(Problem 5) Refer to the Background Story 1.3.2.

Frankly, John F.'s only really interested in hands of five cards which have exactly probability $\frac{1}{2}$.

Rither implement the mess that John F. uses or show, by example, that there might be moreclever ways toaccomlish this. (I don't recommend the implementation idea.)

Specifically, find the shortest reasonable description of a type of hand involving five randomly dealt cards (no replacement) which occurs with probability exactly $\frac{1}{2}$. Demonstrate that your answer is correct.

One of the best choosing criteria can be: more red cards than black cards.

Both proofs I will propose will show that the number of hands in which the red cards are more than the black card are the same as the opposite (more black than red). From that it is immediate to see that the probability is $\frac{1}{2}$.

Method 1.

I will call $N(r)$ the number of hands such that the number of red cards is exactly r.

$$N(r) = P(26, r)\ P(26, 5 - r) = N(5 - r)$$

So

$$N(3) + N(4) + N(5) = N(2) + N(1) + N(0)$$

Method 2.

Let A be any hand with more red than black cards. If we change the suit of each card with the following substitutions: clubs $\leftrightarrow$ diamonds, spades $\leftrightarrow$ hearts, we obtain a hand B where there are more black than red cards. Analogally from any hand with more black than red cards, with the same substitution we obtain a hand with more red than black cards.

This shows us that there is a 1-to-1 correspondence between hands with more red than black cards and hands with more black than red cards. Consequently the number of hands with more red than black cards is the same of the number of hands with more black than red cards.