i+=3;}
int z = i-j;
int x=0, y=0, w=0;
assume( $\phi_1$ );
while(*) {
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- **Idea:** Decorate program with assume statements containing placeholders (e.g., ϕ_1, ϕ_2)

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- Generate VCs over unknowns ϕ_1 and ϕ_2

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$$(x = 0 \wedge y = 0 \wedge w = 0 \wedge \phi_1) \Rightarrow x = y$$

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- VC 2:

$$(\phi_2 \wedge x = y) \Rightarrow wp(\sigma, x = y)$$

Example cont.

Program Decoration and VC Generation

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- VC 2: **NOT VALID**

$$(\phi_2 \wedge x = y) \Rightarrow wp(\sigma, x = y)$$

Example, cont.

Lemma Inference using Abduction

- Fix VC2 using abduction:

$$\phi_2 : (w + z)\%2 = 1$$

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- Subgoal 1:** Prove $x = y$ using $(w + z) \% 2 = 1$

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int x=0, y=0, w=0;

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  assume((w+z)%2=1);
  z+=x+y+w;
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```

Example, cont.

Lemma Inference using Abduction

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$$\phi_2 : (w + z) \% 2 = 1$$

- Now use **circular compositional reasoning**

- Subgoal 1:** Prove $x = y$ using $(w + z) \% 2 = 1$

- Subgoal 2:** Prove ϕ_2 assuming $x = y$

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Example, cont.

Lemma Inference using Abduction

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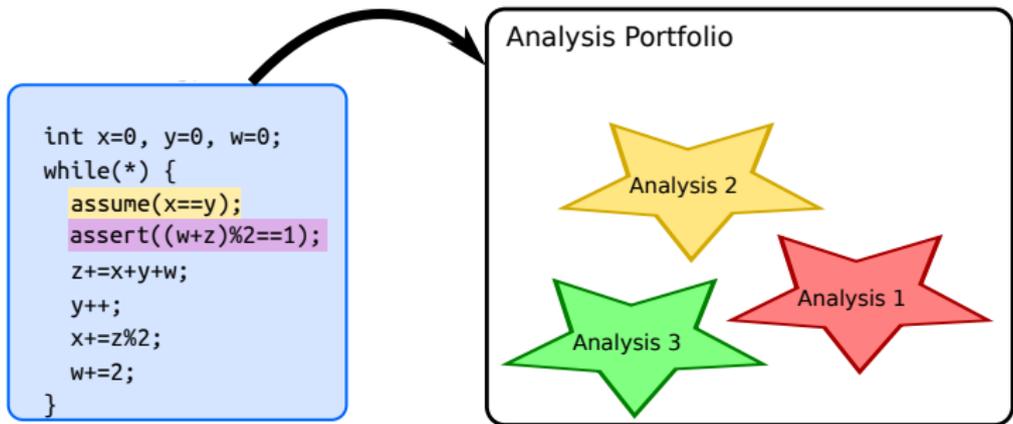
- Now use **circular compositional reasoning**

- Subgoal 1:** Prove $x = y$ using $(w + z) \% 2 = 1$
- Subgoal 2:** Prove ϕ_2 assuming $x = y$
- Subgoal 1 is immediately discharged; focus on subgoal 2

```
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while(*) {j++; i+=3;}
int z = i-j;
int x=0, y=0, w=0;

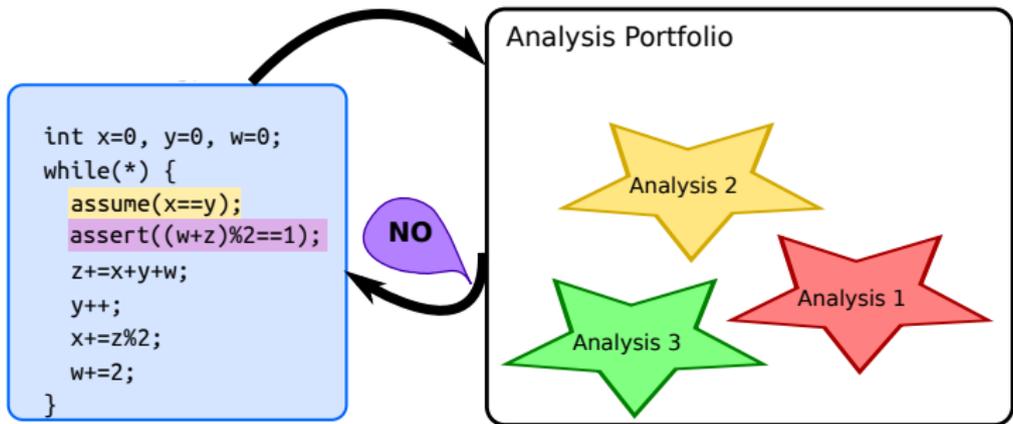
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  assume(x==y);
  assert((w+z)%2=1);
  z+=x+y+w;
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}
```

Example, cont.



- Invoke client analyses to discharge proof subgoal

Example, cont.



- Invoke client analyses to discharge proof subgoal
- No client can prove it because initial value of z unconstrained

Example, cont.

```
int i=1, j=0;
while(*) {j++; i+=3;}
int z = i-j;
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- Go back to lemma inference and annotate program with unknown precondition

Example, cont.

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- Go back to lemma inference and annotate program with unknown precondition
- Generate VC and solve for unknown ϕ_1 :

$$\phi_1 : z \% 2 = 1$$

Example, cont.

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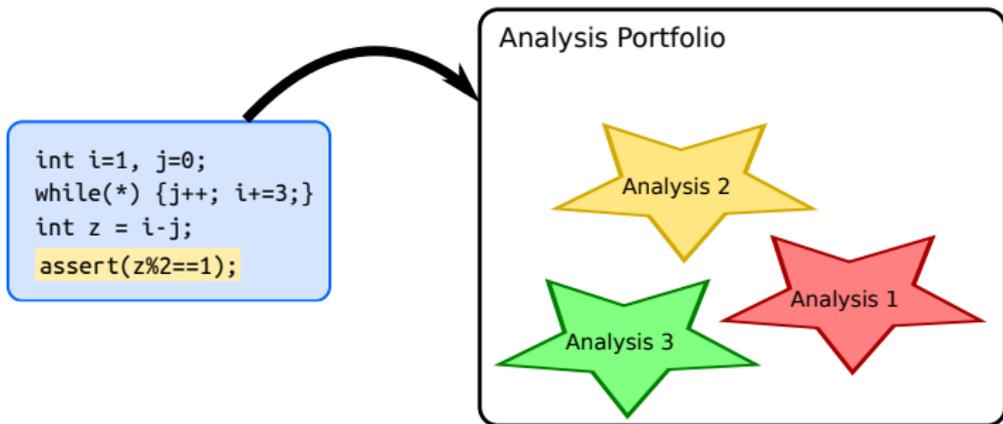
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- Now, ϕ_1 becomes a lemma (assertion) to be proven

Example, cont.

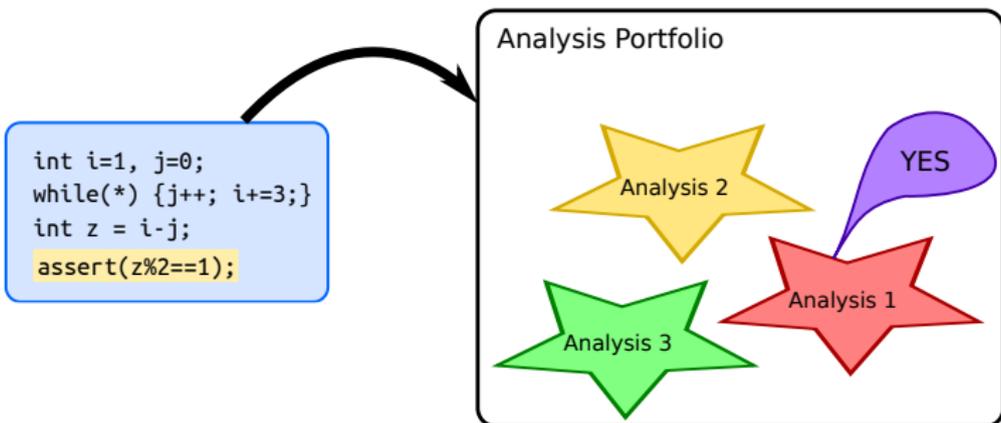
Invoking Client Analyses



- Now, annotate first fragment with assertion and invoke clients

Example, cont.

Invoking Client Analyses



- Now, annotate first fragment with assertion and invoke clients
- Can be shown by any client analysis that can establish $i = 3j + 1$

Example, cont.

Final Piece

```
int x=0, y=0, w=0;
assume(z%2==1);
while(*) {
    assume(x==y);
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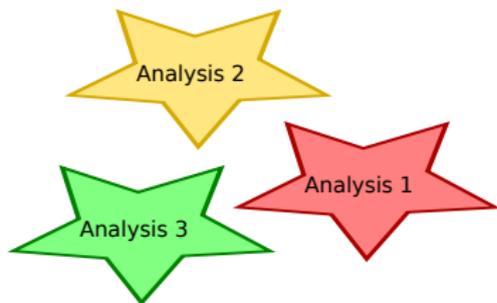
- Now, add this as assumption to second fragment

Example, cont.

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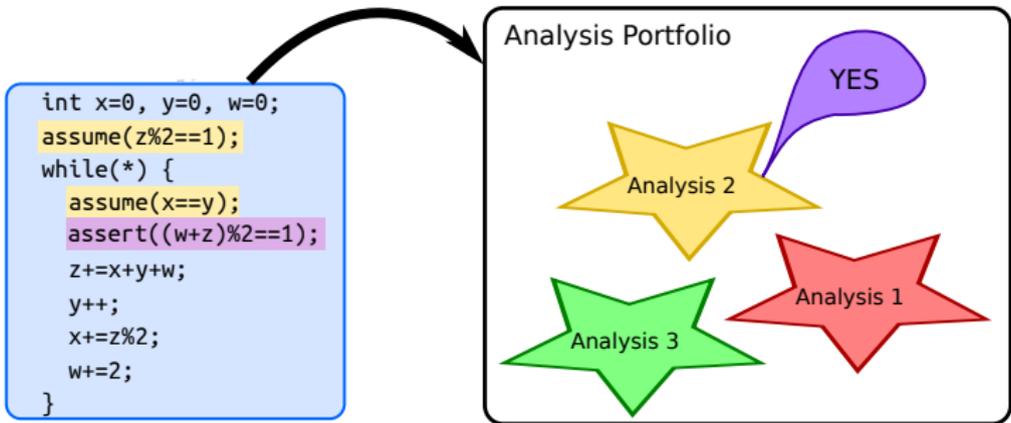
Analysis Portfolio



- Now, add this as assumption to second fragment
- Again, invoke client analyses to verify second fragment

Example, cont.

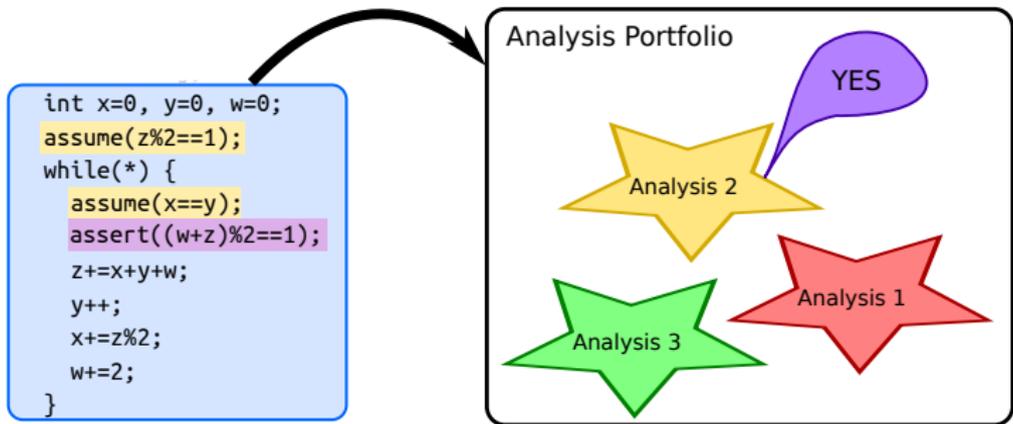
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- Now, add this as assumption to second fragment
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Example, cont.

Final Piece



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- Again, invoke client analyses to verify second fragment – can be proven using linear congruences

We have now proven the original assertion!

Essence of Technique

- Approach involves two key ingredients: **assertion elimination** and **assertion introduction**

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- Approach involves two key ingredients: **assertion elimination** and **assertion introduction**
- Assertions introduced using abductive inference
- Assertions eliminated using client analyses and circular compositional reasoning
- Similar to SMT solver – core part performs VC gen + abduction



- Client analyses similar to theory solvers

Experiments

- Used this technique to verify safety properties in C programs

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| Name | LOC | Time (s) | # queries | Avg # vars in query | Avg LOC in query |
|------------------------|------|----------|-----------|---------------------|------------------|
| Wizardpen Linux Driver | 1242 | 3.8 | 5 | 1.5 | 29 |
| OpenSSH clientloop | 1987 | 2.8 | 3 | 2.3 | 5 |
| Coreutils su | 1057 | 3.0 | 5 | 1.7 | 6 |
| GSL Histogram | 526 | 0.6 | 4 | 3.6 | 15 |
| GSL Matrix | 7233 | 16.9 | 8 | 1.8 | 7 |

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**Property can be proven using our technique,
but not using individual clients**

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Verification time reasonable (0.6-16.9s)

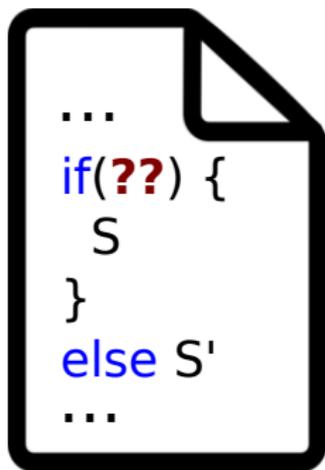
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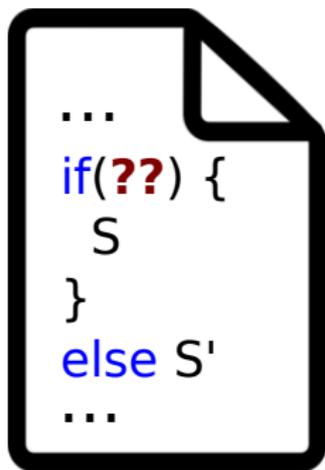
Fragments extracted for queries small in practice

Application #3: Automated Guard Synthesis



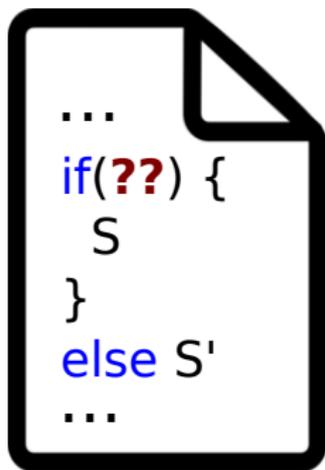
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Application #3: Automated Guard Synthesis



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- Program synthesizer completes the holes in a way that satisfies specification

Application #3: Automated Guard Synthesis



- In **program sketching**, programmer writes a draft program with “holes”
- Program synthesizer completes the holes in a way that satisfies specification
- Abduction is useful for synthesizing unknown guards in program sketches

Concrete Use Case: Memory Safety



- Programmers often write checks to prevent memory safety errors (buffer overruns, null dereferences, ...)

```
if(C) {R} else { /* handle error */}
```

Concrete Use Case: Memory Safety



- Programmers often write checks to prevent memory safety errors (buffer overruns, null dereferences, ...)

```
if(C) {R} else { /* handle error */ }
```

- Such checks are tedious to write and error-prone (e.g, off-by-one errors common cause of buffer overflows)

Guard Synthesis for Memory Safety

Key Idea: Program synthesis to guarantee memory safety

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if(???) {R} else { /* handle error */ }
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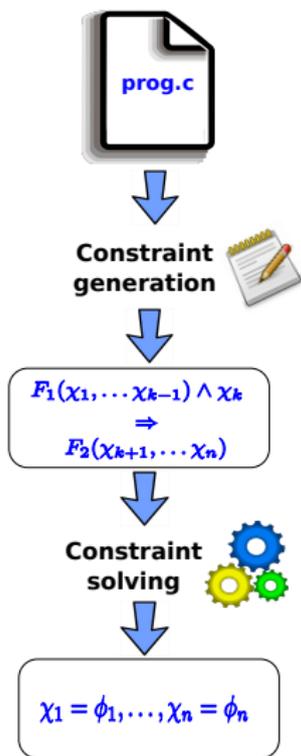
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Guard Synthesis for Memory Safety

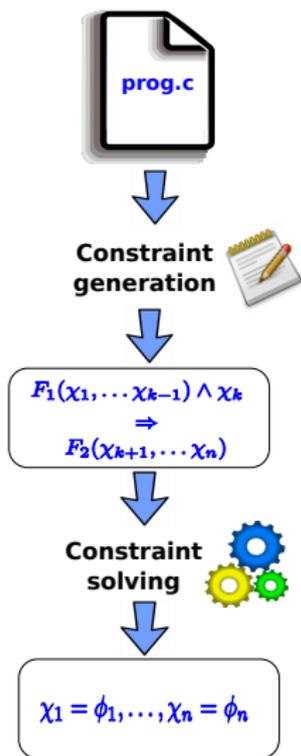
Key Idea: Program synthesis to guarantee memory safety

```
if(???) {R} else { /* handle error */ }
```

- 1 Programmer specifies **which** parts of the program should be protected and how to handle error
- 2 Technique **synthesizes** guards that guarantee memory safety
 - Guards should be as permissive and concise as possible
 - Key ingredient of synthesis algorithm is abduction

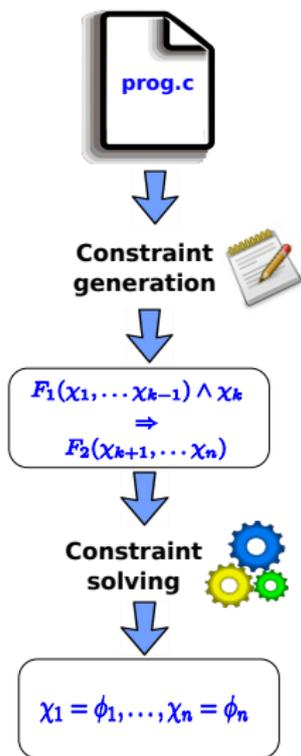


1 Constraint Generation:



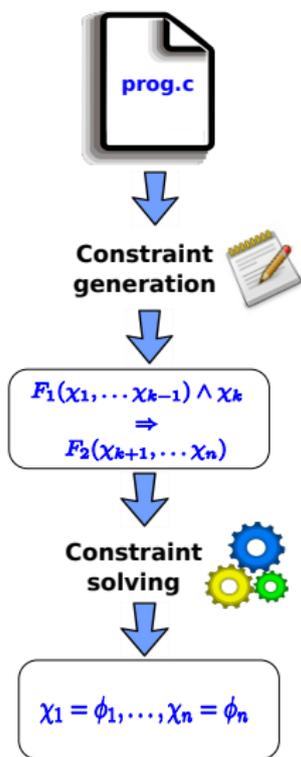
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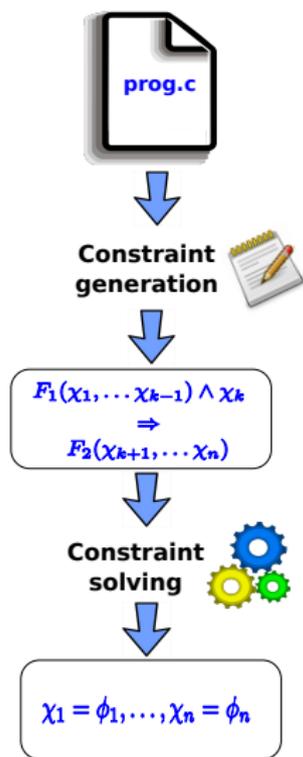
- Represent unknown guards using placeholders
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2 Constraint Solving:



1 Constraint Generation:

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2 Constraint Solving:

- An extended abduction algorithm for solving constraint system with multiple unknowns

- Generate one constraint per unknown

ϕ

```
{  
  ...  
}
```

ψ

```
if(??)  
{  
  ...  
}
```

In More Detail

- Generate one constraint per unknown
- Compute postcondition ϕ of code before unknown

ϕ

```
{  
  ...  
}
```

ψ

```
if(??)  
{  
  ...  
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```

In More Detail

- Generate one constraint per unknown
- Compute postcondition ϕ of code before unknown
- Compute safety precondition ψ of code nested inside unknown

ϕ

```
{  
  ...  
}
```

ψ

```
if(??)  
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  ...  
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```

In More Detail

ϕ

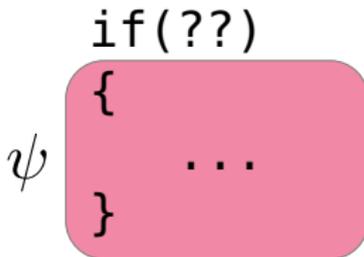
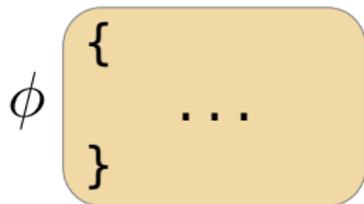
```
{  
  ...  
}
```

ψ

```
if(??)  
{  
  ...  
}
```

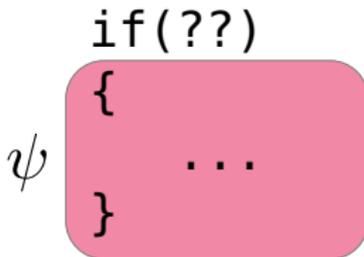
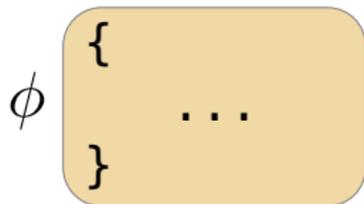
- Generate one constraint per unknown
- Compute postcondition ϕ of code before unknown
- Compute safety precondition ψ of code nested inside unknown
- To guarantee memory safety, find $??$ such that $\phi \wedge ?? \models \psi$

In More Detail



- Generate one constraint per unknown
- Compute postcondition ϕ of code before unknown
- Compute safety precondition ψ of code nested inside unknown
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- Generate one constraint per unknown
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- To guarantee memory safety, find ?? such that $\phi \wedge ?? \models \psi$
- This is almost an abduction problem, but ϕ, ψ can have other unknowns
- Impose ordering on constraints and reduce to standard abduction

Example

- Code snippet from Unix Coreutils with protected memory access

```
int main(int argc,
        char** argv)
{
    if(argc<=1) return -1;
    argv++; argc--;

    optind=0;
    while(...) {
        optind++;
        if(*) {argv++;
              argc--;}
    }
    if(??) {
        argv[optind+1]=...;
    }
}
```

Example

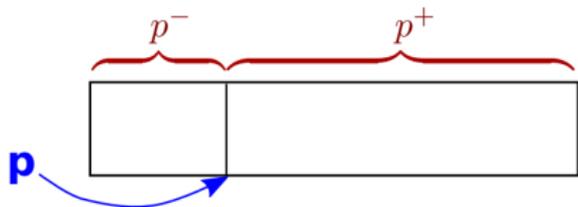
- Code snippet from Unix Coreutils with protected memory access
- **Convention:** For pointer p :

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Example

- Code snippet from Unix Coreutils with protected memory access
- **Convention:** For pointer p :
 - p^+ represents distance to end of memory block

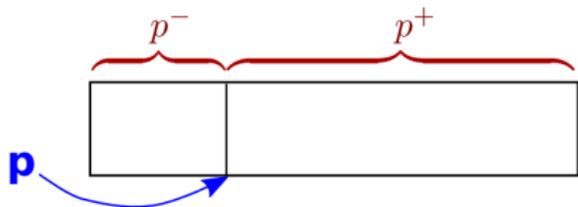


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```

Example

- Code snippet from Unix Coreutils with protected memory access
- **Convention:** For pointer p :
 - p^+ represents distance to end of memory block
 - p^- represents distance from beginning of memory block



```
int main(int argc,
         char** argv)
{
    if(argc<=1) return -1;
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    optind=0;
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```

Example Cont.

- **First Step:** Compute what is known at ?? \Rightarrow **postcondition** ϕ

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int main(int argc,
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Example Cont.

- **First Step:** Compute what is known at ?? \Rightarrow **postcondition** ϕ
 - From language semantics:

$$argv^+ = argc \wedge argv^- = 0$$

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Example Cont.

- **First Step:** Compute what is known at ?? \Rightarrow **postcondition** ϕ

- From language semantics:

$$argv^+ = argc \wedge argv^- = 0$$

- From computing the strongest postcondition:

$$argv^+ = argc \wedge \\ argv^- \geq 1 \wedge optind \geq 0$$

```
int main(int argc,
         char** argv)
{
    if(argc<=1) return -1;
    argv++; argc--;

    optind=0;
    while(...) {
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```

Example Cont.

- **Second Step:** Compute what needs to hold at ?? to ensure memory safety
⇒ precondition ψ

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int main(int argc,
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```

Example Cont.

- **Second Step:** Compute what needs to hold at ?? to ensure memory safety
⇒ precondition ψ
- Buffer access:

$$\begin{aligned}optind + 1 &< argv^+ \wedge \\optind + 1 &\geq -argv^-\end{aligned}$$

```
int main(int argc,
         char** argv)
{
    if(argc<=1) return -1;
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Example Cont.

- Solve abduction problem

$\phi \wedge ?? \models \psi$ where

$$\phi : \quad \begin{array}{l} \text{argv}^+ = \text{argc} \wedge \\ \text{argv}^- \geq 1 \wedge \text{optind} \geq 0 \end{array}$$

$$\psi : \quad \begin{array}{l} \text{optind} + 1 < \text{argv}^+ \wedge \\ \text{optind} + 1 \geq -\text{argv}^- \end{array}$$

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$$\psi : \quad \begin{array}{l} \text{optind} + 1 < \text{argv}^+ \wedge \\ \text{optind} + 1 \geq -\text{argv}^- \end{array}$$

- **Solution:** $\text{argc} - \text{optind} > 1$

```
int main(int argc,
          char** argv)
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    if(argc<=1) return -1;
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- Evaluated technique on the Unix Coreutils and parts of OpenSSH



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- Evaluated technique on the Unix Coreutils and parts of OpenSSH
- Removed conditionals used to prevent memory safety errors
- Used our new technique to synthesize the missing guards



Experiments Cont.

| Program | Lines | # holes | Time (s) | Memory | Synthesis successful? | Bug? |
|-------------------------|-------|---------|----------|--------|-----------------------|------|
| Coreutils hostname | 160 | 1 | 0.15 | 10 MB | Yes | No |
| Coreutils tee | 223 | 1 | 0.84 | 10 MB | Yes | Yes |
| Coreutils runcon | 265 | 2 | 0.81 | 12 MB | Yes | No |
| Coreutils chroot | 279 | 2 | 0.53 | 23 MB | Yes | No |
| Coreutils remove | 710 | 2 | 1.38 | 66MB | Yes | No |
| Coreutils nl | 758 | 3 | 2.07 | 80 MB | Yes | No |
| SSH - sshconnect | 810 | 3 | 1.43 | 81 MB | Yes | No |
| Coreutils mv | 929 | 4 | 2.03 | 42 MB | Yes | No |
| SSH - do_authentication | 1,904 | 4 | 3.92 | 86 MB | Yes | Yes |
| SSH - ssh_session | 2,260 | 5 | 4.35 | 81 MB | Yes | No |

Used technique to synthesize 27 unknown guards in real C programs

Experiments Cont.

| Program | Lines | # holes | Time (s) | Memory | Synthesis successful? | Bug? |
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In 21 out of 27 cases, tool inferred same predicate as programmer

Experiments Cont.

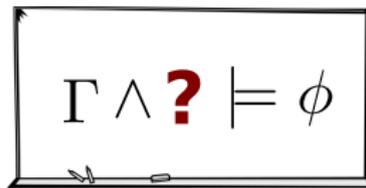
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In 4 cases, syntactically different, but semantically equivalent guards

Experiments Cont.

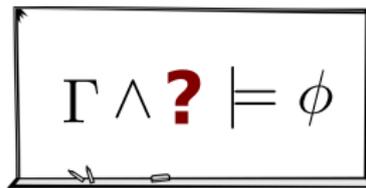
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**In 2 cases, guards did not match
⇒ bug in original program!**



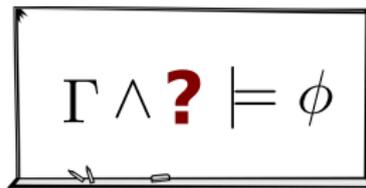
A hand-drawn whiteboard with a black border and a small eraser at the bottom. The whiteboard contains the logical expression $\Gamma \wedge ? \models \phi$. The Greek letter Γ is black, the \wedge is black, the question mark is red, the \models is black, and the ϕ is black.

- Abduction = logical formulation of “guessing”

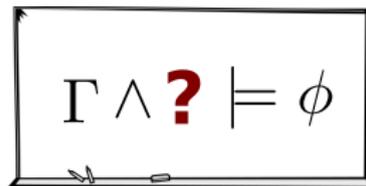


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- Abduction = logical formulation of “guessing”
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- Easy to use: `expl = conclusion.abduce(premises);`

Limitations and Future Work

- ⚠ Abduction algorithm uses quantifier elimination \Rightarrow limited scalability and requires to theories that admit QE

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 - **Future work:** Alternative algorithms that don't use QE
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 - **On-going work:** Multi-abduction algorithm to simultaneously infer multiple unknowns



Questions?