CS344M
Autonomous Multiagent Systems

Patrick MacAlpine

Department of Computer Science
The University of Texas at Austin
Good Afternoon, Colleagues

Are there any questions?
Good Afternoon, Colleagues

Are there any questions?

- How can we apply game theory to RoboCup?
- Examples of game theory that aren’t modeled as a matrix?
- What about irrational agents?
- Pure vs mixed strategy?
Logistics

- Progress reports due in 2 weeks
Logistics

- Progress reports due in 2 weeks
Game Theory Premises

• Simultaneous actions: (mutual exclusivity?)
Game Theory Premises

- Simultaneous actions: (mutual exclusivity?)
- No communication
- Outcome depends on combination of actions
- Utility (payoff) encapsulates everything about preferences over outcomes
Solution Concepts

- Dominant strategy
- Nash equilibrium
- Pareto optimality
- Maximum social welfare
- Maximin strategy
Prisoner’s Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
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<tbody>
<tr>
<td></td>
<td>C(1)</td>
<td>D(2)</td>
</tr>
<tr>
<td>C(1)</td>
<td>3,3</td>
<td>0,5</td>
</tr>
<tr>
<td>D(2)</td>
<td>5,0</td>
<td>1,1</td>
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</table>

Patrick MacAlpine
# Chicken

<table>
<thead>
<tr>
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<tr>
<td>C(1)</td>
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<thead>
<tr>
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<tbody>
<tr>
<td>C(1)</td>
<td>1,5</td>
</tr>
<tr>
<td>D(2)</td>
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</table>
Bach/Stravinsky

- My wife and I agree to meet at a concert
Bach/Stravinsky

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- Unfortunately, there are 2: Bach and Stravinsky
Bach/Stravinsky

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  - If not, so distraught we don’t care what we’re listening to
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- Propose a payoff matrix
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<tr>
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<th>B</th>
</tr>
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<tbody>
<tr>
<td>Wife</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>Me</td>
<td>0,0</td>
<td>1,2</td>
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Nash Equilibrium

- Does every game have a pure strategy Nash equilibrium?
Matching Pennies

- We each put a penny down covered
- If they match, I win, if they don’t, you win
Matching Pennies

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<tr>
<td>H</td>
<td>1,−1</td>
<td>−1,1</td>
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Player 1

Player 2
Matching Pennies

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Nash equilibrium?
Nash Equilibrium

- Every game has at least one Nash equilibrium
Nash Equilibrium

- Every game has at least one Nash equilibrium
  - Nobel prize and academy award!
Nash Equilibrium

• Every game has at least one Nash equilibrium
  – Nobel prize and academy award!

• Not known if complexity of finding one is NP-complete or in P
Some theory

- Prove that if each player plays a dominant strategy, the result is a Nash equilibrium
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- Are all Nash equilibria the result of playing dominant strategies?
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• Is the outcome of a Nash equilibrium necessarily Pareto optimal?

• Is a Pareto optimal outcome necessarily the result of Nash equilibrium strategies?

• Is the maximum social welfare outcome necessarily Pareto optimal?
Some theory

• Prove that if each player plays a dominant strategy, the result is a Nash equilibrium.
• Are all Nash equilibria the result of playing dominant strategies?
• Is the outcome of a Nash equilibrium necessarily Pareto optimal?
• Is a Pareto optimal outcome necessarily the result of Nash equilibrium strategies?
• Is the maximum social welfare outcome necessarily Pareto optimal?
• If both players play maximin, is it necessarily a Nash equilibrium?
Mixed strategy equilibrium

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- What if player 2 picks action 1 3/4 of the time?
Mixed strategy equilibrium

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- What if player 2 picks action 1 3/4 of the time? 1 = 3.5, 2 = 4.5
## Mixed strategy equilibrium

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- What if player 2 picks action 1 1/4 of the time?
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- What if player 2 picks action 1 3/4 of the time? 1 = 3.5, 2 = 4.5
- What if player 2 picks action 1 1/4 of the time? 1 = 2.5, 2 = 1.5
• Player 1 must be indifferent between actions 1 and 2
- Player 1 must be indifferent between actions 1 and $24q+2-2q = 6q$
● Player 1 must be indifferent between actions 1 and 24q+2-2q = 6q
● Player 2 must be indifferent between actions 1 and 2
• Player 1 must be indifferent between actions 1 and 2
  \[ 24q + 2 - 2q = 6q \]
• Player 2 must be indifferent between actions 1 and 2
  \[ 28p + 2 - 2p = 8 - 8p \]
• Player 1 must be indifferent between actions 1 and 2
  \[ 24q + 2 - 2q = 6q \]
• Player 2 must be indifferent between actions 1 and 2
  \[ 28p + 2 - 2p = 8 - 8p \]
Rock/Paper/Scissors

- Nash equilibrium?
Rock/Paper/Scissors

- Nash equilibrium?
- Why is anything else **not** an equilibrium?
Correlated Equilibria

Sometimes mixing isn’t enough: Bach/Stravinsky

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<tr>
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<td>S 2,1 B 0,0</td>
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Want only S,S or B,B - 50% each