

**Introduction to Parsing**  
(adapted from CS 164 at Berkeley)

**Outline**

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- Parser overview
- Context-free grammars (CFG's)
- Derivations
- Syntax-Directed Translation

**The Functionality of the Parser**

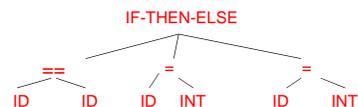
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- **Input:** sequence of tokens from lexer
- **Output:** abstract syntax tree of the program
- **One-pass compiler:** directly generate assembly code
  - This is what you will do in the first assignment
  - Bali → SaM code

**Example**

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- Pyth: `if x == y: z = 1`  
`else: z = 2`
- Parser input: `IF ID == ID : ID = INT ELSE : ID = INT`
- Parser output (*abstract syntax tree*):



### Why A Tree?

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- Each stage of the compiler has two purposes:
  - Detect and filter out some class of errors
  - Compute some new information or translate the representation of the program to make things easier for later stages
- Recursive structure of tree suits recursive structure of language definition
- With tree, later stages can easily find "the else clause", e.g., rather than having to scan through tokens to find it.

### Notation for Programming Languages

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- Grammars:
$$\begin{aligned} E &\rightarrow \text{int} \\ E &\rightarrow E + E \\ E &\rightarrow E * E \\ E &\rightarrow ( E ) \end{aligned}$$
- We can view these rules as rewrite rules
  - We start with E and replace occurrences of E with some right-hand side
- $E \rightarrow E * E \rightarrow ( E ) * E \rightarrow ( E + E ) * E \rightarrow \dots$   
 $\rightarrow (\text{int} + \text{int}) * \text{int}$

### Context-Free Grammars

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- A CFG consists of
  - A set of *non-terminals*  $N$ 
    - By convention, written with capital letter in these notes
  - A set of *terminals*  $T$ 
    - By convention, either lower case names or punctuation
  - A *start symbol*  $S$  (a non-terminal)
  - A set of *productions*
- Assuming  $E \in N$ 
$$\begin{aligned} E &\rightarrow \varepsilon && \text{, or} \\ E &\rightarrow Y_1 Y_2 \dots Y_n && \text{where } Y_i \in N \cup T \end{aligned}$$

### Examples of CFGs

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Simple arithmetic expressions:

- $$\begin{aligned} E &\rightarrow \text{int} \\ E &\rightarrow E + E \\ E &\rightarrow E * E \\ E &\rightarrow ( E ) \end{aligned}$$
- One non-terminal: E
  - Several terminals: int, +, \*, (, )
    - Called terminals because they are never replaced
  - By convention the non-terminal for the first production is the start one

### Key Idea

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1. Begin with a string consisting of the start symbol
2. Replace any *non-terminal*  $X$  in the string by a right-hand side of some production
$$X \rightarrow Y_1 \dots Y_n$$
3. Repeat (2) until there are only terminals in the string
4. The successive strings created in this way are called *sentential forms*.

### The Language of a CFG (Cont.)

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Write

$$X_1 \dots X_n \rightarrow^* Y_1 \dots Y_m$$

if

$$X_1 \dots X_n \rightarrow \dots \rightarrow Y_1 \dots Y_m$$

in 0 or more steps

### The Language of a CFG

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Let  $\mathcal{G}$  be a context-free grammar with start symbol  $S$ . Then the language of  $\mathcal{G}$  is:

$$L(\mathcal{G}) = \{ a_1 \dots a_n \mid S \rightarrow^* a_1 \dots a_n \text{ and every } a_i \text{ is a terminal} \}$$

### Examples:

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- $S \rightarrow 0$  also written as  $S \rightarrow 0 \mid 1$   
 $S \rightarrow 1$   
Generates the language { "0", "1" }
- What about  $S \rightarrow 1 A$   
 $A \rightarrow 0 \mid 1$
- What about  $S \rightarrow 1 A$   
 $A \rightarrow 0 \mid 1 A$
- What about  $S \rightarrow \epsilon \mid ( S )$

## Derivations and Parse Trees

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- A *derivation* is a sequence of sentential forms resulting from the application of a sequence of productions

$S \rightarrow \dots \rightarrow \dots$

- *Parse tree*: summary of derivation w/o specifying completely the order in which rules were applied
  - Start symbol is the tree's root
  - For a production  $X \rightarrow Y_1 \dots Y_n$  add children  $Y_1, \dots, Y_n$  to node  $X$

## Derivation Example

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- Grammar  
 $E \rightarrow E + E \mid E * E \mid (E) \mid \text{int}$
- String  
 $\text{int} * \text{int} + \text{int}$

## Derivation in Detail (1)

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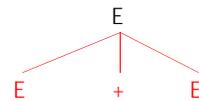
$E$

$E$

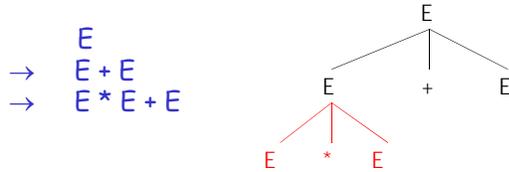
## Derivation in Detail (2)

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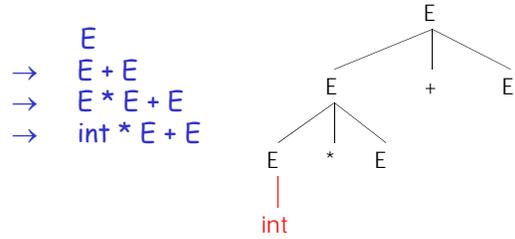
$\rightarrow \begin{array}{l} E \\ E + E \end{array}$



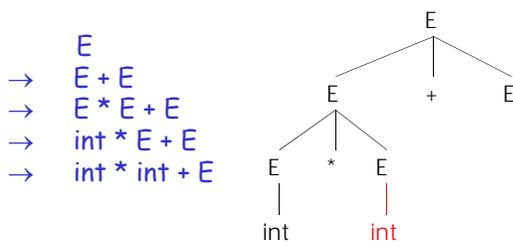
### Derivation in Detail (3)



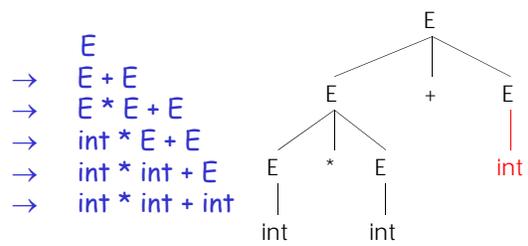
### Derivation in Detail (4)



### Derivation in Detail (5)



### Derivation in Detail (6)



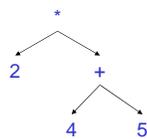
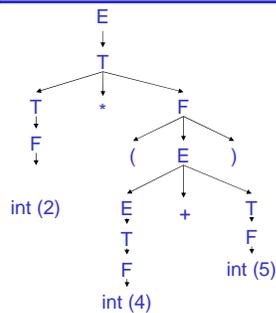
### Notes on Derivations

- A parse tree has
  - Terminals at the leaves
  - Non-terminals at the interior nodes
- A left-right traversal of the leaves is the original input
- The parse tree shows the association of operations, the input string does not !
  - There may be multiple ways to match the input
  - Derivations (and parse trees) choose one

### AST vs. Parse Tree

- AST is condensed form of a parse tree
  - operators appear at *internal* nodes, not at leaves.
  - "Chains" of single productions are collapsed.
  - Lists are "flattened".
  - Syntactic details are omitted
    - e.g., parentheses, commas, semi-colons
- AST is a better structure for later compiler stages
  - omits details having to do with the source language,
  - only contains information about the *essential* structure of the program.

### Example: 2 \* (4 + 5) Parse tree vs. AST



### Summary of Derivations

- We are not just interested in whether  $s \in L(G)$
- Also need derivation (or parse tree) and AST.
- Parse trees slavishly reflect the grammar.
- Abstract syntax trees abstract from the grammar, cutting out detail that interferes with later stages.
- A derivation defines a parse tree
  - But one parse tree may have many derivations
- Derivations drive translation (to ASTs, etc.)
- Leftmost and rightmost derivations most important in parser implementation

## Ambiguity

- Grammar

$$E \rightarrow E + E \mid E * E \mid (E) \mid \text{int}$$

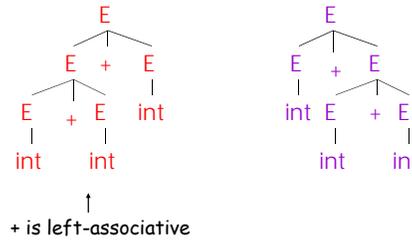
- Strings

int + int + int

int \* int + int

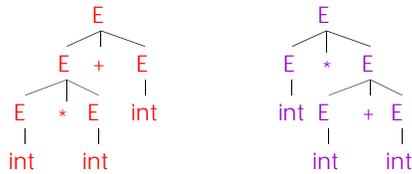
## Ambiguity. Example

The string `int + int + int` has two parse trees



## Ambiguity. Example

The string `int * int + int` has two parse trees



## Ambiguity (Cont.)

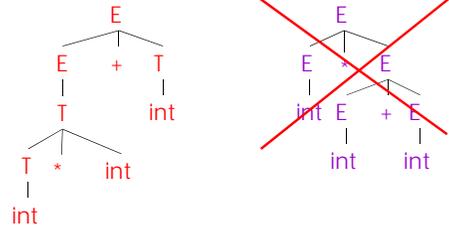
- A grammar is *ambiguous* if it has more than one parse tree for some string
  - Equivalently, there is more than one rightmost or leftmost derivation for some string
- Ambiguity is *bad*
  - Leaves meaning of some programs ill-defined
- Ambiguity is *common* in programming languages
  - Arithmetic expressions
  - IF-THEN-ELSE

## Dealing with Ambiguity

- There are several ways to handle ambiguity
- Most direct method is to rewrite the grammar unambiguously
  - $E \rightarrow E + T \mid T$
  - $T \rightarrow T * \text{int} \mid \text{int} \mid (E)$
- Enforces precedence of  $*$  over  $+$
- Enforces left-associativity of  $+$  and  $*$

## Ambiguity. Example

The  $\text{int} * \text{int} + \text{int}$  has only one parse tree now

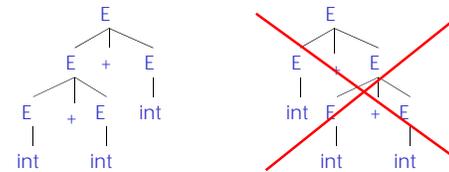


## Ambiguity

- Impossible to convert automatically an ambiguous grammar to an unambiguous one
- Used with care, ambiguity can simplify the grammar
  - Sometimes allows more natural definitions
  - But we need disambiguation mechanisms
- Instead of rewriting the grammar
  - Use the more natural (ambiguous) grammar
  - Along with disambiguating declarations
- Most tools allow *precedence and associativity declarations* to disambiguate grammars
- Examples ...

## Associativity Declarations

- Consider the grammar  $E \rightarrow E + E \mid \text{int}$
- Ambiguous: two parse trees of  $\text{int} + \text{int} + \text{int}$



- Left-associativity declaration: `%left '+'`

## Summary

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- Grammar is specified using a context-free language (CFL)
- Derivation: starting from start symbol, use grammar rules as rewrite rules to derive input string
  - Leftmost and rightmost derivations
- Parse trees and abstract syntax trees
- Ambiguous grammars
  - Ambiguity should be eliminated by modifying grammar, by specifying precedence rules etc. depending on how ambiguity arises in the grammar
- Remaining question: how do we find the derivation for a given input string?