(Problem 9) Let $\mathcal{M}$ be the set of all Turing machines computable functions which take a natural number as input and produce a boolean as output. Interestingly, $\mathcal{M}$ is easily shown to be a countable set.

What goes wrong then you attempt to use a diagonalization argument to show that $\mathcal{M}$ is uncountable.

(Refer to problem 8) Everythink works well until we have that to consider if F is in $\mathcal{M}$. We cannot say if F is an element of $\mathcal{M}$.

(Problem 10) Can you fix the argument so it provides another result, perhaps a statement that some task is not possible? You may sketch a proof here, skipping over the tedious details (there are many, many such details).

A result is that not all functions from naturals to booleans are computable. In fact, if all of them were computable, we would be able to say that $F \in \mathcal{M}$, and then that $\mathcal{M}$ is not countable, that is impossible.