Announcements

The mean grade in the midterm was 73. I expected it to be about 83. I conclude that Midterm 1 was too long.

I will curve the grades of Midterm 1 so that the mean is 83.
I will announce the curving mechanism on Thursday. For the moment, just understand that the posted grades are raw scores and they will be adjusted upwards.
T1: \[(\text{equal} \ (\text{rev} \ (\text{app} \ x \ y)) \ \ (\text{app} \ (\text{rev} \ y) \ (\text{rev} \ x)))\]

T2: \[(\text{iff} \ (\text{true-listp} \ (\text{app} \ x \ y)) \ \ (\text{true-listp} \ y))\]

T3: \[(\text{implies} \ (\text{true-listp} \ x) \ \ (\text{equal} \ (\text{rev} \ (\text{rev} \ x)) \ x))\]
T1: \((\text{rev} \ (\text{app} \ x \ y)) = (\text{app} \ (\text{rev} \ y) \ (\text{rev} \ x))\)

T2: \((\text{true-listp} \ (\text{app} \ x \ y)) \leftrightarrow (\text{true-listp} \ y)\)

T3: \((\text{true-listp} \ x) \rightarrow (\text{rev} \ (\text{rev} \ x)) = x\)
In some of your classes, professors will introduce notation of their own. For example, they might say, “if \( x \) and \( y \) are sequences then \( x \diamond y \) denotes the concatenation of \( x \) followed by \( y \), and \( \overline{x} \) denotes the reverse of \( x \).” They might also say “Let \( S \) be the set of all sequences” and assume implicitly that sequences are true-lists. Given such conventions, “\( x \in S \)” means “\( x \) is an element of the set \( S \)” or “(true-listp \( x \)).”
T1: $x \Diamond y = \overline{y} \Diamond \overline{x}$.

T2: $(x \Diamond y) \in S \iff (y \in S)$

T3: $x \in S \rightarrow \overline{x} = x$

However it is written, you should understand the logical meaning of these sentences to be:
T1: \((\text{rev } (\text{app } x \ y)) = (\text{app } (\text{rev } y) \ (\text{rev } x))\)

T2: \((\text{true-listp } (\text{app } x \ y)) \leftrightarrow (\text{true-listp } y)\)

T3: \((\text{true-listp } x) \rightarrow (\text{rev } (\text{rev } x)) = x\)
Theorem:
(implies (and (true-listp a) (true-listp b))
 (true-listp (rev (app (rev a) b)))))
(implies (and (true-listp a) (true-listp b))
  (true-listp (app (rev b)
                  (rev (rev a))))))
(implies (and (true-listp a) (true-listp b))
  (true-listp (rev (rev a)))))
(implies (and (true-listp a) (true-listp b))
  (true-listp a))
(implies (and (true-listp a) (true-listp b)) t)
The Rules of Inference are precisely described in Section 4.4.

The reason they’re described precisely is so you can learn to do proofs without making mistakes.

I don’t care if you learn the “implementation” of the rules. Who cares what $\pi$ is in your steps? I don’t!

But you must learn how to use the rules flawlessly and naturally.
Theorem:

\( (\text{implies} \ (\text{and} \ (\text{true-listp} \ a) \ (\text{true-listp} \ b)) \ \ (\text{true-listp} \ (\text{rev} \ (\text{app} \ (\text{rev} \ a) \ b)))) \)
Transformation 1 (Rewrite: Steps 1 and 2):
(implies (and (true-listp a) (true-listp b))
  (true-listp (rev (app (rev a) b))))
↑ π

Rewrite at π
Transformation 1 (Rewrite: Steps 1 and 2):
(implies (and (true-listp a) (true-listp b))
 (true-listp (rev (app (rev a) b))))

Rewrite at $\pi$ with
T1: (equal (rev (app x y))
 (app (rev y)
 (rev x)))
Transformation 1 (Rewrite: Steps 3 and 4):
(implies (and (true-listp a) (true-listp b))
  (true-listp (rev (app (rev a) b))))

Rewrite at $\pi$ with
T1: (implies t
  (equal (rev (app x y)) ; $\phi_h$
    (app (rev y) ; $\alpha =$
      (rev x)))
  (rev x)))

$eqv = equal$

$\sigma = \{ x \leftarrow (rev a), y \leftarrow b \}$
Transformation 1 (Rewrite: Steps 5 and 6):
(implies (and (true-listp a) (true-listp b))
    (true-listp (rev (app (rev a) b)))))

Rewrite at $\pi$ with
T1: (implies t
    (equal (rev (app x y)) ; $\alpha$
        (app (rev y) ; $\beta$
            (rev x))))

$eqv = equal$

$\sigma = \{x \leftarrow (rev\ a), y \leftarrow b\}$

Prove: ((true-listp a) $\land$ (true-listp b)) $\rightarrow$ t
Transformation 1 (Rewrite: Step 7):
\[
(\text{implies} \ (\text{and} \ (\text{true-listp} \ a) \ (\text{true-listp} \ b)) \ \\
(\text{true-listp} \ (\text{rev} \ \text{(app} \ (\text{rev} \ a) \ b)))
\]

Rewrite at $\pi$ with

T1: (implies t  ; $\phi_h$

(\text{equal} \ (\text{rev} \ \text{(app} \ x \ y))  ; $\alpha$

(\text{app} \ (\text{rev} \ y)  ; $\beta$

(\text{rev} \ x))))

$eqv = equal$

$\sigma = \{x \leftarrow (\text{rev} \ a), y \leftarrow b\}$

$\beta/\sigma = (\text{app} \ (\text{rev} \ b)  \\
(\text{rev} \ (\text{rev} \ a)))$
Transformation 1 (Rewrite: Step 7):
(implies (and (true-listp a) (true-listp b))
  (true-listp (app (rev b)
                (rev (rev a))))))

Rewrite at $\pi$ with
T1: (implies t ; $\phi_h$
           (equal (rev (app x y)) ; $\alpha$
                     (app (rev y) ; $\beta$
                     (rev x)))))

eqv = equal
$\sigma=${$x \leftarrow (\text{rev } a)$, $y \leftarrow b$}
$\beta/\sigma = (\text{app } (\text{rev } b)$
                 (rev (rev a)))
Transformation 2:
(implies (and (true-listp a) (true-listp b))
   (true-listp (app (rev b)
                  (rev (rev a))))))
Transformation 2 (Rewrite: Steps 1, 2, 3, 4, 5, 6):

\[(\text{implies (and (true-listp } a \text{) (true-listp } b))\]
\[
(\text{true-listp (app (rev } b)\]
\[
(\text{true-listp (rev (rev } a))\])\]

Rewrite with

T2: \[(\text{iff (true-listp (app } x \ y)) \; \alpha \leftrightarrow \]
\[
(\text{true-listp } y)) \; \beta\]

\[eqv = \text{iff}\]

\[\sigma = \{x \leftarrow (\text{rev } b), \; y \leftarrow (\text{rev (rev } a))\}\]

\[\beta/\sigma = (\text{true-listp (rev (rev } a))\]

Prove \((\text{true-listp } a) \land (\text{true-listp } b) \rightarrow t\)
Transformation 2 (Rewrite: Steps 1,2,3,4,5,6):
(implies (and (true-listp a) (true-listp b))
  (true-listp (rev (rev a))))

Rewrite with
T2: (iff (true-listp (app x y)) ; α ←
  (true-listp y)) ; β

equiv = iff
σ = {x←(rev b), y←(rev (rev a))}
β/σ = (true-listp (rev (rev a)))
Prove (true-listp a) ∧ (true-listp b) → t
Transformation 3:
(implies (and (true-listp a) (true-listp b))
  (true-listp (rev (rev a))))
Transformation 3:
(implies (and (true-listp a) (true-listp b))
  (true-listp (rev (rev a)))))
Transformation 3 (Rewrite: Steps 1,2,3,4,5):  
(implies (and (true-listp a) (true-listp b))  
  (true-listp (rev (rev a))))

Rewrite with
T3:(implies(true-listp x)  ; \phi_h
    (equal (rev (rev x)) x)); \alpha = \beta

eqv=equal
\sigma={x \leftarrow a}
\beta/\sigma=a
\phi_h/\sigma = (true-listp a)
Transformation 3 (Rewrite: Steps 1,2,3,4,5):
(implies (and (true-listp a) (true-listp b))
  (true-listp (rev (rev a)))))

Rewrite with
T3: (implies (true-listp x) ; \( \phi_h \)
  (equal (rev (rev x)) x)); \( \alpha = \beta \)

eqv = equal
\( \sigma = \{x \leftarrow a\} \)
\( \beta / \sigma = a \)
\( \phi_h / \sigma = (true-listp a) \)
Prove \((true-listp a) \land (true-listp b) \rightarrow (true-listp a)\)
Transformation 3 (Rewrite: Steps 1,2,3,4,5):
(implies (and (true-listp a) (true-listp b))
 (true-listp a))

Rewrite with
T3:(implies(true-listp x) ; \phi_h
 (equal (rev (rev x)) x)); \alpha = \beta

eqv=equal
\sigma=\{x \leftarrow a\}
\beta/\sigma=a
\phi_h/\sigma = (true-listp a)
Prove (true-listp a) \land (true-listp b)
 \rightarrow (true-listp a)
Transformation 4:
(implies (and (true-listp a) (true-listp b))
  (true-listp a))
Transformation 4:
(implies (and (true-listp a) ; $\alpha \leftrightarrow \beta$ ($\beta = t$
 (true-listp b))
 (true-listp a)) ; $\alpha$

Use Hyp 1, $\delta = (\text{true-listp } a)$
$\alpha = (\text{true-listp } a), \beta = t, eqv = iff$
Transformation 4:
(implies (and (true-listp a) ; $\alpha \leftrightarrow \beta \ (\beta = t$
               (true-listp b)))
           t) ; $\beta$

Use Hyp 1, $\delta = (\text{true-listp } a)$
$\alpha = (\text{true-listp } a), \beta = t, eqv = iff$
Transformation 5:
(implies (and (true-listp a)
               (true-listp b))
          t)

Taut: (implies p t)
\(\sigma = \{ p \leftarrow (\text{and } (\text{true-listp a}) (\text{true-listp b}))\}\)
Transformation 6:

\[ t \]
Thm \((\text{implies } p \ t)\)
Proof:
\((\text{implies } p \ t)\)
\[= \{\text{rewrite with def implies}\}\]
\((\text{if } p \ (\text{if } t \ t \ \text{nil}) \ t)\)

Case 1: \(p=\text{nil}\)
\((\text{if } p \ (\text{if } t \ t \ \text{nil}) \ t)\)
\[= \{\text{by hyp 1}\}\]
\((\text{if } \text{nil} \ (\text{if } t \ t \ \text{nil}) \ t)\)
\[= \{\text{by comp}\}\]
\(t\)
Case 2: $p \neq \text{nil}$ ($p \leftrightarrow t$)

$(\text{if } p (\text{if } t t \text{ nil}) t)$

$= \{\text{by hyp}\}$

$(\text{if } t (\text{if } t t \text{ nil}) t)$

$= \{\text{by comp}\}$

$t$

Q.E.D.