CS 378 - Game Technology
Linear Algebra Question Set

The following questions are presented for you to work on your own. There may be similar questions in the midterm and the final. You do not have to provide the numerical answer to these questions: For example, if the answer is a solution to a linear system ($Ax = b$), you can just say given A and b, we obtain the solution by XXX. Make sure the answer is the solution to your linear system.

1) Show that the determinant of the following matrix is 0.

$$\begin{bmatrix} sin(a) & cos(a) & sin(a+d) \\ sin(b) & cos(b) & sin(b+d) \\ sin(c) & cos(c) & sin(c+d) \end{bmatrix}$$

2) Compute the projection of $(2,1)$ onto the line $y = x$ in 2D.

3) Compute the projection of $(2,1,2)$ onto the plane $x+y+2z-1=0$ in 3D.

4) Compute the intersection of the sphere $x^2+y^2+z^2+9=0$ with the parametrized line $(1+t, 0.5t, 2t)$ (for any value of t, this equation gives you a point on the line).

5) You are writing a 3D shooter. The characters in your game are represented with their position and orientation in a global coordinate frame. Every character is modeled as a sphere with known radius for collision purposes. Lets say the gravity is $(0,-10,0)$ and the ground plane is represented with $y-5=0$. What would be the 3D position of a character at (a,b,c) with radius d be ?. What if the ground plane was $x+5y+2z-25=0$? (Project the character onto the plane along the gravity vector such that the character's distance to the plane is d).

6) Given the following 4 points: $(2,0),(2,5),(4,5),(4,0)$, apply the following homogeneous transformations from right to left (i.e., represent these points with homogeneous column vectors).

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -5 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Is this transformation invertible ?

7) Given the following 4 points: $(2,0),(2,5),(4,5),(4,0)$, compute the 3×3 homogeneous transformation matrix T that would transform them to $(1,1),(6,1),(6,-1),(1,-1)$. Again represent these 2D points as column vectors.

8) You have a triangle in 2D with following vertex-color pairs (float RGB [0,1]):

(0,0) - (1,0,0)
(0,1) - (0,0,1)
(1,1) - (0,1,0)

Use linear interpolation to find the color at (0.5,0.5), at (0.25,0.75) and at (0.67, 0.33)?

9) Your 3D game character is located at $(5,1,3)$ in the world coordinate system. It is facing the direction $(1,0,0)$ (along the positive x axis) and the up direction is $(0,1,0)$. The height of the character is 2 units (you can assume the character is a sphere with radius 2). You would like to place a camera and setup a perspective projection such that the character's projection is looking to the left and the character entirely fits into the image but it also not too small. Describe the steps you would take compute the world to camera transformation matrix and the camera's perspective projection matrix.
(Hint: Represent the camera as $r = P\times T\times x$ where x is a point in the world coordinate system, T is world to camera transformation matrix, P is the camera perspective projection matrix and r is the screen coordinates. Derive T and P by thinking about what positions (x) should project to what locations in the image (r).)

Can you do the same construction if I gave you the direction that the character is facing (and the up vector) as arbitrary 3D vectors ?