Automated Certified Hybrid System Safety Verification

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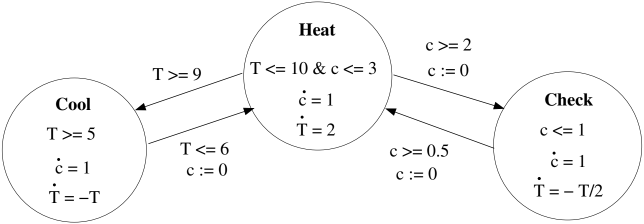
Background

- C-CoRN: Coq library of constructive mathematics
- 2000, Milad Niqui: Constructive real numbers
  - Computation impractical ☹
- 2007, Russell O'Connor: Re-implementation
  - Computation practical! ☺
- "Let's find an application that calls for certified proof-by-computation with reals!"

  $\rightarrow$ Automated hybrid system safety verification

Hybrid system: Basics

- Model of *software* interacting with *environment*
- Running example: Thermostat
- **Software**: Finite state automaton
  - Thermostat:



- **Environment**: Continuous space (typically $\mathbb{R}^n$).
  - Thermostat: $\mathbb{R}^2$ (= Temperature $\times$ clock)
- State of hybrid system: software state $\times$ environment state

Hybrid system: Behaviour

System state can change in two ways:

1. Discrete transition:
   - Instantaneous jump to different software state
   - “Guarded” by condition on environment state
2. Continuous transition (‘passage of time’):
   - Environment state (point in continuous space) changes according to *flow*
   - One flow function per location: solution to differential equations on continuous space:

$$flow : SoftState \to Duration \to Point \to Point$$

Execution “trace”: sequence of these transitions

Hybrid system: Safety

Given:

1. designated set of *initial* states;
2. designated set of *unsafe* states
   - thermostat: states with temperature < 4.5

Safety problem:

Any unsafe states reachable from initial states?

- Undecidable in general
- Manual approach: find system invariant
- Better: Do it automatically (using heuristics)!

The predicate abstraction method (Alur, 2006)

Idea:

- Partition continuous space into finite set of *regions*

  Abstract system state: software state $\times$ region
- **Compute** *abstract* discrete/continuous transitions...
- ... such that resulting graph *respects* original system:

  If $a \rightsquigarrow b$ in concrete system, then $abs(a) \rightsquigarrow abs(b)$ in abstract system
- **Compute** reachable states in abstract system
- If no unsafe ones among them, system is safe!

Alur's implementation

Alur's implementation is pragmatic:

- Nice language for hybrid system specification
- Integration with existing tools
- Modest preconditions on hybrid systems (linear flow/guards/etc)
- Sophisticated optimizations and data structures

But... does *not* produce fully verified safety proofs:

- Abstract system not provably respectful
- Uncertified implementation
- Floating point approximations of real numbers

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Our development

Our goal: *do* produce *fully verified* safety proofs.

- Formalize hybrid systems in Coq
- Reimplement abstraction method in Coq
- Keep it simple (for now)
- Different algorithm for abstract transition computation
  $\rightarrow$ to make respect provable
- Stronger preconditions on hybrid systems
- Use O'Connor's "efficient" computable reals in C-CoRN

Abstract system construction: Region partitioning

- Regions in $\mathbb{R}^n$: products of n intervals in $\mathbb{R}$
  - Thermostat: rectangles
- Interval bound selection (Alur):
  1. Start with constants occurring in guards/invariants (e.g. thermostat temperature intervals: 0, 4.5, 5, etc)
  2. Refine if safety unprovable for resulting abstract system
  3. Repeat

In our development:

- Automatic refinement not yet implemented
- For thermostat: refinement needed because constants from guards/invariants don't immediately work
- Ad-hoc solution: "right" interval bounds given by user

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Abstract system construction: Continuous transitions

Question:

Given regions A and B and flow function f, is there flow from (a point in) A to (a point in) B?

- If *no*: no abstract transition
- If *yes* (or not sure): emit transition

Alur's heuristic:

- Calculate flow at rectangle corners after $r, 2r, 3r, ..., nr$
- Use `d/dt` tool to compute convex hull overapproximation
- Determine intersections with other regions (rectangles)

Abstract system construction: Continuous transitions

We use a different approach:

- Require separability of flow functions:

$$f_s(d,(x,y)) = (f_{s,X}(d,x), f_{s,Y}(d,y))$$

- Require flow inverses: $f_{s,X}(f_{s,X}^{-1}(x,x'),x) = x'$
- Decide region-flowability by computing:
  - for each dimension, inverses between region bounds;
  - if no non-negative overlap: omit transition
  - otherwise: emit transition

Computable reals

Deciding interval overlap:

- Boils down to deciding if $a < b$ for $a, b \in \mathbb{R}$
- Or equivalently: deciding if $0 < a - b$

Can't do it for arbitrary *computable* reals!

- Can only observe arbitrarily close $\mathbb{Q}$ approximations of $a - b$

Hence, cannot *decide* overlap in general ☹

But we don't *need* full decidability!

- We only need "best effort" semi-deciders

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Best-effort semi-deciders:

- Underestimation for proposition *P*: term of type *option P*
- Naturally gives underestimators for non-overlap and flow absence

Used at higher levels, too, because abstraction method can fail:

- poor partitioning of continuous space;
- epsilon too big;
- unsafe system.

Toplevel result: *option TheSystemIsSafe*.

Local classical reasoning

In Coq's constructive logic: no PEM for arbitrary propositions ☹

But we *do* have it under double negation: $\neg\neg(P \vee \neg P)$

1. $DN\ P := \neg\neg P$ is a monad
2. For some P, $P \leftrightarrow DN\ P$
   These *stable* propositions can escape from *DN*!

So we get to use PEM when proving stable propositions ☺

In our development, we:

- introduce strategic *DN* annotations and stability req's;
- ... to make PEM (and e.g. $a < b$ decisions in $\mathbb{R}$) available in their proofs

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Conclusions

- It works: we produce *fully certified*, *formal* proofs of hybrid system safety, in acceptable time
- Nice use case for proof-by-computation-with-reals
- *Constructive* reals do complicate theory and implementation
- ... but this can be dealt with systematically:
  - “estimators” to make “tactics” without dropping to meta-level (Ltac)
  - Double negation monad

Conclusions (cont'd)

Development works, but...

- Still very much a prototype
- No nice interface for defining hybrid system
- Strong restrictions on hybrid systems...
- ... some of which require additional proofs from user (e.g. flow invertibility)
- No automatic refinement
- Less efficient than Alur's implementation

Future work

Continue work to get best of both worlds:

- Ease restrictions on hybrid systems:
  - Better heuristics that don't require flow separability
  - ODE solver instead of making user provide solution
- Nicer user interface / specification language
- Implement automatic partitioning refinement
- Make C-CoRN reals faster
- Conditional guarantees that safety can be determined
- Failure traces