In traditional proof systems, everything that is inferred, is logically implied by the premises.

\[
\begin{align*}
    C \lor I & \quad \neg I \lor D \quad \text{(res)} \\
    \hline 
    C \lor D \\
\end{align*}
\]

\[
\begin{align*}
    A & \quad A \rightarrow B \quad \text{(mp)} \\
    \hline 
    B \\
\end{align*}
\]
Traditional Proofs vs. Interference-Based Proofs

- In traditional proof systems, everything that is inferred, is logically implied by the premises.

\[
\frac{C \lor l}{C \lor D} \quad \frac{\neg l \lor D}{C \lor D} \quad \frac{A}{A \rightarrow B} \quad \frac{A}{B} \quad \text{(res)} \quad \text{(mp)}
\]

- Inference rules reason about the presence of facts.
  - If certain premises are present, infer the conclusion.
Traditional Proofs vs. Interference-Based Proofs

- In traditional proof systems, everything that is inferred, is logically implied by the premises.

\[
\frac{C \lor I}{C \lor D} \quad \frac{\neg I \lor D}{C \lor D} \quad (\text{res}) \quad \frac{A}{B} \quad (\text{mp})
\]

- Inference rules reason about the presence of facts.
  - If certain premises are present, infer the conclusion.

- Different approach: Allow not only implied conclusions.
  - Require only that the addition of facts preserves satisfiability.
  - Reason also about the absence of facts.
  - This leads to interference-based proof systems.
Interference-Based Proof Systems

- **Interference-based proof systems** generalize traditional proof systems.

- **An interference-based proof** is a sequence of clauses.
  
  - **Idea**: Clauses are added to the formula or deleted from it step-by-step.
  
  - Added clauses need not be implied, but their addition must preserve satisfiability:
    
    - If the formula is satisfiable, then the formula obtained by adding a clause is also satisfiable.
    
    - If the (unsatisfiable) empty clause, $\bot$, can be added, then the original formula must be unsatisfiable.
      
      - The empty clause is unsatisfiable because it has no literal that could be true.
Proof Checking whether modifications preserve satisfiability should be efficient. Clauses that can be added or removed are called redundant. Idea: Allow only modifications that fulfill an efficiently checkable redundancy criterion. Idea: Showing satisfiability equivalence with the empty formula allows proving satisfiability.
Interference-Based Proofs

Checking whether modifications preserve satisfiability should be efficient. Clauses that can be added or removed are called redundant. Idea: Allow only modifications that fulfill an efficiently checkable redundancy criterion. Idea: Showing satisfiability equivalence with the empty formula allows proving satisfiability.
Interference-Based Proofs

Formula

\[
\begin{align*}
1 & \equiv 1 \\
2 & \equiv 2 \\
3 & \equiv 3 \\
4 & \equiv 4
\end{align*}
\]

\[
\begin{align*}
5 & \equiv 5
\end{align*}
\]

Proof

\[
\begin{align*}
6 & \perp \\
\perp
\end{align*}
\]
Interference-Based Proofs

Proof Checking whether modifications preserve satisfiability should be efficient. Clauses that can be added or removed are called redundant. Idea: Allow only modifications that fulfill an efficiently checkable redundancy criterion. Idea: Showing satisfiability equivalence with the empty formula allows proving satisfiability.

Proof
Interference-Based Proofs

Proof

Checking whether modifications preserve satisfiability should be efficient. Clauses that can be added or removed are called redundant.

Idea: Allow only modifications that fulfill an efficiently checkable redundancy criterion.

Idea: Showing satisfiability equivalence with the empty formula allows proving satisfiability.
Interference-Based Proofs

Checking whether modifications preserve satisfiability should be efficient.

Clauses that can be added or removed are called redundant.

> Idea: Allow only modifications that fulfill an efficiently checkable redundancy criterion.

> Idea: Showing satisfiability equivalence with the empty formula allows proving satisfiability.
The interference-based proof system DRAT is the de-facto standard in SAT solving.

- DRAT is based on the addition of RAT clauses and on deletion.

QRAT, the extension of DRAT for the satisfiability problem of quantified Boolean formulas (QSAT), is very powerful.

- It is the only QBF proof system that can succinctly express virtually all preprocessing techniques.

At CADE we present new interference-based proof systems for SAT that allow for short proofs without new variables.
Almost all proof systems reason only about the presence of premises.

What prevents us, in particular theoreticians, from reasoning about their absence?

The best known interference-based system, extended resolution, allows exponentially smaller proofs compared to resolution.

Other interference rules such as blocked-literal addition in QSAT also facilitate short proofs for hard problems.
Deletion of clauses can be a powerful technique.

In SAT, clause deletion provides efficiency.

In QSAT, it provides a way to prove satisfiability.

**What can clause deletion offer in first-order logic?**
The Potential of Interference-Based Proof Systems

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August 6, 2017