Exercises #1

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These exercises are intended to reinforce the lecture material. Sample solutions will be handed out on January 29. It is recommended that you consider the problems on your own before reading the sample solutions.

1. State a Θ-bound for $T(n)$ in parts (b), (d), (f), (h), and (j) of Problem 4–3, page 108 (second edition: Problem 4–4, page 86; parts (b) and (d) are slightly different).

2. Give an $O(\lg n)$-time comparison-based algorithm to solve the selection problem assuming that the input is provided in the form of 100 sorted arrays of total length n. You may assume that the length of each of the 100 arrays is specified in an auxiliary integer array of length 100.

3. Let us define the *external path length* of a binary tree as the sum of the depths of all of its leaves. (The root is at depth 0, the children of the root are at depth 1, the grandchildren of the root are at depth 2, et cetera.)

   (a) Prove that any ℓ-leaf binary tree has external path length $\Omega(\ell \lg \ell)$.

   (b) Use the result of part (a) to prove that the average-case complexity of any comparison-based sorting algorithm is $\Omega(n \lg n)$.

4. This question is related to Exercise 5.4–4 on page 142 (second edition: page 117). Suppose that k balls are thrown into n bins (independently and uniformly at random).

   (a) Give an exact expression for the expected number of "tri-collisions", i.e., the number of triples of balls that land in the same bin.

   (b) For any given number of bins n, let $f(n)$ denote the minimum k such that the expected number of tri-collisions is at least 1. State a Θ-bound for $f(n)$.

   (c) For any given number of bins n, let $g(n)$ denote the minimum k such that the probability there are no tri-collisions is at most $1/2$. Prove that $g(n) = O(f(n))$.

5. For any positive constant ε, prove that there is a randomized selection algorithm that uses at most $(\frac{3}{2} + o(1))n$ comparisons with probability at least $1 - \exp(-n^{1-\varepsilon})$.

6. Exercise 31.4–4, page 950 (second edition: page 872). Note: The question implicitly assumes that the given polynomial is not identically zero, i.e., that at least one of the coefficients f_i is nonzero.

7. Exercise 11.3–5, page 269 (second edition: page 236).

8. Exercise 11.3–6, page 269 (second edition: page 237; the RHS of the displayed equation should be taken modulo p).

9. Exercise 11.5–1, page 282 (second edition: page 249).

10. Problem 11–4, page 284 (second edition: page 251).