(Problem 8) Let S be the set of all functions which take a natural number as input and produce a boolean as output.

Use a diagonalization argument to prove that S is, in fact, uncountable.

Suppose S is countable. Then there is a 1-to-1 correspondence between the elements of S and the natural numbers. Suppose that that correspondence is expressed such that the first element of S is f_1, the second f_2, and so on. So S is $\{f_1,\ f_2,\ \ldots\}$.

Now I define a function from naturals to booleans F such that $F(n) = \neg f_n(n)$.. This function is different from any f_i, because $F(i) \neq f_i(i)$, but F is an element of S as S contains all functions from naturals to booleans. This means that F has not been "counted", and then that S in uncountable.