Financial Development, Intellectual Property Rights, and Growth *

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** Preliminary Work - Please Do Not Distribute **

Abstract

This paper explores the interaction between long run economic growth, financial development, and intellectual property rights through a general equilibrium, over-lapping generation, infinite horizon, endogenous R&D model with heterogeneous agents. I find that a reduced collateral constraint leads to both an ability to engage more heavily in research and a reduced incentive to engage in research. I show that intellectual property rights become relatively more important as countries become more financially developed and, therefore, less collateral constrained. This has implications both on policy development and empirical research on the effects of financial development. I find that as countries develop financially it is relatively more important to consider legislating explicit intellectual property rights.

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1 Introduction

The question of what effects long run economic growth is an important one. Historically, income per capita has been primarily due to the long run growth rate of the economy instead of business cycle fluctuations. One area in which a lot of theoretical and empirical research has focused is the role that financial development plays in this long run growth rate. However, the role of collateral constraints, while being the central cost of finance in many theoretical models (e.g., Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Banerjee and Newman (1993)), has not been well explored in either the theoretical or empirical literature on long run economic growth, though it has been documented that there is significant variations in collateral requirements across countries of different levels of financial development. Further, Aghion and Howitt, among others, argue that the most reasonable way to understand economic growth is through an endogenous research and development decision at the firm level. In order to correctly incentivize a firm to engage in research there must be some level of protection of property rights, either implicit through the inability to reproduce the innovation or explicit through a patent system. As countries become more financially developed, and therefore less collateral constrained, I intend to argue that the implicit protection of intellectual property rights becomes weaker and explicit protection becomes relatively more important.

The role of financial development on long run economic growth has a long empirical and theoretical history. Goldsmith (1969) and McKinnon (1973) document that there is a close positive tie between economic development and financial development. However, it was long debated as to whether financial development caused economic growth, is simply correlated with it, or lags it. It wasn’t until Rajan and Zingales (1998) that someone was able to show convincing empirical evidence that financial development contributes to long run economic growth. Due to the difficulty of collecting the data, the role of collateralizable assets in financial development has gone unexplored until Liberti and Mian (2009) where they demonstrate a strong correlation between traditional measures of economic growth and the strictness of collateral requirements. Demirguc-Kunt and Maksimovic (1999) show that firms in financially undeveloped countries tend to have a much higher proportion of fixed assets than their counterparts in more developed countries, potentially because fixed assets are easier to collateralize. In addition, La Porta et. al. (1998) finds that financial development is correlated with the development of a strong legal framework, which is a necessary requirement for intellectual property protection. Therefore, understanding the interaction between collateral requirements and this long run growth rate seems to be important.

Further, as countries become more financially developed and the access to capital increases, a greater
number of entrepreneurs can enter the marketplace. This has implications towards the incentive to engage in R&D. As competition increases, it is reasonable to expect that the ability to protect non-exclusionary property (like intellectual property) should decrease, so there should be a reduced incentive to engage in the production of intellectual property. Therefore, explicit intellectual property rights become relatively more important.

In this paper, I develop a general equilibrium, over-lapping generation, infinite horizon, endogenous R&D model with heterogeneous agents in order to demonstrate a mechanism through which financial development can lead to greater competition and therefore, a reduced incentive to engage in R&D. My model follows the Schumpeterian notion of technological progress as creative destruction. More specifically, I follow in the line of endogenous growth through creative destruction models that was started by Aghion and Howitt (1988 and 1992). There are two necessary features of these models. First, there must be some notion of technological progress through R&D that makes the production of goods either less capital or less labor intensive. Second, there must be some barrier to entry so that firms find it profitable to engage in R&D because they are able to obtain some ex-post economic profit. I use a constraint on the amount of financing a firm can receive in order to provide a barrier to entry. The amount of financing a firm can receive is constrained by the amount of collateralizable assets the firm holds, and R&D is intangible capital intensive. I model progress in financial development as a slackening of the collateral constraint. The state of financial development is given as exogenous, but economic growth is endogenous. Furthermore, I introduce heterogeneous agents so that as the collateral constraint is slackened, competition increases.

I am interested in financially undeveloped economies, so I model technological progress as copying some frontier technology. Cohen and Levinthal (1990) and Griffith, Redding, and Van Reenen (2004) argue that the adoption of technologies developed by others is a key function of R&D. Also, Carlin and Meyer (2003) and Hall(2005) show that R&D is highly sensitive to the financial environment, so it is reasonable to look at the effect of financial development on the R&D decision of the firm. Further, Terleckyj (1980) shows that R&D intensity is positively related to productivity growth at the industry level, so using the R&D choice to drive the output of the economy is an empirically reasonable way to model this problem.

After I derive a solution for the general model, I then introduce the notion of explicit intellectual property rights. I model these intellectual property rights as the ability to sell the results of your research. They differ from the commonly held view of patents in that they do not provide any notion of rights to the downstream products that are producible from your level of technology. So, while my paper is related to the debate over patents, it is not as divisive as it might appear. Boldrin and Levine, who are well known as opponents
of intellectual property rights, acknowledge in their 2002 paper that having the right to the intellectual property you produce directly is essential to incentivize firms to engage in R&D. They only argue against downstream licensing and excessive breadth of intellectual property rights. I am not attempting to assert whether or not stronger intellectual property rights are welfare improving, in fact my results are very ambiguous on that front, but I do find that as countries become more financially developed, it becomes more important to consider strong protection of intellectual property. This result has obvious implications from a political economics perspective, implying that the need to consider explicit intellectual property rights becomes more urgent as countries develop financially.

My model implies that as countries become more financially developed we see two competing effects. The first is that companies can shift closer towards an optimal mix of tangible and intangible capital because the incentive to invest strictly in tangible capital is reduced. The second is a competing effect where the incentive to engage in research is reduced through increased competition. In most parameter specifications that I looked at, the first effect dominated, however, there are situations under which the second effect is stronger and there is an overall drop in the level of research.

My paper is related to a few different strands of literature. It is most closely related to the endogenous growth through endogenous R&D literature started by Aghion and Howitt (1988). It follows the focus of a financial constraint being the main barrier to entry as in Aghion, Dewatripont, and Rey (1999). It is most closely related to Aghion, Howitt, and Meyer-Foulkes (2005) which examines the relationship between financial development and cross-country convergence. However, there paper differs from mine in that I am chiefly concerned with the role of the negative effects of increased competition on growth. In methodology my model is most closely related to Ilyina and Samaniego (2009), but they examine the role of a financial constraint on cross sectional industry characteristics within country. My paper is also related to the strand of literature that focuses on a collateral or net worth constraint as the major cost of financing. Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Banerjee and Newman (1993), and Rampini and Viswanathan (2009) are all papers that argue that a collateral constraint is an important determinant of firm characteristics. However, my paper focuses on the long run economic growth implication of a collateral constraint instead of the firm specific capital structure. There is also an extensive intellectual property rights literature that is relevant to my paper. For those arguing for intellectual property rights there are Nordhaus (1969), Scherer (1972), Klemperer (1990), and Gilbert and Shapiro (1990). Then Boldrin and Levine (2008) argue that intellectual property rights are unnecessary and potentially economically harmful. However, all of these papers look at intellectual property rights that are either horizontally broad so that you can exclude similar
products from being produced or vertically broad so that you can exclude future products that are derived from your technology. My assumption about intellectual property rights is significantly simpler in that I model explicit intellectual property rights as the right to sell your specific production technology, but arbitrarily similar production technologies as well as innovations to the current technology are not excluded. Further, I don’t argue for the benefit or harm of intellectual property rights; I argue for the relative importance of intellectual property rights as countries financially develop.

The paper proceeds as follows. The next section presents the model. Then I present the analytical results that can be obtained from the model and the intuition for those results. The following section details an explicit specification for a numerical solution, and I present some results from the numerical solution. Next, I introduce intellectual property rights and I present the solution to the model with intellectual property rights. I then conclude.

2 Model

There are several elements that are essential to understanding the tradeoff between financial development and economic growth. First, there must be some notion of a firm being collateral constrained, otherwise, a slackening of the constraint means nothing. Second, there must be more firms entering the market or, at least, some notion of increased competition as the collateral constraint is slackened, otherwise, the results become trivial since a reduction in the constraint implies a more optimal overall level of production and R&D. Third, there must be a channel through which technological progress affects total output of the economy. Fourth, there must be some way in which agents are able to obtain ex-post economic profits so that there is an incentive to engage in research. I will show that my model possesses these features.

My model is a small open economy, general equilibrium, over-lapping generation, infinite horizon model with heterogeneous agents. While my model seems complex at first, I believe it to be at least an approximation of the simplest model that is able to address the relevant features. Throughout the next few subsections I will describe the model in detail.

2.1 Economic Agents

There is a mass 1 of heterogeneous agents indexed by \( n \in [0,1] \). The agents are heterogeneous over their intertemporal consumption preferences. I use the convention to denote the period in which the agents were born by a superscript and the period that the variable references with a subscript. All agents live for two
periods, and have the following preferences:

\[ U_t(n_t, n_{t+1}) = \max_{c_t^t, c_{t+1}^t} c_t^t + \beta_t c_{t+1}^t \]  

(1)

\( \beta_t \) is a strictly increasing continuous function of \( t \) that is bounded above by 1 and below by 0. This implies that as \( t \) increases the agent is more patient. Every agent has an initial endowment of 0, and has an endowment of 1 unit of labor in each period. As a matter of convenience, I will refer to any agent in the first period of life as young and in the second period as old.

The agents in each period can either sell their labor or choose to start a firm which requires the full input of their labor. However, due to the collateral constraint, unless an agent possesses some economic wealth, a firm will not be profitable. Therefore, in the first period all agents sell their labor since their initial endowment is 0. The consumption good is the numeraire. The agents face the following budget constraint when they are young:

\[ c_t^t + i_t^t \leq w_t \]  

(2)

where \( w_t \) is the wage rate at time \( t \), \( c_t^t \) is the consumption of agent \( t \) at time \( t \), and \( i_t^t \) is the amount that agent \( t \) chooses to invest in a firm if he starts one. When agent \( t \) is old, assuming he doesn’t choose to start a firm, he will face the following constraint:

\[ c_{t+1}^t \leq w_{t+1} \]  

(3)

and if he does start a firm,

\[ c_{t+1}^t \leq V(i_t^t, A_t^t) \]  

(4)

where \( V(i_t^t, A_t^t) \) is the realized profits from starting a firm, and \( A_t^t \) is the level of technology of that firm after engaging in R&D. Notice that there is no savings technology for the agents from the young stage of life to the old. This is without loss of generality because \( \beta_t \leq 1 \), and it is always optimal to consume as early as possible.

The agents make the choice on whether to start a firm or not based on the expected value of starting a firm. The profitableness of a firm depends on not only the initial investment but also a random draw from a technology distribution, which is determined by the previous period, and a stochastic research process. An agent who chooses to start a firm will be referred to as an entrepreneur.
The timing is as follows:

Young agents: 1. The agent is born and receives his type.
2. The agent sells his labor for the first period.
3. The agent decides whether or not to start a firm based on the expected value of starting a firm.
4. If he starts a firm:
   i. He decides how much of his own net worth to invest (and current period consumption to forego), then receives a draw from the previous period’s productivity distribution.
   ii. He chooses how much tangible/intangible capital to buy and he finances his investment.
   iii. He then sees the outcome of his research decision.
5. If he doesn’t start a firm, he eats his wage.

Old agents: 1. If he didn’t start a firm, he sells his labor and then consumes his wage.
2. If he did start a firm he produces the amount that his research outcome and investment decision allows. Then he sells his good, sells his depreciated tangible capital, and consumes.

I will describe the production decision and the evolution of the technology distribution in more detail in the following two sections.

2.2 The Final Good and the Distribution of Technology

There is a single final good that uses as an input an intermediate good and labor. The production of the final good, $Y_t$, follows

$$Y_t = A_t^* F(L_t, X_t)$$

(5)

where $F(L_t, X_t)$ is concave and increasing in both $L_t$ and $X_t$. $L_t$ is the total labor provided by both young agents and old agents who decide not to start a firm, $X_t$ is the total amount of intermediate goods used in production of the final good, and $A_t^*$ is an exogenously given frontier technology level. The final good is used for consumption and production on a one to one basis for both tangible capital and intangible capital. So, the aggregate constraint is:

$$Y_t = C_t + I_t$$

(6)

where $C_t$ is the aggregate consumption of all agents and $I_t$ is the aggregate investment of all agents who decide to start a firm. I assume that the technology for the production of the final good is available to
everyone, so the price of inputs is the equal to the marginal productivity of an additional unit of the input.

The frontier level of technology at time \( t \), \( A_t^* \) is given exogenously, and it evolves at rate \( g \), i.e. \( A_{t+1}^* = g A_t^* \).

The goal of this model is to understand how financially undeveloped countries are affected by financial development, so the way to interpret this exogenous frontier technology is that it is the level of technology endogenously produced by some unconstrained developed economy, something like the U.S., that the underdeveloped economy is imitating. In order for the intermediate good to be utilized by this frontier level of technology, it must be customized through the R&D technology of the intermediate good producer. When an agent decides to start a firm he receives a random draw from the previous period's technology distribution. The technology that the entrepreneur inherits affects the cost of engaging in R&D, with a less advanced technology draw being more costly to customize to the frontier level of technology.

Figure 1: Evolution of the Technology Distribution

<table>
<thead>
<tr>
<th>Frontier</th>
<th>( A_t^* )</th>
<th>( A_{t+1}^* )</th>
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<tbody>
<tr>
<td>One-Step Behind</td>
<td>( A_{t-1} )</td>
<td>( A_{t+1} )</td>
</tr>
<tr>
<td>Two-Steps Behind</td>
<td>( A_{t-2} )</td>
<td>( A_{t+1} )</td>
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In order to simplify my model, I assume that there are only two possible levels of technology that the entrepreneur can inherit. These are the two previous periods' frontier level of technology. The technology distribution is determined by the previous period's percentage of firms that successfully customize their intermediate goods. If a firm in the previous period that is one step behind the frontier level of technology is unsuccessful at customizing his intermediate good, that firm's technology will automatically be two steps behind the next period's technology. While if a firm was two steps behind the frontier level of technology and was unsuccessful at customizing their good, their technology would become three steps behind in the next period. I assume that their technology is "dragged" along at the two step behind level. This is equivalent to the two step behind technology being in the public domain, while the one step behind technology is exclusive.
to those who have developed it. I am going to denote a one step behind technology for an entrepreneur born in period \( t \) as \( A_{t-1} \) and a two step behind technology for the same entrepreneur as \( A_{t-2} \). However, the way to interpret this technology is \( A_{t-1} = A_t^* \), this is due to the entrepreneur producing in the period after he was born, so the frontier technology is \( A_{t+1}^* \) in the period in which he produces. This process of updating the technology distribution is illustrated graphically in Figure 1. I will denote the proportion of firms that are two steps behind the frontier level of technology at time \( t \) (before the distribution is updated through R&D) by \( \pi_t^{-2} \) and the one step behind as \( \pi_t^{-1} \). I define a measure of the economy wide level of technology as \( A_t = A_t^* \pi_t^{-1} + A_{t-1}^* \pi_t^{-2} \).

These assumptions assure several desirable characteristics of the model. First, the distribution of technologies is guaranteed to be stationary. Second, there is still dispersion in technologies, so the ability to sell a technology has both a meaning and purpose in this model. Third, it is reasonable to talk about the level of technology, which is the proportion of firms that are one step behind versus two steps behind. Fourth, it is not necessary to restrict the model in order that firms do not permanently fall behind to a point in which they never have an incentive to engage in R&D.

2.3 Production of Intermediate Goods

Intermediate goods are produced by entrepreneurs. In order for an intermediate good to be employed in the production of the final good at the frontier level of efficiency, \( A_t^* \), it must be customized. Therefore, any intermediate goods that are not customized will be worthless. When the entrepreneur chooses to start a firm, he makes a decision to invest in R&D to allow for the customization of the intermediate goods that his firm produces. R&D is a stochastic process where the probability of success depends on how much is invested in the technology. If he is successful at R&D he sells his customized intermediate good. If he is unsuccessful at R&D, he cannot sell his intermediate good. The production of intermediate goods is a function of how much tangible capital, \( k_{tn}^f \), an entrepreneur of type \( n \) and born in period \( t \) employs, and R&D is a function of intangible capital, \( k_{tn}^i \). The value function that the entrepreneur that chooses to invest \( i_n^t \) and that receives
a draw from the technology distribution of $A$ faces is as follows:

$$V_{t+1}(A, i^t_n) = \max_{b, k^t_{In}, k^t_{tn}} \left( \pi_{t+1} \left( \frac{k^t_{In}}{A^*_t + 1} \right) p_{t+1} f_{t+1} \left( \frac{k^t_{tn}}{A^*_t + 1} \right) \right) + (1 - \delta) k^t_{tn} - R b^t_n$$  \tag{7}

s.t.

$$k^t_{In} + k^t_{tn} \leq b^t_n + i^t_n$$  \tag{8}

$$R b^t_n \leq \theta (1 - \delta) k^t_{tn}$$  \tag{9}

$$\pi_{t+1} \left( \frac{k^t_{In}}{A^*_t + 1}, A \right) \in [0, 1]$$  \tag{11}

where $A$ is the technology that is inherited when the entrepreneur chooses to start a firm, $A^*_t + 1$ is the current frontier level of technology, $b^t_n$ is the amount that the entrepreneur borrows, $p_{t+1}$ is the price for which the entrepreneur can sell the intermediate good, $\pi_{t+1} \left( \frac{k^t_{In}}{A^*_t + 1}, A \right)$ is the probability of research being successful given the investment in intangible capital, $f_{t+1} \left( \frac{k^t_{tn}}{A^*_t + 1} \right)$ is the production technology for the firm, $\delta$ is the rate of depreciation, $R$ is the interest rate, and $\theta$ is a parameter that determines the amount of collateral necessary to borrow additional funds. The interest rate $R$ is determined exogenously because entrepreneurs have access to international credit markets. Since there are a continuum of firms, the market is perfectly competitive and they take the price of the intermediate good as given. Note that the time subscripts are $t + 1$; this is because an agent born in period $t$ produces in period $t + 1$.

There are several things that are worth discussing about the production and R&D technology. $\pi_{t+1}$ is increasing in $\frac{A}{A^*_t + 1}$ and increasing and concave in $k^t_{In}$, while $f_{t+1}$ is increasing and concave in $k^t_{tn}$. The inputs for both the firms production and R&D technology are scaled by the frontier level of technology. The way to interpret this is that as the production of the final good becomes more complex (i.e. as the technology frontier advances) the production of the intermediate good and the R&D process becomes proportionally more capital intensive. This is a standard assumption in the growth literature in order to ensure that all variables grow at a single constant rate in the steady state. The cost of R&D is increasing as the technology of the firm becomes farther away from the frontier. Also of note is that while both $\pi_{t+1}$ and $f_{t+1}$ are independently concave in their respective capital inputs, production can still be locally convex in $i^t_n$. However, since $\pi_{t+1}$ is bounded above by 1, the value function is guaranteed to be concave for a large enough value of $i^t_n$.

The interesting situation for the purposes of this paper is for the entrepreneurs to be collateral constrained. In order to assure this, I will assume that the function $f_{t+1}$ is such that this is always the case. Since all entrepreneurs are always collateral constrained and hence quantity constrained in output, though the market
is perfectly competitive, they are able to make an economic profit because they are in a quantity constrained Cournot equilibrium. This potential for profit is what gives incentives to the entrepreneurs to engage in R&D. Without the collateral constraint, firms would produce up to the point where all economic profits would be erased and no one would have any incentive to engage in R&D or start a firm. However, since only tangible capital is able to be collateralized, the need for financing creates a distortion in the optimal ratio of tangible and intangible capital, and this distortion becomes less as the collateral constraint is reduced by increasing $\theta$.

3 Solution of the Model in the Steady State

At this point, I am only interested in deriving the steady state solution. At some point, it may be worthwhile to examine the transition dynamics for an economy that is not in the steady state, but as the model stands, it is not conducive to an analytical treatment of a non-steady state solution.

A steady state solution will be characterized by all non-stationary variables increasing at the rate $g$, which is the rate of growth of the frontier technology. There will be a critical value of $\beta_n$ above which everyone will choose to start a firm, I will denote this value by $\tilde{\beta}_n$ and the corresponding value of $n$ as $\bar{n}$. Also, I will indicate the optimal level of investment for agent $n$ conditional on starting a firm as $i^*_n$. The model cannot be fully solved analytically in this form, but I can completely characterize the solution with a system of equations. For a proof of the following proposition see the appendix.

**Proposition 1.** If in the steady state equilibrium, $\pi_{t+1}\left(\frac{k_{t+1}^n}{A_{t+1}}, A\right) \in (0, 1)$ and (10) binds for all entrepreneurs, then the steady state equilibria is characterized by the solution to the following system of equations:
\[ k_{tn}^* = \frac{(i_{tn}^{*} - k_{tn}^{l})R}{R - \theta(1 - \delta)} \]  

(12)

\[ 0 = \frac{d}{dk_{tn}^{l}} \left[ \pi_{t+1} \left( k_{tn}^{l} \frac{A_{t+1}}{A_{t+1}^*} \right) p_{t+1} f_{t+1} \left( \frac{k_{tn}^{l}}{A_{t+1}^*} \right) + (1 - \theta)(1 - \delta)k_{tn}^{l} \right] \]  

(13)

\[ \pi_{t+1}^{-1} = \pi_{t+1}^{-1} \int_{\bar{n}}^{1} \pi_{t+1} \left( k_{tn}^{l} \frac{A_{t+1}^*}{A_{t+1}^*} \right) dn + \pi_{t+1}^{-2} \int_{\bar{n}}^{1} \pi_{t+1} \left( k_{tn}^{l} \frac{A_{t+1}^*}{A_{t+1}^*} \right) dn \]  

(14)

\[ \pi_{t+1}^{-1} = 1 - \pi_{t+1}^{-2} \]  

(15)

\[ \pi_{t+1}^{-1} = \pi_{t+1}^{-1} \]  

(16)

\[ w_t + \bar{\beta} \gamma w_t = w_t - i_{tn}^{*} + \bar{\beta} \gamma (gw_t)E_t V(A, i_{tn}^{*}) \]  

(17)

\[ w_t = \frac{\partial}{\partial L_t} A_t^* F(L_t, X_t) \]  

(18)

\[ p_t = \frac{\partial}{\partial X_t} A_t^* F(L_t, X_t) \]  

(19)

\[ L_t = 1 + \bar{n} \]  

(20)

If \( V_t(A, \bar{i}) \) is concave in \( i \), then for all \( n \in [\bar{n}, 1] \) if \( i_{tn}^{*} \in [0, w_t) \)

\[ \frac{d}{di_{tn}^{*}} E_t V(A, i_{tn}^{*}) = 1/\bar{\beta} \]  

(21)

and if \( \frac{d}{di_{tn}^{*}} E_t V(A, w_t) > 1/\bar{\beta} \), then \( i_{tn}^{*} = w_t \).

If \( V_t(A, \bar{i}) \) is locally convex in \( i \), then for all \( n \in [\bar{n}, 1] \)

\[ i_{tn}^{*} = w_t \]  

(22)

Obviously, with this being a steady state solution all time subscripts can be dropped. However, since this is an over-lapping generation model, I think understanding is impeded when time subscripts are dropped. Very little can be said about the characteristics of the solution from this general specification. The following corollary provides one general result. Again, the proof is in the appendix.

**Corollary 1.** If \( \frac{\partial^2}{\partial L_t \partial X_t} Y_t \geq 0 \), then \( p_t(\theta) \) is decreasing in \( \theta \), i.e. \( \frac{d}{d\theta} p_t(\theta) \leq 0 \).

While the result from Corollary 1 is not surprising, it is not immediately obvious. As the collateral constraint is reduced two things can happen: either profits for entrepreneurs can increase due to increased access to collateral, or profits can decrease due to increased competition reducing profits to the extent that
it overcomes the positive effect of the reduced collateral constraint. However, Corollary 1 indicates that in both situations, the price must decrease.

4 Numerical Solution and Results

4.1 Assumptions

Due to the intractability of a general solution, I have made the following specific assumptions about the model:

\[ F(L_t, X_t) = -(L_t)^2 + \alpha L_t - (X_t)^2 + \gamma X_t \]  \hspace{1cm} (23)

\[ f_t \left( \frac{k_{t+n}^k}{A_{t+1}^*} \right) = \frac{k_{t+n}^k}{A_{t+1}^*} \]  \hspace{1cm} (24)

\[ \pi_{t+1} \left( \frac{k_{t+n}^l A}{A_{t+1}^* A_{t+1}^*} \right) = \begin{cases} \frac{A}{A_{t+1}^* A_{t+1}^*} & \text{if } k_{t+n}^l \leq \frac{(A_{t+1}^*)^2}{A} \\
1 & \text{if } k_{t+n}^l \geq \frac{(A_{t+1}^*)^2}{A} \end{cases} \]  \hspace{1cm} (25)

\[ \beta_n = n \]  \hspace{1cm} (26)

Notice that in (23) I define the final good production function to be quadratic in both arguments and separable. The advantage of a quadratic utility function is that (in addition to having a separable specification) first derivatives are linear and second derivatives are constant. However, I am only interested in the portion of the production function for which the first derivative (and hence the price of the input) is positive. This is ensured by setting \( \alpha \) and \( \gamma \) high enough such that for \( \theta \in [0, 1] \), \( L_t \leq \alpha \) and \( X_t \leq \gamma \). The separability does not affect the solution to the model in any qualitatively significant manner, but it reduces both the number and nonlinearity of the equations that must be solved numerically.

Also, (23) and (24) seem at first glance to be nonstandard. Notice that both (23) and (24) satisfy the properties described in section 2.3: both functions are concave in their respective inputs, but they are not strictly concave everywhere. Further, they imply that the value function is no longer concave, so (22) holds. This specific assumption about the production and R&D implies that the value function is locally quadratic in investment. However, the function is not globally convex, because once \( k_{t+n}^l \geq \frac{(A_{t+1}^*)^2}{A} \), where \( A \) is the level of technology that the entrepreneur inherits, \( \pi_{t+1} \left( \frac{k_{t+n}^l}{A_{t+1}^* A_{t+1}^*} \right) = 1 \), and the value function becomes linear in investment. Since the interesting region for this model is the region where the entrepreneur must make a choice between R&D and production, for the numerical solution, I will parameterize the model such that
Given these assumptions about the model, the set of equations (12)-(22) reduces to a significantly more tractable set of two polynomials which when solved for \( w_t \) and \( p_t \), are fourth order in \( p_t \) and third order in \( w_t \). I am able to solve this set of equations for a unique solution. I can then use the values of \( w_t \) and \( p_t \) to solve for the remaining variables.

For the parameters I make the following assumptions: \( \delta = .1, \ g = 1.1, \ R = 1.05, \ \alpha = 2, \) and \( \gamma = 4. \) I choose all of the parameters except for \( \alpha \) and \( \gamma \) to be roughly consistent with the standard values in the literature. \( \alpha \) and \( \gamma \) are chosen to ensure that the level of production is increasing as the inputs increase. Figure 2 plots both the amount of intermediate goods and labor employed in the production of the final good. Notice that \( X_t < 4 \) and \( L_t < 2 \), so the quadratic production function is still increasing throughout the relevant domain. While I only report results from this parameter specification, these results are robust to changes in the parameter specifications with the stipulation that the production function is increasing throughout the domain.

4.2 Numerical Solution

In this section, I will present the numerical results from the model and provide an interpretation of my findings. I normalize all non-stationary variables by the frontier level of technology \( A_t^* \), which implies that they become stationary. In Figure 3, I plot the economy wide level of technology. There are several interesting features about this solution. First, the level of technology is a concave function of \( \theta \). The concavity implies that an incremental increase in the ability to borrow funds has a less positive effect on the aggregate level of
technology as the economy becomes more financially developed. This is unsurprising when R&D technology is strictly concave because the marginal return to an extra unit of intangible capital is decreasing. However, by (25), the R&D technology is linear in intangible capital (up to a point, which my parameterization is everywhere below). Therefore, the marginal benefit of employing an extra unit of intangible capital for R&D remains constant, but the marginal benefit of an extra unit of research decreases. This concavity is strictly due to an increase in competition. This can be seen from Figure 2, because the number of firms at time $t$ is equal to $2 - L_t$. Until approximately $\theta = .75$, competition is increasing in level of financial development. Then a shift occurs, and the price of the intermediate good starts to fall too rapidly, and there is a reduced incentive to start a firm or engage in R&D with a slackened financial constraint because competition has increased too much.

One sees the same pattern in the expected profit from starting a firm, Figure 4. The expected profit from starting a firm should increase approximately quadratically if there is no change in price given that the value function is quadratic in investment. For low values of $\theta$, this appears to be roughly true. However, $\theta$ reaches a critical level past which the competition, both within industry and through the expansion of the number of firms, starts increasing at too fast of a rate.
The decline in price is concave, which can be seen in Figure 5. This is unsurprising given that the production technology for intermediate goods is roughly quadratic in investment and the production for the final goods is quadratic, so even given that there is no increase in competition we would expect to see a concave price function. However, the third derivative of price with respect to \( \theta \) (not shown) is also negative until competition starts decreasing, then it becomes positive. This is due to the additional downward pressure that increased competition puts on price.

So far, this model has not made any statements about the effect of strictness in the collateral requirement on aggregate welfare. Unfortunately, this model is not well equipped to address such questions since we have over-lapping generations of a continuum of heterogeneous agents. However, I think that the welfare implications are important, so I will define a measure of aggregate welfare by the equal weighted lifetime utilities in a single generation. Denoting welfare at time \( t \) by \( W_t \):

\[
W_t = \int_0^1 U_t^n(c_{nt}^t, c_{nt+1}^t) \, dn
\]  

(27)

Figure 6 plots the level of aggregate welfare versus \( \theta \). Notice that Figure 6 looks very similar to Figure 4.
Figure 5: Price - $p_t$

Figure 6: Aggregate Welfare - $W_t$
This is because the vast majority of agents in this parameter specification choose to start a firm (at most only 10% of agents choose to sell their labor in the second period). Since entrepreneurs invest the entirety of their first period wages in the firm, their lifetime utility is the discounted value of starting a firm. However, Figure 6 indicates that my model does show given some parameter specifications financial development may have a negative effect on overall welfare by encouraging an excessive degree of competition.

5 Intellectual Property Rights

As was shown by Figure 3 and Figure 6, as the collateral constraint is slackened the relative benefit of that increased slack is not only relatively less beneficial, but without the necessary property rights, it may be harmful. The question, logically, then becomes, if we reduce competition through increased intellectual property rights, can we increase overall welfare? While my model is unable to assign either a strictly positive or negative value to the change in level of aggregate welfare due to explicit intellectual property protection, I can describe the predicted effects of strengthening intellectual property rights though incorporating a notion of patents into the existing framework.

The way I model patents is relatively non-standard. Previous work in patents, such as Nordhaus (1969) and Gilbert and Shapiro (1990), have focused on the depth or breadth of a patent. That is, their work has focused on the optimal duration or breadth of horizontal product differentiation in order to best induce R&D. I look at a much simpler formulation. I model patents as the ability to sell your level of technology to the next generation. There is no notion of explicit firm exclusion that is necessary to talk about the breadth of a patent, and neither is their any exclusion from building off the current level of technology because a two period lived agent does not care about the next periods R&D choice. However, their is implicit firm exclusion, so competition will be decreased. To see why this is, I will formally introduce the notion of patents as a modification to the entrepreneurs problem:
$V_{t+1}(A, i^t_n) = \max_{b_k, k_{Tn}, k_t} \pi_{t+1} \left( \frac{k_{Tn}}{A_t^{*+1}}, \frac{A}{A_t^{*+1}} \right) \left( pt+1 f_{t+1} \left( \frac{k_{Tn}}{A_t^{*+1}} \right) + P_{t+1} \right) + (1 - \delta) k_{Tn} - Rb_t$ \quad (28)

s.t.

\begin{align*}
&k_{Tn} + k_t \leq b_t + i^n_t \quad \text{(29)}

&Rb_t \leq \theta (1 - \delta) k_{Tn} \quad \text{(30)}

&\pi_{t+1} \left( \frac{k_{Tn}}{A_t^{*+1}}, A \right) \in [0, 1] \quad \text{(32)}
\end{align*}

$P_{t+1}$ is the price of selling a frontier technology level to the next generation. It’s important to remember that possessing a frontier level of technology when the entrepreneur chooses to start a firm does not mean that he does not have to engage in R&D to produce in the next period, because he will still be one step behind if he doesn’t engage in R&D. However, R&D will be cheaper for him. Therefore, all entrepreneurs possess a strictly positive value for inheriting the higher level of technology, and all property rights will be sold for a strictly positive value. For a strictly concave value function in investment, determining an equilibrium value for $P_{t+1}$ is difficult since everyone agent will place a different value on the technology, and the ones who value it most highly will buy it at the price of the second highest valuation (since agents are continuously heterogeneous, the second highest valuation will be their own valuation). Therefore, I will restrict my analysis to the case of a convex value function in investment.

**Proposition 2.** If $V_{t+1}(A, i^t_n)$ is convex in $i^t_n$, in equilibrium $V_{t+1}(A_{t-2}^t, i_{t-1}^t - P_t) = V_{t+1}(A_{t-2}^t, i_{t-1}^t)$.\)

What Proposition 6 states is that, all benefits from having a higher state of technology go to the entrepreneur who engaged in the R&D project. Without the ability to sell the intellectual property rights to the young generation, the young generation was free riding on the technology distribution that was passed down to them by the old generation. Now, ex-ante everyone expects to earn the amount that someone with a low draw from the technology distribution would. The way I model intellectual property rights does reduce the level of competition in the economy, and it does this through a reduced ex-ante profit from starting a firm. Therefore, less patient agents (those with a lower $n$) will be less willing to become entrepreneurs, and there will be fewer entrepreneurs starting firms. Therefore, relative to the situation in which there are no patents, the profit from starting a firm with a two step behind technology will be strictly greater for any given level of investment when there are patents.

However, there is a competing effect which defies a simple analysis of the relative levels of variables for the
two cases. As the incentive to start a firm decreases, the number of agents who decide to sell their labor in
the second period becomes greater, which drives down the equilibrium wage rate. Since with a convex value
function in investment, all of the young agent’s wages are invested in the firm, the individual, as well as the
aggregate, level of investment in the production of intermediate goods goes down. However, this decrease in
the production of the intermediate good is partially offset by the increase in labor used in the production of
the final good.

5.1 Numerical Solution with Intellectual Property Rights

Figure 7: Difference in Economy Wide Level of Technology - $A_t$ w/ patents - $A_t$ w/out patents

I assume that (23)-(26) hold as before. Therefore, the value function is concave and the above discussion
holds. The shape of the numerical solution is nearly qualitatively identical to the model without patents, so it
is instead more informative to plot the difference between the steady state solution with patents and without
patents. I will always use this convention, so a positive value means the solution with patents is larger than
the solution without patents. In Figure 7, I see that the introduction of patents leads to a decrease in the
economy wide level of technology, but the difference in levels of technology becomes less as the economy
becomes more financially developed. The level of the difference in technology is dependent upon the specific

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parameterization, and I can find parameterizations for which the difference is positive for all values of $\theta$. However, the upward slope is a general feature of this model. The reason is the same as the reason for the level of technology of the economy as a whole declining. As $\theta$, and therefore competition, increases, the incentive to engage in R&D decreases. Since patents further incentivizes R&D, the patent system becomes relatively more important to the aggregate level of technology.

The level of technology is not the same as the welfare of the economy, and a welfare statement is significantly stronger. I calculate welfare according to (27), and I plot the differences in Figure 8. Like before, the number of agents who choose not to start a firm is small, so aggregate welfare is dominated by the profitability of starting firms. There are two distinct regions in the figure. First, until approximately $\theta = .32$, the level of aggregate welfare with patents is falling relative to the case without patents. When $\theta$ is low, and it’s difficult to start a firm it is beneficial to free ride off of the R&D of the previous generation, and as $\theta$ increases, the R&D effort of the previous generation increases. However, at a certain point, the decreased competition starts to become relatively more important and welfare starts to increase with $\theta$. Again, this welfare result is limited to a specific weighting function for welfare. This definition of aggregate welfare is equivalent to aggregate wealth with the second period wealth discounted by the appropriate discount factor,
this is equivalent to the economic goals of many developing economies. This metric doesn’t include a penalty for the inequality of wealth, and intellectual property rights will concentrate wealth in the hands of successful entrepreneurs, so the policy implications for a country that is concerned both with the level of wealth and the inequality among citizens are not as clear.

As stated previously, this definition of intellectual property rights is not able to speak to the optimal depth or breadth of the rights. Further work is needed to determine how intellectual property rights should be optimally structured during the evolution of a developing economy. However, it is clear that the importance of intellectual property rights should increase as a country develops financially.

6 Conclusion

I am able to construct a model that, for a wide set of parameter values, shows that a weak form of explicit intellectual property protection becomes relatively more important to consider as the level of financial development, proxied by the strictness of a collateral constraint, increases. While my model is very specific in its specifications, and I currently do not have analytical results for the generality of my findings, my result seems to be robust to a fairly wide set of parameter values. My model does not have the power to say whether or not intellectual property rights are beneficial, that must be determined on a case-by-case basis. It can only speak to the relative importance of explicit intellectual property rights. For some specifications, I find that they are beneficial in terms of overall welfare, but not beneficial for the aggregate level of technology. The general result is that intellectual property rights become relatively more beneficial as countries become more financially developed.

If financial development is only optimally beneficial in the presence of a corresponding development in intellectual property rights, it has implications both in policy and empirical research. The policy implication is straightforward: as a country develops financially, the importance for policy makers to be discussing and considering legislating explicit intellectual property rights increases. For empirical research on financial development, it may be important to consider the level of intellectual property rights. There is a documented correlation between financial development and intellectual property rights, which is consistent with my model, but it is an open question as to whether that correlation is due to the policy makers acknowledgement of the increased importance of intellectual property rights with increased levels of financial development or is due to some underlying factor, such as a better developed legal system.

Future work will consist of modifying the model such that it is more conducive to a general analytical
solution, as well as modifying the definition of aggregate welfare to take into account a preference for a more equal wealth distribution. As of now, intellectual property rights are modeled in a very simplistic binary manner, but the observed intellectual property rights are significantly more complex allowing for a degree of protection in both a vertical and horizontal sense. More work must be done to determine which of these dimensions is more important for a developing economy. In addition, empirical work must be done to determine the relative magnitude of the effect. A reduced form analysis of this effect would be unlikely to establish causation, so a modification of the model presented here in order to be able to structurally estimate the effect would be beneficial.
Appendix

**Proof of Proposition 1.** Notice that in the entrepreneurs problem, (9) always binds since tangible capital has a purely positive value, debt is costly, and investment in the firm is costly through being forced to consume later. Further, I assume that (10) binds since it is the interesting constraint in this model. Therefore, (12) holds by combining (9) and (10), solving for tangible capital. Also, (13) holds by the first order condition for the entrepreneurs problem after substituting in (10).

The assumption of a steady state imposes (16), and (14) is the average, weighted by the previous periods technology distribution, of the firms that are successful at R&D. (15) is a consequence of there only being two technology states, and (15) and (16) implies that $\pi_{t+1} = \pi_t$.

(17) is the point at which an entrepreneur is indifferent between choosing to start a firm or sell his labor in the second period. Notice that all agents of type greater than $\bar{n}$ will choose to start a firm since the value function is strictly increasing in investment, investment is at least weakly increasing in $n$ by (21) and (22), and $\beta_n$ is increasing in $n$. (18) and (19) hold since the production technology for the final good is public knowledge. (20) holds since all young agents sell their labor and any old agents of type less than $\bar{n}$ will also sell their labor. (21) and (22) are solutions to the entrepreneurs investment problem.

**Proof of Corollary 1.** Assume that this is not true, i.e. there exists a $\theta$ such that $\frac{\partial}{\partial \theta} p_t(\theta) > 0$. There are two possibilities: either profit is weakly increasing in $\theta$ or it’s strictly decreasing in $\theta$. If profit is weakly increasing in $\theta$, then as $\theta$ is decreased, the reduced financial constraint allows for a strictly larger production of intermediate goods by each individual firm, and the expected profit from starting a firm is increasing. Therefore, by Proposition 1, labor supply is weakly decreasing. Since $\frac{\partial^2}{\partial X_t \partial X_t} Y_t \geq 0$, by Proposition 1

$p_t = \frac{\partial}{\partial X_t} A_t^* F(L_t, X_t), \text{ and } Y_t \text{ is concave in } X_t, p_t \text{ must decrease, a contradiction.}$

Now, assume that the profit for a firm is strictly decreasing. Since the original production amount of intermediate goods by each firm is still an option to the entrepreneurs, $p_t$ must decrease for firm profit to decrease, a contradiction.

Therefore, $\frac{\partial}{\partial \theta} p_t(\theta) \leq 0$

**Proof of Proposition.** Since the value function is assumed to convex in investment, by Proposition 1, all entrepreneurs find it optimal to invest the entirety of their wages in starting a firm. Therefore, all entrepreneurs value the different levels of technology equally. So, competition will drive the price of the high technology to the point where entrepreneurs are indifferent between the two levels of technology.
References


